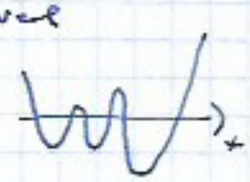
Perturbed Gaussian measure

consider more general distribution $\boxed{\dfrac{e^{-A(\bar{x},\lambda)}}{Z(\lambda)}}$

where $\boxed{A(\bar{x},\lambda) = A(\bar{x}) + \lambda V(\bar{x})}$

polynomial in x_i , $\lambda > 0$
such that integral converges



normalization

$$\boxed{Z(\lambda) = \int d^n x \, e^{-A(\bar{x},\lambda)}}$$

The integral is computed by expanding in λ

$$Z(\lambda) = \sum_{k=0}^{\infty} \frac{(-\lambda)^k}{k!} \int d^n x \, V^k(\bar{x}) \, e^{-A(\bar{x})} =$$

$$\boxed{= Z(0) \sum_{k=0}^{\infty} \frac{(-\lambda)^k}{k!} \langle V^k(\bar{x}) \rangle_0}$$

the average with respect to
$e^{-A(\bar{x})}$
Gaussian integral

This can be computed with Wick's theorem.

using $\quad \langle F(\bar{x}) \rangle = F\left(\frac{\partial}{\partial \bar{b}}\right) e^{\Delta(\bar{b})} \Big|_{\bar{b}=0}$

with $\quad F = e^{-\lambda V}$

we obtain a formal result

$$\boxed{\frac{Z(\lambda)}{Z(0)} = \left[e^{-\lambda V\left(\frac{\partial}{\partial \bar{b}}\right)} \right] e^{\Delta(\bar{b})} \Big|_{\bar{b}=0}}$$

Example $\quad V(\bar{x}) = \frac{1}{4!}\sum_{i=1}^{n} x_i^4$

up to $O(\lambda^2)$ order we get $\quad (A\Delta = \mathbb{1})$

$$\boxed{\frac{Z(\lambda)}{Z(0)}} = 1 - \frac{1}{4!}\lambda \sum_i \langle x_i^4 \rangle_0 + \frac{1}{2!\,4!}\lambda^2 \sum_{ij} \langle x_i^4 x_j^4 \rangle_0 + O(\lambda^3)$$

$$\boxed{\begin{aligned} &= 1 - \frac{1}{8}\lambda \sum_i \Delta_{ii}^2 + \frac{1}{128}\lambda^2 \sum_i \Delta_{ii}^2 \sum_j \Delta_{jj}^2 + \\ &+ \lambda^2 \sum_{ij}\left(\frac{1}{16}\Delta_{ii}\Delta_{jj}\Delta_{ij}^2 + \frac{1}{48}\Delta_{ij}^4\right) + O(\lambda^3) \end{aligned}}$$

In case of one variable $(n=1)$

$$\frac{Z(\lambda)}{Z(0)} = 1 - \frac{1}{8}\lambda + \frac{35}{384}\lambda^2 + O(\lambda^3)$$