General Gaussian integral

$$Z(\mathbb{A},\bar{b}) = \int d^n x \, e^{-\mathbb{A}(\bar{x}) + \bar{b}\cdot\bar{x}}$$

we look at the minimum of $\mathbb{A}(\bar{x}) - \bar{b}\cdot\bar{x}$

$$\frac{\partial}{\partial x_k}\left(\sum_{ij=1}^{n} \frac{1}{2} x_i A_{ij} x_j - \sum_{i=1}^{n} b_i x_i \right) = \sum_{j=1}^{n} A_{kj} x_j - b_k = 0$$

Introducing the inverse matrix

$$\Delta = \mathbb{A}^{-1}$$

we solve

$$x_i = \sum_{j=1}^{n} \Delta_{ij} b_j$$

Next, we change the variable

$$x_i = \sum_{j=1}^{n} \Delta_{ij} b_j + y_i$$

and get

$$Z(\mathbb{A},\bar{b}) = e^{\Delta(\bar{b})} \int d^n y \, e^{-\mathbb{A}(\bar{y})}$$

with

$$\Delta(\bar{b}) = \frac{1}{2} \sum_{ij=1}^{n} b_i \Delta_{ij} b_j$$

Thus we find

$$\boxed{Z(\mathbb{A},\bar{b}) = (2\pi)^{n/2} \left(\det \mathbb{A}\right)^{-1/2} e^{\Delta(\bar{b})}}$$