§5. NON-EQUILIBRIUM PROPERTIES

To study dynamics of FL we need to extend the definition of δE to <u>non-equilibrium states</u>.

Assume that the properties vary at <u>macroscopic distances</u> $\gg k_F^{-1}$ $\rightarrow$ semiclassical approach valid

Df. $\boxed{\text{Local quasi-particle distribution function } n_{\bar{k}\sigma}(\bar{r},t) \text{ giving the density of quasiparticles with momentum } \bar{k} \text{ and spin } \sigma \text{ in the vicinity of point } \bar{r} \text{ and time } t.}$

<u>Time dependent functional</u>

$$\delta E[\delta n, t] = \sum_{\bar{k}\sigma} \int d_3 r \; \varepsilon_{\bar{k}} \, \delta n_{\bar{k}\sigma}(\bar{r},t) +$$

$$+ \frac{1}{2V} \sum_{\substack{\bar{k}\bar{k}' \\ \sigma\sigma'}} \int d_3 r \int d_3 r' \; f_{\sigma\sigma'}(\bar{k},\bar{k}'; \bar{r}-\bar{r}')\, \delta n_{\bar{k}\sigma}(\bar{r},t)\, \delta n_{\bar{k}'\sigma'}(\bar{r}',t)$$

For homogeneous system:

$\varepsilon_{\bar{k}}$ - spin independent, position $\bar{r}$ independent

$f_{\sigma\sigma'}(\bar{k},\bar{k}', \bar{r}-\bar{r}')$ - instantaneous, short-range

reasonable approximation

$$f_{\sigma\sigma'}(\bar{k},\bar{k}') = \int d_3 r' \; f_{\sigma\sigma'}(\bar{k},\bar{k}', \bar{r}-\bar{r}')$$

notation $\quad x = (\bar{r}, t)$

$$\boxed{\delta E[\delta n, t] = \sum_{\bar{k}\sigma} \int d_3 r \, \varepsilon_{\bar{k}} \, \delta n_{\bar{k}\sigma}(x) + + \frac{1}{2V} \sum_{\substack{\bar{k}\bar{k}' \\ \sigma\sigma'}} \int d_3 r \, f_{\sigma\sigma'}(\bar{k},\bar{k}') \, \delta n_{\bar{k}\sigma}(x) \, \delta n_{\bar{k}'\sigma'}(x)}$$

We need a kinetic (Boltzmann) equation

The time-dependent quasiparticle energy is

$$\tilde{\varepsilon}_{\bar{k}\sigma}(x) = \frac{\delta E[n,t]}{\delta n_{\bar{k}\sigma}(x)} = \varepsilon_{\bar{k}} + \frac{1}{V} \sum_{\bar{k}'\sigma'} f_{\sigma\sigma'}(\bar{k},\bar{k}') \delta n_{\bar{k}'\sigma'}(x)$$

We use this as a <u>quasi-classical Hamiltonian</u>

<u>Equations of motion</u> Hamilton-Jacobi Eq.

$$\hbar = 1 \quad \begin{cases} \dfrac{\partial}{\partial t} \bar{r} = \bar{\nabla}_{\bar{k}} \tilde{\varepsilon}_{\bar{k}\sigma}(x) = \bar{v}_{\bar{k}} \\ \dfrac{\partial \bar{k}}{\partial t} = - \bar{\nabla}_{\bar{r}} \tilde{\varepsilon}_{\bar{k}\sigma}(x) = \bar{F} \end{cases}$$

In semi-classical dynamics in phase space

$$\underset{\text{time } t}{(\bar{r}, \bar{k})} \longrightarrow \underset{\text{time } t+dt}{(\bar{r} + \bar{v}_{\bar{k}} dt, \bar{k} + \bar{F} dt)}$$

$$\delta n_{\bar{k}\sigma}(x) \rightarrow \delta n_{\bar{k}}(\bar{r}, \bar{k}, t)$$

$$\delta n_{\bar{k}}(\bar{r} + \bar{v}_{\bar{k}} dt, \bar{k} + \bar{F} dt, t + dt) \equiv \delta n_{\bar{k}}(\bar{r}, \bar{k}, t) + I \, dt$$

