Ideal fermions in a Zeeman magnetic field

$$H_{zeeman} = -g\,\vec{\mu}\cdot\vec{B}$$ uniform external magnetic field

spin magnetic moment of electrons

$\vec{B} = (0,0,B)$

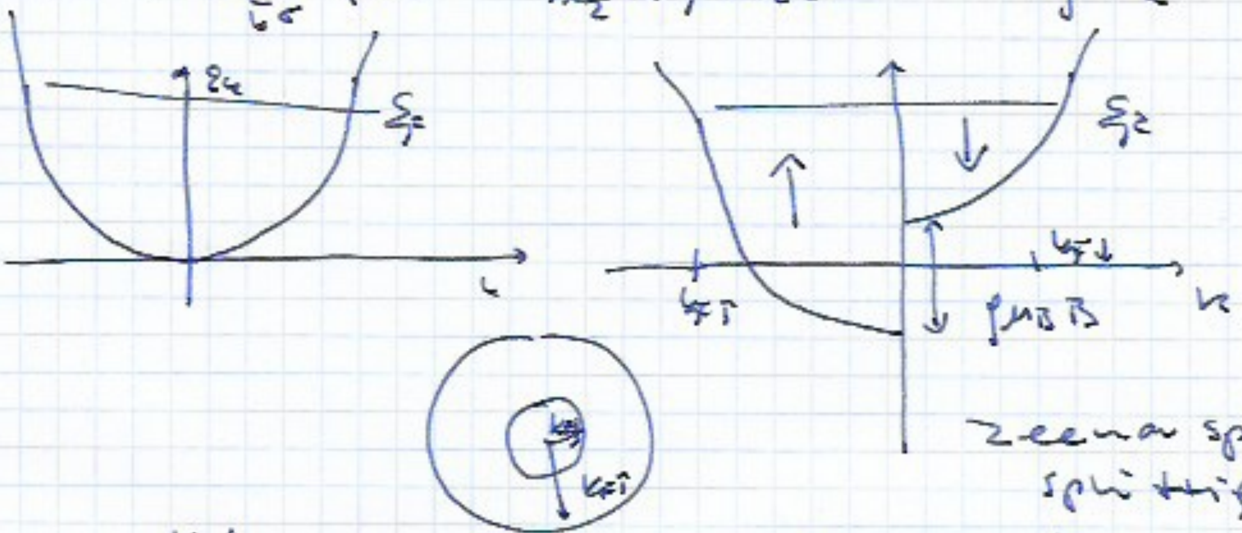
$H_{zeeman} = -g\,\mu_z B$ $\qquad \mu_z = \pm \frac{\mu_B}{2}$ spin magnetic moment for up/down spins

$$\hat{H} = \sum_{k\sigma}\left(\frac{\hbar^2 k^2}{2m} - \sigma g \frac{\mu_B}{2} B\right)\hat{a}^{\dagger}_{k\sigma}\hat{a}_{k\sigma}$$

$\mu_B = \frac{e\hbar}{2mc}$ Bohr magneton

$$\hat{H} = \sum_{k\sigma}\left(\varepsilon_k - \sigma g \frac{\mu_B}{2} B\right)\hat{n}_{k\sigma}$$

$g = 2$



Zeeman spin splitting

$$N_\sigma = \frac{V}{(2\pi)^3}\int d^3k\;\theta(k_{F\sigma} - |k|) = V\frac{1}{(2\pi)^3}\cdot 4\pi\int_0^{k_{F\sigma}} dk\, k^2 =$$

$$= V\frac{1}{2\pi^2}\frac{1}{3}k_{F\sigma}^3$$

$$\boxed{n_\sigma = \frac{N_\sigma}{V} = \frac{1}{6\pi^2}k_{F\sigma}^3}$$

density $\qquad n = n_\uparrow + n_\downarrow = \frac{1}{6\pi^2}\left(k_{F\uparrow}^3 + k_{F\downarrow}^3\right)$

magnetization $\qquad m = \mu_B\frac{n_\uparrow - n_\downarrow}{2} = \frac{\mu_B}{2}\left(\frac{1}{6\pi^2}\right)\left(k_{F\uparrow}^3 - k_{F\downarrow}^3\right)$

