Non-interacting fermions

$$\boxed{\hat{H} = \sum_k \varepsilon_k \hat{c}_k^+ \hat{c}_k} \qquad c_k(t) = e^{\frac{i\hat{H}t}{\hbar}} c_k e^{-\frac{i\hat{H}t}{\hbar}}$$

$$i\hbar \frac{d}{dt} c_k(t) = i\hbar\left(\frac{i\hat{H}}{\hbar} c_k(t) - \frac{i}{\hbar}\hat{c}_k(t)\hat{H}\right) = [\hat{c}_k(t), \hat{H}] =$$

$$= [\hat{c}_k(t), \hat{H}(t)] = \sum_{k'} \varepsilon_{k'} [\hat{c}_k, \hat{c}_{k'}^+ c_{k'}] =$$

$$= \sum_{k'} \varepsilon_{k'} \Big(\underbrace{\hat{c}_k \hat{c}_{k'}^+}_{\delta_{kk'} - c_{k'}^+ c_k} \hat{c}_{k'} - \hat{c}_{k'}^+ \hat{c}_{k'} c_k\Big) = \sum_{k'} \varepsilon_{k'} \Big(\delta_{kk'} c_{k'} - c_{k'}^+ \overset{(-)}{c_k c_{k'}} - c_{k'}^+ c_{k'} c_k\Big) =$$

EOM $$= \varepsilon_k \hat{c}_k$$

$$\frac{d\theta}{dt} = \delta(t)$$

$$G^R_{kk'}(t) = \langle\langle c_k(t) | c_{k'}^+(0)\rangle\rangle = -i\theta(t)\langle\{c_k(t), c_{k'}^+(0)\}\rangle$$

$$i\hbar \frac{d}{dt}\langle\langle c_k(t)|c_{k'}^+(0)\rangle\rangle = \hbar\delta(t)\langle\underbrace{\{c_k, c_{k'}^+\}}_{=\delta_{kk'}}\rangle - i\theta(t)(\hbar i)\langle\{\frac{dc_k(t)}{dt} | c_{k'}^+(0)\rangle =$$

$$= \hbar\delta(t)\delta_{kk'} + \varepsilon_k \langle\langle c_k(t)|c_{k'}^+(0)\rangle\rangle$$

$$i\hbar \frac{d}{dt} G^R_{kk'}(t) = \hbar\delta(t)\delta_{kk'} + \varepsilon_k G^R_{kk'}(t)$$

(*) $$G^R_{kk'}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\, e^{-i\omega t} G^R_{kk'}(\omega), \qquad \delta(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\, e^{-i\omega t}$$

$$\theta(t) = \int_{-\infty}^{t} \delta(t')dt'$$

$$i\hbar \frac{d}{dt} G^R_{kk'}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\, (i\hbar)(-i\omega) e^{-i\omega t} G^R_{kk'}(\omega) =$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\, e^{-i\omega t} \hbar\omega G^R_{kk'}(\omega)$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\, e^{-i\omega t} \hbar\omega G^R_{kk'}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\, e^{-i\omega t} \hbar\delta_{kk'} + \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\, e^{-i\omega t} \varepsilon_k G^R_{kk'}(\omega)$$

$$\Rightarrow \quad \hbar\omega G^R_{kk'}(\omega) = \hbar\delta_{kk'} + \varepsilon_k G^R_{kk'} G^R_{kk'}(\omega)$$

$$\boxed{G^R_{kk'}(\omega) = \frac{\hbar\delta_{kk'}}{\hbar\omega - \varepsilon_k}}$$

$\delta_{kk'}$ – momentum conservation

(*) where $\omega = \omega + i0^+ = \omega + i\eta$

$$[G] = \frac{[J\cdot s]}{[J\cdot s\cdot \frac{1}{s} - J]} = [s]$$

From definition

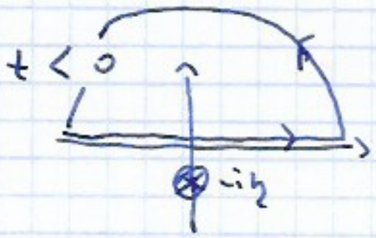
$$i\hbar \frac{d}{dt} c_k(t) = \varepsilon_k c_k(t) \rightarrow c_k(t) = c_k e^{-i\frac{\varepsilon_k t}{\hbar}}$$