I - collision integral

$$\boxed{\frac{d}{dt} n_{\bar{k}\sigma}(x) = I}$$

Expanding LHS

$$\delta n_\sigma(\bar r,\bar k,t) + \frac{\partial n_\sigma(\bar r,\bar k,t)}{\partial \bar r}\cdot\frac{\partial \bar r}{\partial t}\,dt +$$

$$\frac{\partial n_\sigma(\bar r,\bar k,t)}{\partial \bar k}\,\frac{\partial \bar k}{\partial t}\,dt + \frac{\partial n_\sigma(\bar r,\bar k,t)}{\partial t}$$

$$= \delta n_\sigma(\bar r,\bar k,t) + \bar\nabla_{\bar r} n_{\bar k}(x)\,\bar\nabla_{\bar k}\tilde\varepsilon_{\bar k\sigma}(x)\,dt$$

$$+ \frac{\partial n_\sigma(x)}{\partial t} - \bar\nabla_{\bar k}\, n_{\bar k\sigma}(x)\,\bar\nabla_{\bar r}\tilde\varepsilon_{\bar k\sigma}(x)\,dt = \delta n + I\,dt$$

To first order in $\delta n_{\bar k\sigma}(x) = n_{\bar k\sigma}(x) - n^0_{\bar k}$

$$\frac{\partial\,\delta n_{\bar k\sigma}(x)}{\partial t} - \bar\nabla_{\bar k}\,\delta n_{\bar k\sigma}(x)\cdot\bar\nabla_{\bar r}\tilde\varepsilon_{\bar k\sigma}(x) +$$

$$\bar\nabla_{\bar r}\,\delta n_{\bar k\sigma}(x)\cdot\underbrace{\bar\nabla_{\bar k}\,\varepsilon_{\bar k}}_{\bar v^*_{\bar k}} = I$$

$\Rightarrow$

$$\boxed{\begin{aligned}&\frac{\partial}{\partial t}\delta n_{\bar k\sigma}(x) + \bar v_k^*\cdot\bar\nabla_{\bar r}\,\delta n_{\bar k\sigma}(x) + \\ &+\frac{1}{V}\sum_{\bar k'\sigma'} f_{\sigma\sigma'}(\bar k,\bar k')\,\bar\nabla_{\bar r}\,\delta n_{\bar k'\sigma'}(x)\cdot\bar v_k^*\,\delta(\xi_k) = I\end{aligned}}$$

kinetic Boltzmann equation for quasiparticles near the FS.

$*)$ $\bar\nabla_{\bar k}\,\delta n_{\bar k\sigma}(x)\cdot\bar\nabla_{\bar r}\tilde\varepsilon_{\bar k\sigma}(x) =$

$$= \underbrace{\bar\nabla_{\bar k}\,\delta n_{\bar k\sigma}(x)}_{-\delta(\xi_k)\,\underbrace{\bar\nabla_{\bar k}\varepsilon_{\bar k}}_{\bar v_k^*}}\cdot\bar\nabla_{\bar r}\frac{1}{V}\sum_{\bar k'\sigma'} f_{\sigma\sigma'}(\bar k,\bar k')\,\delta n_{\bar k'\sigma'}(x)$$

•) Conservation laws

Particle number conservation

total number of particles

$$\sum_{\vec{k}\sigma} \frac{d n_{\vec{k}\sigma}(x)}{dt} = \sum_{\vec{k}\sigma} I[n_{\vec{k}\sigma}(x)] = 0$$

This can be written as continuity equation

$$\boxed{\frac{\partial}{\partial t} n(x) + \vec{\nabla} \cdot \vec{j}(x) = 0}$$

where $n(x) = \frac{1}{V} \sum_{\vec{k}\sigma} n_{\vec{k}\sigma}(x)$ particle density

$$\vec{j}(x) = \frac{1}{V} \sum_{\vec{k}\sigma} n_{\vec{k}\sigma}(x) \vec{\nabla}_{\vec{k}} \tilde{\varepsilon}_{\vec{k}\sigma}(x)$$

current density

Indeed, note that

$$\vec{\nabla}_{\vec{r}} n_{\vec{k}\sigma}(x) \cdot \vec{\nabla}_{\vec{k}} \tilde{\varepsilon}_{\vec{k}\sigma}(x) - \vec{\nabla}_{\vec{k}} n_{\vec{k}\sigma}(x) \cdot \vec{\nabla}_{\vec{r}} \tilde{\varepsilon}_{\vec{k}\sigma}(x) =$$

$$= \vec{\nabla}_{\vec{r}} \left(n_{\vec{k}\sigma}(x) \vec{\nabla}_{\vec{k}} \tilde{\varepsilon}_{\vec{k}\sigma}(x) \right) - \vec{\nabla}_{\vec{k}} \left(n_{\vec{k}\sigma}(x) \vec{\nabla}_{\vec{r}} \tilde{\varepsilon}_{\vec{k}\sigma}(x) \right)$$