Fermi energy

$$\varepsilon_F = \frac{\hbar^2 k_{F\uparrow}^2}{2m} - \frac{g\mu_B B}{2} = \frac{\hbar^2 k_{F\downarrow}^2}{2m} + \frac{g\mu_B B}{2}$$

$$k_{F\uparrow}^2 = \frac{2m}{\hbar^2}\left(\varepsilon_F + \frac{g\mu_B B}{2}\right)$$

$$k_{F\downarrow}^2 = \frac{2m}{\hbar^2}\left(\varepsilon_F - \frac{g\mu_B B}{2}\right)$$

$$m = \frac{\mu_B}{2}\left(\frac{1}{6\pi^2}\right)\left(\frac{2m}{\hbar^2}\right)^{3/2}\left[\left(\varepsilon_F + \frac{g\mu_B B}{2}\right)^{3/2} - \left(\varepsilon_F - \frac{g\mu_B B}{2}\right)^{3/2}\right]$$

at small B we can expand

$$m = \frac{\mu_B}{2}\left(\frac{1}{6\pi^2}\right)\left(\frac{2m}{\hbar^2}\right)^{3/2}\varepsilon_F^{3/2}\left[\left(1 + \frac{g\mu_B B}{2\varepsilon_F}\right)^{3/2} - \left(1 - \frac{g\mu_B B}{2\varepsilon_F}\right)^{3/2}\right] \simeq$$

$$\simeq \frac{\mu_B}{2}\left(\frac{1}{6\pi^2}\right)\left(\frac{2m}{\hbar^2}\right)^{3/2}\varepsilon_F^{3/2}\left[\cancel{1} + \frac{3}{2}\frac{g\mu_B B}{2\varepsilon_F} - \cancel{1} + \frac{3}{2}\frac{g\mu_B B}{2\varepsilon_F} + \dots\right]$$

$$= \frac{\mu_B}{2}\left(\frac{1}{6\pi^2}\right)\left(\frac{2m}{\hbar^2}\right)^{3/2}\varepsilon_F^{3/2}\cdot 3\,\frac{g\mu_B B}{2\varepsilon_F}$$

Pauli susceptibility $\chi = \frac{m}{B}\Big|_{B\to 0}$

$$\chi = 3\frac{\mu_B}{2}\left(\frac{1}{6\pi^2}\right)\left(\frac{2m}{\hbar^2}\right)^{3/2}\varepsilon_F^{3/2}\,\frac{g\mu_B}{2\varepsilon_F}$$

$$\boxed{\chi = 3\,\frac{g}{4}\,\mu_B^2\left(\frac{2m}{\hbar^2}\right)^{3/2}\varepsilon_F^{1/2}} \qquad \mu_B = \frac{e\hbar}{2mc}$$

$$\varepsilon_F = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3}$$

$$\chi \sim \mu_B^2\left(\frac{m}{\hbar^2}\right)^{3/2}\cdot\varepsilon_F^{1/2} = \frac{e^2\hbar^2}{2m^2c^2}\left(\frac{m}{\hbar^2}\right)^{3/2}\cdot\sqrt{\frac{\hbar^2}{2m}}\,(3\pi^2 n)^{1/3} \simeq$$

$$\simeq \frac{e^2\hbar^2}{m^2c^2}\,\frac{m^{3/2}}{\hbar^3}\,\frac{\hbar}{m}\,n^{1/3} \simeq O(\hbar^0)$$

why no $\hbar$ present?

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Van Leeuwen theorem - in classical physics $m \underset{\hbar \to 0}{\longrightarrow} 0$ and $\chi \underset{\hbar \to 0}{\longrightarrow} 0$

What is wrong?

Limits $\cdot)\ \underset{\downarrow \atop 0}{k_B T} \ll \varepsilon_F = \dfrac{\hbar^2 n^{2/3}}{m} \to 0$

ok

$\cdot\cdot)\ \mu_B B \ll \varepsilon_F$

$$\frac{e\hbar}{mc} B \ll \frac{\hbar^2}{2m} n^{2/3}$$

or $B \ll \dfrac{\hbar}{e} c n^{2/3} \underset{\hbar \to 0}{\longrightarrow} 0$

In a constant magnetic field one can not take $\hbar \to 0$ limit !