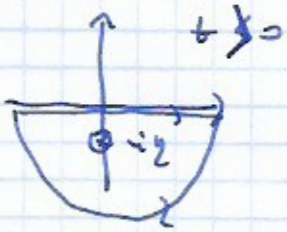
$$G^R_{kk'}(t) = -i\,\theta(t) \langle \{ c_k e^{-i\frac{\varepsilon_k t}{\hbar}}, c^+_{k'} \} \rangle =$$

$$= -i\,\theta(t) e^{-i\frac{\varepsilon_k t}{\hbar}} \delta_{kk'} \qquad [G(t)] = [1]$$

using representation $\theta(t) = -\frac{1}{2\pi i}\int_{-\infty}^{\infty} d\omega \frac{e^{-i\omega t}}{\omega + i\eta}$

$\eta \to 0^+$





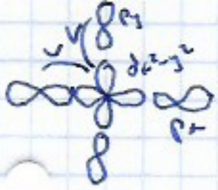
$$\boxed{\oint dz \frac{e^{-izt}}{z} \quad z = \rho e^{i\theta}}$$

$$G^R_{kk'}(t) = -i\left(-\frac{1}{2\pi i}\right)\int_{-\infty}^{\infty} d\omega \frac{e^{-i\omega t}}{\omega + i\eta} e^{-i\frac{\varepsilon_k t}{\hbar}} \delta_{kk'} =$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega \frac{e^{-i(\omega + \frac{\varepsilon_k}{\hbar})t}}{\omega + i\eta} \delta_{kk'} = \left\{ \omega + \frac{\varepsilon_k}{\hbar} = \tilde{\omega} \right\}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} d\tilde{\omega}\, e^{-i\tilde{\omega} t} \frac{\delta_{kk'}}{\tilde{\omega} - \frac{\varepsilon_k}{\hbar} + i\eta} =$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\, e^{-i\omega t} \underbrace{\frac{\hbar\,\delta_{kk'}}{\hbar\omega - \varepsilon_k + i\eta}}_{G^R_{kk'}(\omega)}$$

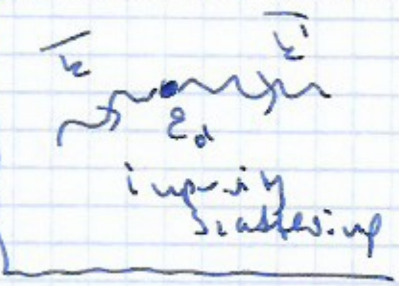


$$\boxed{H = \sum_k \varepsilon_k c_k^\dagger c_k + \varepsilon_d d^\dagger d + \sum_k \left(V_k c_k^\dagger d + V_k^* d^\dagger c_k \right)}$$

Anderson-Fano-Friedrichs-Lee model

$$i\hbar \frac{d c_k(t)}{dt} = [c_k(t), H] = \varepsilon_k c_k + V_k d$$

$$i\hbar \frac{d\, d(t)}{dt} = [d, H] = \varepsilon_d d + \sum_k V_k^* c_k$$



$$\langle\langle c_k(t) | c_{k'}^\dagger(0) \rangle\rangle, \quad \langle\langle d(t) | d^\dagger(0) \rangle\rangle, \quad \langle\langle d(t) | c_{k'}^\dagger(0) \rangle\rangle$$
$$\langle\langle c_k(t) | d^\dagger(0) \rangle\rangle$$

$$i\hbar \frac{d}{dt} \langle\langle c_k(t) | c_{k'}^\dagger \rangle\rangle = \hbar\delta(t) \langle \{ c_k, c_{k'}^\dagger \} \rangle + \varepsilon_k \langle\langle c_k(t) | c_{k'}^\dagger \rangle\rangle + V_k \langle\langle d(t) | c_{k'}^\dagger \rangle\rangle$$

$$i\hbar \frac{d}{dt} \langle\langle d(t) | c_{k'}^\dagger \rangle\rangle = \hbar\delta(t) \langle \overbrace{\{ d, c_{k'}^\dagger \}}^{=0} \rangle + \varepsilon_d \langle\langle d(t) | c_{k'}^\dagger \rangle\rangle + \sum_k V_k^* \langle\langle c_k(t) | c_{k'}^\dagger \rangle\rangle$$

$$i\hbar \frac{d}{dt} \langle\langle d(t) | d^\dagger \rangle\rangle = \hbar\delta(t) \langle \{ d, d^\dagger \} \rangle + \varepsilon_d \langle\langle d(t) | d^\dagger \rangle\rangle + \sum_k V_k^* \langle\langle c_k(t) | c_{k'}^\dagger \rangle\rangle$$

$$i\hbar \frac{d}{dt} \langle\langle c_k(t) | d^\dagger \rangle\rangle = \hbar\delta(t) \langle \overbrace{\{ c_k, d^\dagger \}}^{=0} \rangle + \varepsilon_k \langle\langle c_k(t) | d^\dagger \rangle\rangle + V_k \langle\langle d(t) | d^\dagger \rangle\rangle$$