Taking a sum $\sum_{\vec{k}\sigma}$ with $\sum_{\vec{k}\sigma} \vec{\nabla}_{\vec{k}} \left(n_{\vec{k}\sigma}(x) \vec{\nabla}_{\vec{r}} \tilde{\varepsilon}_{\vec{k}\sigma}(x) \right) = 0$

$$\sum_{\vec{k}\sigma} I[n_{\vec{k}\sigma}(x)] = 0$$

we get

$$\frac{\partial}{\partial t} \underbrace{\sum_{\vec{k}\sigma} n_{\vec{k}\sigma}(x)}_{n(x)} + \vec{\nabla}_{\vec{r}} \underbrace{\sum_{\vec{k}\sigma} \overbrace{n_{\vec{k}\sigma}(x) \vec{\nabla}_{\vec{k}} \tilde{\varepsilon}_{\vec{k}\sigma}(x)}^{\vec{j}_{\vec{k}\sigma}(x)}}_{\vec{j}(x)} = 0 \qquad \square$$

To linear order in δn

$$\boxed{\begin{aligned}\vec{j}(x) &= \frac{1}{V}\sum_{\vec{k}\sigma} \delta n_{\vec{k}\sigma}(x)\Big[\vec{v}_{k}^{\,*} + \frac{1}{V}\sum_{\vec{k}'\sigma'} f_{\sigma\sigma'}(\vec{k},\vec{k}')\,\vec{v}_{\vec{k}'}^{\,*}\,\delta(\xi_{\vec{k}'})\Big] \\ &= \frac{1}{V}\sum_{\vec{k}\sigma} \delta n_{\vec{k}\sigma}(x)\,\vec{j}_{\vec{k}}\end{aligned}}$$

the same expression as in discussing m^*.

<u>momentum conservation</u>

we multiply by $\vec{k}$ and take $\sum_{\vec{k}\sigma}$

$$\vec{g}(x) := \frac{1}{V}\sum_{\vec{k}\sigma} \vec{k}\; n_{\vec{k}\sigma}(x)$$

$$\frac{\partial \vec{g}(x)}{\partial t} + \frac{1}{V}\sum_{\vec{k}\sigma} \vec{k}\cdot\Big[\vec{\nabla}_{\vec{r}}\big(n_{\vec{k}\sigma}(x)\,\vec{\nabla}_{\vec{k}}\,\tilde{\varepsilon}_{\vec{k}\sigma}(x)\big) - \vec{\nabla}_{\vec{k}}\big(n_{\vec{k}\sigma}(x)\,\vec{\nabla}_{\vec{r}}\,\tilde{\varepsilon}_{\vec{k}\sigma}(x)\big)\Big] = 0$$

we rewrite

$$\sum_{\vec{k}\sigma} \vec{k}\cdot\vec{\nabla}_{\vec{k}}\big(n_{\vec{k}\sigma}(x)\,\vec{\nabla}_{\vec{r}}\,\tilde{\varepsilon}_{\vec{k}\sigma}(x)\big) \overset{\text{by parts}}{=}$$

boundary term vanishes

$$= -\sum_{\vec{k}\sigma} n_{\vec{k}\sigma}(x)\,\vec{\nabla}_{\vec{r}}\,\tilde{\varepsilon}_{\vec{k}\sigma}(x) =$$

$$= -\vec{\nabla}_{\vec{r}}\sum_{\vec{k}\sigma} n_{\vec{k}\sigma}(x)\,\tilde{\varepsilon}_{\vec{k}\sigma}(x) + \sum_{\vec{k}\sigma}\vec{\nabla}_{\vec{r}}\,n_{\vec{k}\sigma}(x)\,\underbrace{\tilde{\varepsilon}_{\vec{k}\sigma}(x)}_{\frac{\partial E}{\partial n(x)}} =$$

$$= -\vec{\nabla}_{\vec{r}}\Big[\sum_{\vec{k}\sigma} n_{\vec{k}\sigma}(x)\,\tilde{\varepsilon}_{\vec{k}\sigma}(x) - E\Big]$$

