Feynman diagrams

- To each perturbative contribution generated by Wick's theorem, one can associate a graph called a Feynman diagram.
- All contributions can be derived from a subclass that contains only connected terms, represented by connected diagrams.

$$Z(\lambda) = \int d^n x \, e^{-A(\vec{x}) + \lambda V(\vec{x})} =$$

$A(\vec{x}) = \frac{1}{2} \sum_{ij} x_i A_{ij} x_j$

$V(\vec{x})$ - polynomial

$$= \sum_{k=0}^{\infty} \frac{(-\lambda)^k}{k!} \int d^n x \, V(\vec{x})^k e^{-A(\vec{x})} =$$

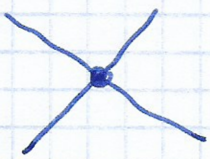
$$= Z(0) \sum_{k=0}^{\infty} \frac{(-\lambda)^k}{k!} \langle V(\vec{x})^k \rangle_0$$

where

$$\langle V(\vec{x})^k \rangle_0 = \frac{1}{Z(0)} \int d^n x \, V(\vec{x})^k e^{-A(\vec{x})}$$

Feynman diagrams

Each monomial contributing to a perturbation $V(\vec{x})$ is represented by a point (a vertex) from which originates a number of lines equal to the degree of the monomial



x^4 vertex in $V(\vec{x}) = \frac{1}{4!} \sum_{i=1}^{n} x_i^4$

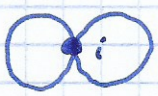
Each pairing is represented by a line joining the vertices to which belong the corresponding variable.

Example $\quad V(\vec{x}) = \frac{1}{4!}\sum_{i=1}^{n} x_i^4$

$$\frac{Z(\lambda)}{Z(0)} = 1 - \frac{1}{4!}\lambda\sum_i \langle x_i^4\rangle_0 + \frac{1}{2!(4!)^2}\lambda^2\sum_{ij}\langle x_i^4 x_j^4\rangle_0 + O(\lambda^3) =$$

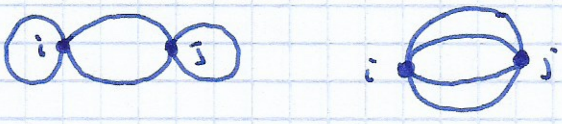
$$= 1 - \frac{1}{8}\lambda\sum_i \Delta_{ii}^2 + \frac{1}{128}\lambda^2\sum_i \Delta_{ii}^2\sum_j \Delta_{jj}^2 +$$

$$+ \lambda^2\sum_{ij}\left(\frac{1}{16}\Delta_{ii}\Delta_{jj}\Delta_{ij}^2 + \frac{1}{48}\Delta_{ij}^4\right) + O(\lambda^3)$$

diagram of order λ:  $\langle x_i^4\rangle_0$

diagrams of order λ^2:

– non-connected contributions to $\langle x_i^4 x_j^4\rangle_0$ (product of sums)



– connected contributions to $\langle x_i^4 x_j^4\rangle_0$

Latter we will not indicate indices of summation