Fourier transformation

$$\begin{cases} \hbar\omega \langle\langle c_k | c_{k'}^+ \rangle\rangle_\omega = \hbar \delta_{kk'} + \varepsilon_k \langle\langle c_k | c_{k'}^+ \rangle\rangle_\omega + V_k \langle\langle d | c_{k'}^+ \rangle\rangle_\omega \\ \hbar\omega \langle\langle d | c_{k'}^+ \rangle\rangle_\omega = \varepsilon_d \langle\langle d | c_{k'}^+ \rangle\rangle_\omega + \sum_k V_k^* \langle\langle c_k | c_{k'}^+ \rangle\rangle_\omega \\ \hbar\omega \langle\langle d | d^+ \rangle\rangle = \hbar + \varepsilon_d \langle\langle d | d^+ \rangle\rangle_\omega + \sum_k V_k^* \langle\langle c_k | d^+ \rangle\rangle_\omega \\ \hbar\omega \langle\langle c_k | d^+ \rangle\rangle = \varepsilon_k \langle\langle c_k | d^+ \rangle\rangle_\omega + V_k \langle\langle d | d^+ \rangle\rangle_\omega \end{cases}$$

(2) $\rightarrow$ $$\langle\langle d | c_{k'}^+ \rangle\rangle_\omega = \frac{1}{\hbar\omega - \varepsilon_d} \cdot \sum_k V_k^* \langle\langle c_k | c_{k'}^+ \rangle\rangle_\omega$$

(4) $\rightarrow$ $$\langle\langle c_k | d^+ \rangle\rangle_\omega = \frac{1}{\hbar\omega - \varepsilon_k} \cdot V_k \langle\langle d | d^+ \rangle\rangle_\omega$$

(3) $\rightarrow$ $$(\hbar\omega - \varepsilon_d) \langle\langle d | d^+ \rangle\rangle = \hbar + \sum_k V_k^* \frac{V}{\hbar\omega - \varepsilon_k} \langle\langle d | d^+ \rangle\rangle_\omega$$

$$\boxed{\underset{G_{dd}(\omega)}{\langle\langle d | d^+ \rangle\rangle_\omega} = \frac{\hbar}{\hbar\omega - \varepsilon_d - \sum_k \frac{|V_k|^2}{\hbar\omega - \varepsilon_k}}}$$

(1) $\rightarrow$ $$(\hbar\omega - \varepsilon_k) \langle\langle c_k | c_{k'}^+ \rangle\rangle_\omega = \hbar \delta_{kk'} + \frac{V_k}{\hbar\omega - \varepsilon_d} \sum_{k''} V_{k''}^* \langle\langle c_{k''} | c_{k'}^+ \rangle\rangle_\omega$$

$$\langle\langle c_k | c_{k'}^+ \rangle\rangle_\omega = \frac{\hbar \delta_{kk'}}{\hbar\omega - \varepsilon_k} + $$

$$+ \frac{1}{\hbar\omega - \varepsilon_d} \frac{V_k}{\hbar\omega - \varepsilon_k} \sum_{k''} V_{k''}^* \langle\langle c_{k''} | c_{k'}^+ \rangle\rangle_\omega$$

iterative solution possible

0 - order

$$\langle\langle c_k | c_{k'}^+ \rangle\rangle = \frac{\hbar\, \delta_{kk'}}{\hbar\omega - \varepsilon_k}$$

1-st order

$$\langle\langle c_k | c_{k'}^+ \rangle\rangle = \frac{\hbar\, \delta_{kk'}}{\hbar\omega - \varepsilon_k} + \frac{V_k}{\hbar\omega - \varepsilon_k} \frac{\hbar}{\hbar\omega - \varepsilon_d} \frac{V_{k'}^*}{\hbar\omega - \varepsilon_{k'}}$$