Using index notation we get

$$\boxed{\frac{\partial}{\partial t} \sum_{\bar{k}\sigma} \left(k_i \, n_{\bar{k}\sigma}(x) \right) + \frac{\partial}{\partial r_j} \left(\Pi_{ij}(x) \right) = 0}$$

where

DP

$$\boxed{\begin{aligned} \Pi_{ij} \equiv \sum_{\bar{k}\sigma} \; k_i \, n_{\bar{k}\sigma}(x) \frac{\partial \tilde{\varepsilon}_{\bar{k}\sigma}(x)}{\partial k_j} + \\ + \delta_{ij} \sum_{\bar{k}\sigma} \left(n_{\bar{k}\sigma}(x) \, \tilde{\varepsilon}_{\bar{k}\sigma}(x) - E \right) \end{aligned}}$$

<u>momentum flux tensor</u>

<u>Energy conservation</u>

multiply by $\tilde{\varepsilon}_{\bar{k}\sigma}$ and sum over $(\bar{k}\sigma)$

$$\frac{\partial}{\partial t} E + \frac{1}{V} \sum_{\bar{k}\sigma} \tilde{\varepsilon}_{\bar{k}\sigma}(x) \Big[\bar{\nabla}_{\bar{r}} \left(n_{\bar{k}\sigma}(x) \, \bar{\nabla}_{\bar{k}} \tilde{\varepsilon}_{\bar{k}\sigma}(x) \right) - \bar{\nabla}_{\bar{k}} \left(n_{\bar{k}\sigma}(x) \, \bar{\nabla}_{\bar{r}} \tilde{\varepsilon}_{\bar{k}\sigma}(x) \right) \Big] = 0$$

where

$$\boxed{\begin{aligned} \frac{\partial}{\partial t} E &= \frac{1}{V} \sum_{\bar{k}\sigma} \frac{dE}{d n_{\bar{k}\sigma}(x)} \frac{\partial n_{\bar{k}\sigma}(x)}{\partial t} = \\ &= \frac{1}{V} \sum_{\bar{k}\sigma} \hat{\varepsilon}_{\bar{k}\sigma}(x) \frac{\partial n_{\bar{k}\sigma}(x)}{\partial t} \end{aligned}}$$

is a time derivative of the energy

Integrating by parts the last term we get

$$\boxed{\frac{\partial}{\partial t} E + \vec{\nabla}_{\vec{r}} \cdot \vec{j}_E(x) = 0}$$

with

$$\boxed{\vec{j}_E(x) = \frac{1}{V} \sum_{\vec{k}\sigma} \tilde{\varepsilon}_{\vec{k}\sigma}(x)\, n_{\vec{k}\sigma}(x)\, \vec{\nabla}_{\vec{k}} \tilde{\varepsilon}_{\vec{k}\sigma}(x)}$$

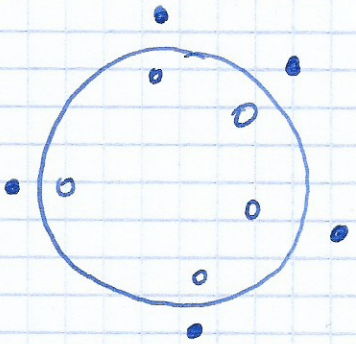
is the <u>energy current</u>

To linear order in δn

$$\boxed{\vec{j}_E(x) = \frac{1}{V} \sum_{\vec{k}\sigma} \varepsilon_{\vec{k}} \Big[\delta n_{\vec{k}\sigma}(x) + \delta(\xi_{\vec{k}}) \cdot \frac{1}{V} \sum_{\vec{k}'\sigma'} f_{\sigma\sigma'}(\vec{k},\vec{k}')\, \delta n_{\vec{k}'\sigma'}(x) \Big] \vec{v}^{*}_{\vec{k}}}$$

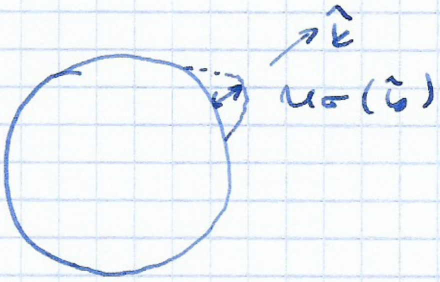
§6. COLLECTIVE MODES

A collective mode with momentum $\vec{q}$ and frequency ω is a coherent superposition of quasi-particle – quasi-hole excitations.



When $\vec{q} \to 0$ the quasi-particles (holes) excitations energy vanishes and the collective modes can be seen as a time-dependent displacement of the Fermi surface

$$u_\sigma(\hat{k})\, e^{i(\bar{q}\cdot\bar{r} - \omega t)} + \text{h.c.}$$



We consider

$$\delta n_{\bar{k}\sigma}(x) = v_F^{*}\, \delta(\xi_{\bar{k}})\, u_\sigma(\hat{k})\, e^{i(\bar{q}\cdot\bar{r} - \omega t)} + \text{h.c.}$$

$u_\sigma(\hat{k}) \in \mathbb{C}$

Expanding $u_\sigma(\hat{k}) = u^s(\hat{k}) + \sigma\, u^a(\hat{k}) =$

$$= \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \left(u^s_{lm} + \sigma\, u^a_{lm} \right) Y_l^m(\theta, \varphi)$$

and using the kinetic equation

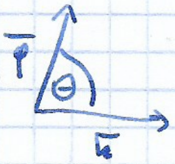
$$\frac{\partial}{\partial t}\delta n_{\bar k\sigma}(x) + \bar v_k^{*}\cdot\bar\nabla_{\bar r}\,\delta n_{\bar k\sigma}(x) +$$

$$+\frac{1}{V}\sum_{\bar k'\sigma'} f_{\sigma\sigma'}(\bar k,\bar k')\,\bar\nabla_{\bar r}\,\delta n_{\bar k'\sigma'}(x)\cdot\bar v_{\bar k}^{*}\,\delta(\xi_{\bar k}) = I[n]$$

we get

$$(\cos\theta - s)\,u^{\nu}(\hat k) + \cos\theta\int\frac{d\Omega_{\bar k'}}{4\pi}F^{\nu}(\bar k,\bar k')\,u^{\nu}(\bar k') = I[u]$$

$(\nu = s, a)$, $s = \dfrac{\omega}{v_F^{*}|\bar q|}$,



and

$$(\cos\theta - s)\sum_{l=0}^{\infty}\sum_{m=-l}^{l} u_{lm}^{\nu}Y_l^m(\hat k) +$$

$$+\cos\theta\sum_{l=0}^{\infty}\frac{F_l^{\nu}}{2l+1}\sum_{m=-l}^{l}u_{lm}^{\nu}Y_l^m(\hat k) = I[u]$$

$F_l^{\nu} = 2N^{*}(0)\,f_l^{\nu}$

Ignoring the collision term $I[u]$ we multiply by $\int d\Omega_{\bar k}\,Y_l^m(\hat k)^{*}$ we get a set of equations on u_{lm}^{ν}.

•) <u>density fluctuations</u> $\quad m=0$

$$\delta n(x) = 2N^*(0)\, v_F^* \int \frac{d\Omega_{\hat{k}}}{4\pi}\, u^s(\hat{k})\, e^{i(\bar{q}\cdot\bar{r}-\omega t)} + h.c. =$$

$$= N^*(0)\, v_F^* \frac{u_{00}^s}{\sqrt{\pi}}\, e^{i(\bar{q}\cdot\bar{r}-\omega t)} + h.c. =$$

$$= \frac{k_F^2}{2\pi^{5/2}}\, u_{00}^s\, e^{i(\bar{q}\cdot\bar{r}-\omega t)} + h.c.$$

the current is longitudinal and

$$\bar{j}(x) = \hat{q}\, N^*(0)\, v_F^{*2} \frac{u_{10}^s}{\sqrt{3\pi}} \left(1+\frac{F_1^s}{3}\right) e^{i(\bar{q}\cdot\bar{r}-\omega t)} + h.c.$$

$$\boxed{\frac{m^*}{m} = 1+\frac{F_1^s}{3}} \quad = \hat{q}\, \frac{k_F^3}{2\sqrt{3}\,\pi^{5/2} m}\, u_{10}^s\, e^{i(\bar{q}\cdot\bar{r}-\omega t)} + h.c.$$

•) <u>zero sound</u> $\quad \frac{1}{T^2} \sim \tau$ - quasiparticle collision time

$m=0$ mode for $\boxed{\omega\tau \gg 1}$ where $I[u] \sim -\frac{u}{\tau}$ and $I(u)$ can be neglected with respect to $\frac{\partial}{\partial t}\delta n_{\bar{k}\sigma}(x)$.