2nd order

$$\langle\langle c_k | c_{k'}^+ \rangle\rangle = \frac{\delta_{kk'}}{\hbar\omega - \varepsilon_k} + \frac{V_k}{\hbar\omega - \varepsilon_k} \frac{1}{\hbar\omega - \varepsilon_d} \frac{V_{k'}^*}{\hbar\omega - \varepsilon_{k'}} +$$

$$+ \frac{V_k}{\hbar\omega - \varepsilon_k} \frac{1}{\hbar\omega - \varepsilon_d} \sum_{k''} \frac{V_{k''}}{\hbar\omega - \varepsilon_{k''}} \frac{\hbar}{\hbar\omega - \varepsilon_d} \frac{V_{k'}^*}{\hbar\omega - \varepsilon_{k'}}$$

3rd order

$$\langle\langle c_k | c_{k'}^+ \rangle\rangle = \frac{\delta_{kk'}}{\hbar\omega - \varepsilon_k} + \frac{V_k}{\hbar\omega - \varepsilon_k} \frac{1}{\hbar\omega - \varepsilon_d} \frac{V_{k'}^*}{\hbar\omega - \varepsilon_{k'}} +$$

$$+ \frac{V_k}{\hbar\omega - \varepsilon_k} \frac{1}{\hbar\omega - \varepsilon_d} \left(\sum_{k''} \frac{|V_{k''}|^2}{\hbar\omega - \varepsilon_{k''}} \right) \frac{\hbar}{\hbar\omega - \varepsilon_d} \frac{V_{k'}^*}{\hbar\omega - \varepsilon_{k'}} +$$

$$+ \frac{V_k}{\hbar\omega - \varepsilon_k} \frac{1}{\hbar\omega - \varepsilon_d} \left| \sum_{k'''} \frac{|V_{k'''}^*|^2}{\hbar\omega - \varepsilon_{k'''}} \right) \frac{\hbar}{\hbar\omega - \varepsilon_d} \left(\sum_{k''} \frac{|V_{k''}|^2}{\hbar\omega - \varepsilon_{k''}} \right) \frac{\hbar}{\hbar\omega - \varepsilon_d} \frac{V_{k'}^*}{\hbar\omega - \varepsilon_{k'}}$$

$$\langle\langle c_k | c_{k'}^+ \rangle\rangle = \frac{\hbar\,\delta_{kk'}}{\hbar\omega - \varepsilon_k} + \frac{V_k}{\hbar\omega - \varepsilon_k}\left[\frac{\hbar}{\hbar\omega - \varepsilon_d} + \right.$$

$$\left. \frac{1}{\hbar\omega - \varepsilon_d}\left(\sum_{k''} \frac{|V_{k''}|^2}{\hbar\omega - \varepsilon_{k''}}\right)\frac{\hbar}{\hbar\omega - \varepsilon_d} + \frac{1}{\hbar\omega - \varepsilon_d}\left(\left(\sum_{k''} \frac{|V_{k''}|^2}{\hbar\omega - \varepsilon_{k''}}\right)\frac{\hbar}{\hbar\omega - \varepsilon_d}\right)^2 + \ldots \right] \cdot \frac{V_{k'}^*}{\hbar\omega - \varepsilon_{k'}} =$$

$$1 + x + x^2 + \ldots = \frac{1}{1-x}$$

$$= \frac{\delta_{kk'}}{\hbar\omega - \varepsilon_k} + \frac{V_k}{\hbar\omega - \varepsilon_k} \frac{\frac{1}{\hbar\omega - \varepsilon_d}}{1 - \frac{1}{\hbar\omega - \varepsilon_d}\sum_{k''} \frac{|V_{k''}|^2}{\hbar\omega - \varepsilon_{k''}}} \frac{V_{k'}^*}{\hbar\omega - \varepsilon_{k'}}$$

$$\overbrace{\langle\langle c_k | c_{k'}^+ \rangle\rangle_\omega}^{G_{kk'}(\omega)} = \frac{\hbar\,\delta_{kk'}}{\hbar\omega - \varepsilon_k} +$$

$$+ \underbrace{\frac{\hbar}{\hbar\omega - \varepsilon_k}}_{G_k^0(\omega)} \underbrace{\underbrace{\left(\frac{V_k V_{k'}^*}{\hbar\omega - \varepsilon_d - \sum_{k''} \frac{|V_{k''}|^2}{\hbar\omega - \varepsilon_{k''}}} \right)}_{V_k V_{k'}^* \langle\langle d | d^+ \rangle\rangle_\omega}}_{T_{kk'}(\omega)} \underbrace{\frac{\hbar}{\hbar\omega - \varepsilon_{k'}}}_{G_{k'}^0(\omega)}$$