This mode is a collective excitation of quasiparticles when the restoring force is due to the quasiparticle interaction $f_{\sigma\sigma'}(\bar{k},\bar{k}')$ - called <u>zero sound</u>

Seen only if $\quad \omega \gg \frac{1}{\tau} \sim T^2$

<u>Ordinary (first) sound</u> has the restoring force due to frequent adiabatic collisions between of quasiparticles $I[\delta n]$ restoring a local equilibrium.

Seen if $\quad \omega \ll \frac{1}{\tau} \sim T^2$

Assume that $f_{\sigma\sigma'}(\hat{k},\hat{k}') = f_0^s$

$u^a = 0$ – in phase for ↑ and ↓

We get
$$\boxed{0 = (\cos\theta - s)\, u^s(\hat{k}) + F_0^s \cos\theta \int \frac{d\Omega_{k'}}{4\pi}\, u^s(\hat{k}')}$$

Eigen value equation with a solution

$$\boxed{u^s(\hat{k}) = \text{const.}\,\frac{\cos\theta}{s+i\eta-\cos\theta}} \qquad \eta \to 0^+$$

with
$$\boxed{\frac{1}{F_0^s} = \int \frac{d\Omega}{4\pi}\, \frac{\cos\theta}{s+i\eta-\cos\theta} = -1 + \frac{s}{2}\ln\left(\frac{s+i\eta+1}{s+i\eta-1}\right)}$$

Solution on $s = \dfrac{\omega}{v_F^* |q|}$

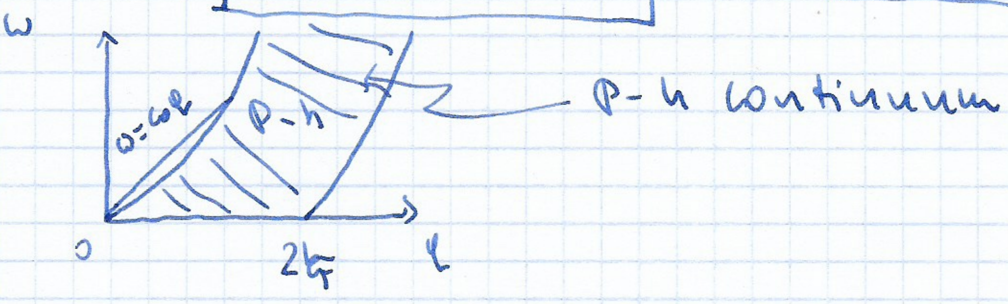
$\boxed{F_0^s > 0} \to s \in \mathbb{R}$ and $s > 1$

$F_0^s \to 0 \Rightarrow s \to 1 + 2e^{-\frac{2}{F_0^s}}$

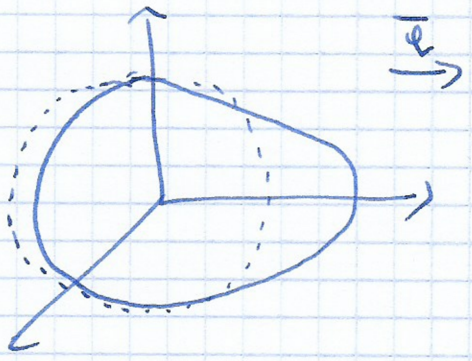
$F_0^s \to \infty \Rightarrow s \to \sqrt{\dfrac{F_0^s}{3}}$

$\mathbb{R} \ni s$ – undamped mode corresponding to zero-sound propagating with velocity

$$\boxed{c_0 = \frac{\omega}{|q|} = s\, v_F^*} \qquad \boxed{c_0 > v_F^*}$$



Plotting $\dfrac{\cos\theta}{s-\cos\theta}$



zero sound

Fermi surface

oscillations

(not spherical)

$\boxed{-1 < F_0^s < 0}$ we find numerically that $s \in \mathbb{C}$

such that $|\mathrm{Re}\, s| < 1$

$\mathrm{Im}\, s < 0$

damped zero sound modes.