Scattering <u>Lippmann-Schwinger</u> equation

$$\boxed{G_{kk'}(\omega) = G_k^0(\omega)\,\delta_{kk'} + G_k^0(\omega)\, T_{kk'}(\omega)\, G_{k'}^0(\omega)}$$

df <u>t-matrix</u> $\boxed{T_{kk'}(\omega) = \frac{V_k V_{k'}^*}{\hbar\omega - \varepsilon_d - \sum_{k''} \frac{|V_{k''}|}{\hbar\omega - \varepsilon_{k''}}}}$

$$\boxed{T_{kk'}(\omega) = V_k\, G_{dd}(\omega)\, V_{k'}^*}$$

Discussion of $G_{dd}(\omega)$

$$\boxed{G_{dd}(\omega) = \frac{\hbar}{\hbar\omega - \varepsilon_d - \sum_k \frac{|V_k|^2}{\hbar\omega - \varepsilon_k}} = \frac{\hbar}{\hbar\omega - \varepsilon_d - \Delta(\omega)}}$$

$$\boxed{\Delta(\omega) = \sum_k \frac{|V_k|^2}{\hbar\omega - \varepsilon_k} = \int d\varepsilon \frac{|V|^2}{\hbar\omega - \varepsilon} \rho(\varepsilon)}$$ – hybridization function

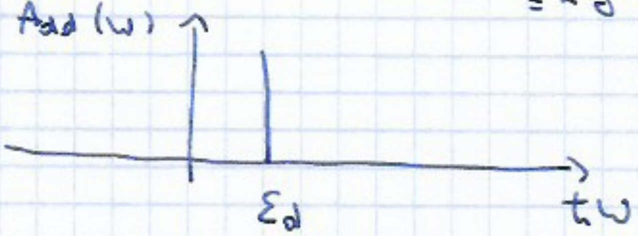
one-particle self-energy

$V_k = 0$

$$G^{(0)}_{dd}(\omega) = \frac{\hbar}{\hbar\omega - \varepsilon_d}$$

$$A^{(0)}_{dd}(\omega) = -\frac{1}{\pi} \lim_{\eta\to 0+} \operatorname{Im} \frac{\hbar}{\hbar\omega - \varepsilon_d + i\eta} = \lim_{\eta\to 0+} \frac{1}{\pi} \frac{\eta\hbar}{(\hbar\omega - \varepsilon_d)^2 + \eta^2} =$$

$$= \hbar\delta(\hbar\omega - \varepsilon_d)$$



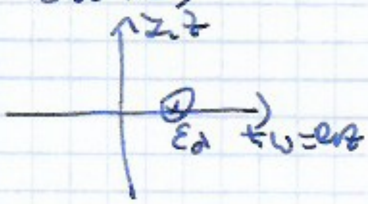
single delta-peak

atomic state with infinite life-time

a pole of $G^{(0)}_{dd}(\omega)$ $\quad$ $\hbar\omega = \varepsilon_d \in \mathbb{R}$