The damping is due to the interaction of the sound with p-h continuum (Landau damping)

$\boxed{F_0^s < -1}$ two pure imaginary solutions

Let $s = i\alpha \rightarrow \dfrac{1}{F_0^s} = -1 + \dfrac{i}{2}\alpha \ln\left(\dfrac{1+i\alpha}{i\alpha-1}\right) = -1 - \alpha\left(\gamma - \dfrac{\pi}{2}\right)$

where $\gamma \in]-\pi, \pi]$ and $1+i\alpha = \sqrt{1+\alpha^2}\, e^{i\gamma}$

$-1+i\alpha = \sqrt{1+\alpha^2}\, e^{i(\pi-\gamma)}$

since $\tan\gamma = \alpha$

$$\frac{1}{F_0^s} = -1 - \alpha\left(\arctan\alpha - \frac{\pi}{2}\right) = -1 + \alpha \arctan\left(\frac{1}{\alpha}\right)$$

There are two solutions with opposite signs.

instability $\rightarrow$ divergent density fluctuations $\rightarrow$

$\rightarrow$ negative compressibility.

•) First (ordinary) Sound

$\omega\tau \ll 1$ - hydrodynamic regime,

the displacement $u_\sigma(\hat{k})$ is extremely small and governed by $I[n]$.

Only u^s_{00} and u^s_{10} are constrained by the conservation of particle number and are not affected by collision term $I[n]$

With $Y^0_0(\hat{k}) = \frac{1}{2\sqrt{\pi}}$, $Y^0_1(\hat{k}) = \cos\theta \sqrt{3}\, Y^0_0(\hat{k})$

the kinetic equation

$$(\cos\theta - s)\left(u^s_{00} + u^s_{10}\sqrt{3}\cos\theta\right) + \cos\theta\left(F^s_0 u^s_{00} + \frac{F^s_1}{\sqrt{3}} u^s_{10}\cos\theta\right) = 0$$

$/ \int d\Omega_{\hat{k}}\, Y^0_0(\hat{k})^*$ $\qquad$ $/ \int d\Omega_{\hat{k}}\, Y^0_1(\hat{k})^*$

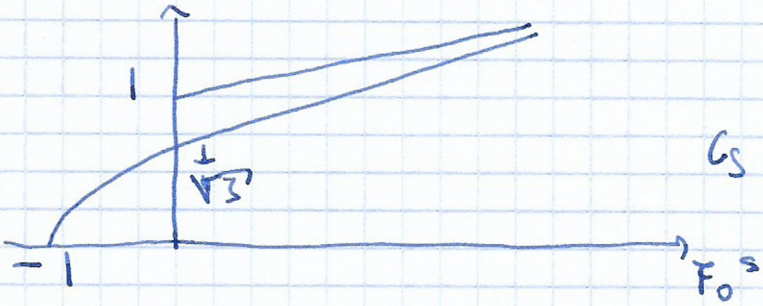
$$\begin{cases} s\, u^s_{00} - \frac{u^s_{10}}{\sqrt{3}}\left(1 + \frac{F^s_1}{3}\right) = 0 & \Leftrightarrow \quad \partial_t n + \vec{\nabla}\cdot\vec{j} = 0 \quad \text{for } m=0 \\ u^s_{00}(1 + F^s_0) - s\sqrt{3}\, u^s_{10} = 0 & \Leftrightarrow \quad m\,\partial_t \vec{j} + \vec{\nabla}\cdot\Pi = 0 \quad m=0 \end{cases}$$

$\Pi_{ij} = \delta_{ij}\Pi(k)$ - diagonal

Solution $\qquad \boxed{s^2 = \frac{\omega^2}{(v^*_F |k|)^2} = \frac{1}{3}(1 + F^s_0)\left(1 + \frac{F^s_1}{3}\right)}$

→ Sound velocity

$$c_s = \frac{1}{\sqrt{\partial e n\, m}} = \frac{v^*_F}{\sqrt{3}}\left[(1 + F^s_0)\left(1 + \frac{F^s_1}{3}\right)\right]^{1/2} \xrightarrow[\text{ideal gas}]{} \frac{v_F}{\sqrt{3}}$$



$$f_{\sigma\sigma'}(\vec{p},\vec{p}') = f_0^s$$

$$c_s \simeq c_0 \simeq v_F^* \left(\frac{F_0^s}{3}\right)^{1/2}$$

$$F_0^s \to \infty$$

o) <u>other solutions</u>

many different solutions:

if $\delta n_\uparrow$ and $\delta n_\downarrow$ oscillate in phase

$\boxed{\text{sound}}$

if $\delta n_\uparrow$ and $\delta n_\downarrow$ oscillate out of phase

$\boxed{\text{spin waves}}$