$z \in \mathbb{C}$

$G^{(0)}_{dd}(z)$



a single pole at $z = \varepsilon_d - i\eta$

infinitesimally below the real axis

$G^{(0)}_{dd}(z)$ is single valued

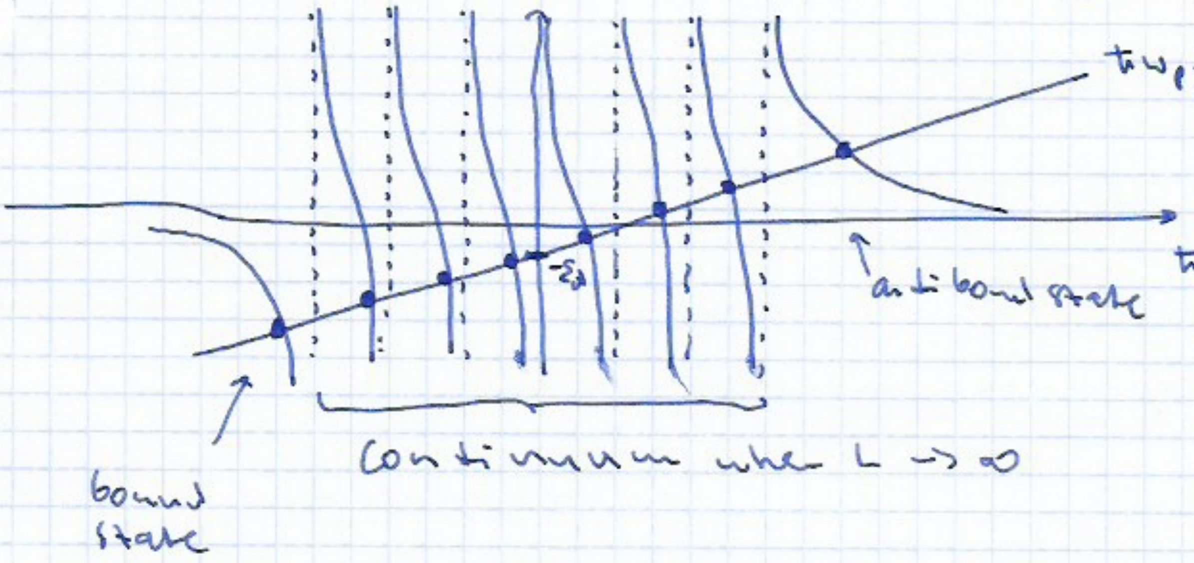
$V_k \neq 0$ $\quad$ $G_{dd}(\omega) = \frac{1}{\hbar\omega - \varepsilon_d - \Delta(\omega)}$

a pole of $G_{dd}(\omega)$

$$\hbar\omega_p - \varepsilon_d - \Delta(\omega_p) = 0$$

$$\hbar\omega_p - \varepsilon_d - \sum_k \frac{V^2}{\hbar\omega_p - \varepsilon_k} = 0 \qquad k = \frac{2\pi}{L}(n_x, n_y, n_z)$$

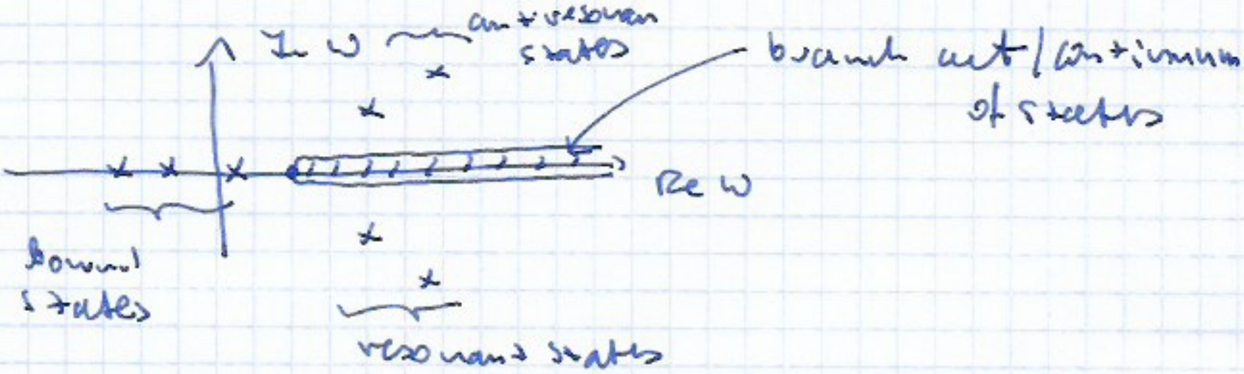
$V_k = V \in \mathbb{R}$ – k independent



poles $\hbar\omega_p^0 = \varepsilon_k$ at $V_k = 0$ are shifted due to finite hybridization

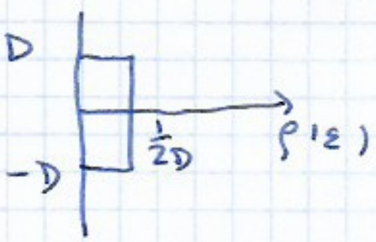
<u>In general, in thermodynamic limit</u>

$\omega \in \mathbb{C}$
$$G_{dd}(\omega) = \sum_n \frac{1}{\omega - E_n + i\Gamma_n} + \int d\varepsilon \frac{A(\varepsilon)}{\omega - \varepsilon}$$



Consider a simple model

$$\rho(\varepsilon) = \frac{1}{2D}\,\theta(D - |\varepsilon|)$$



$$\int d\varepsilon\, \rho(\varepsilon) = 1$$

$V \in \mathbb{R}$

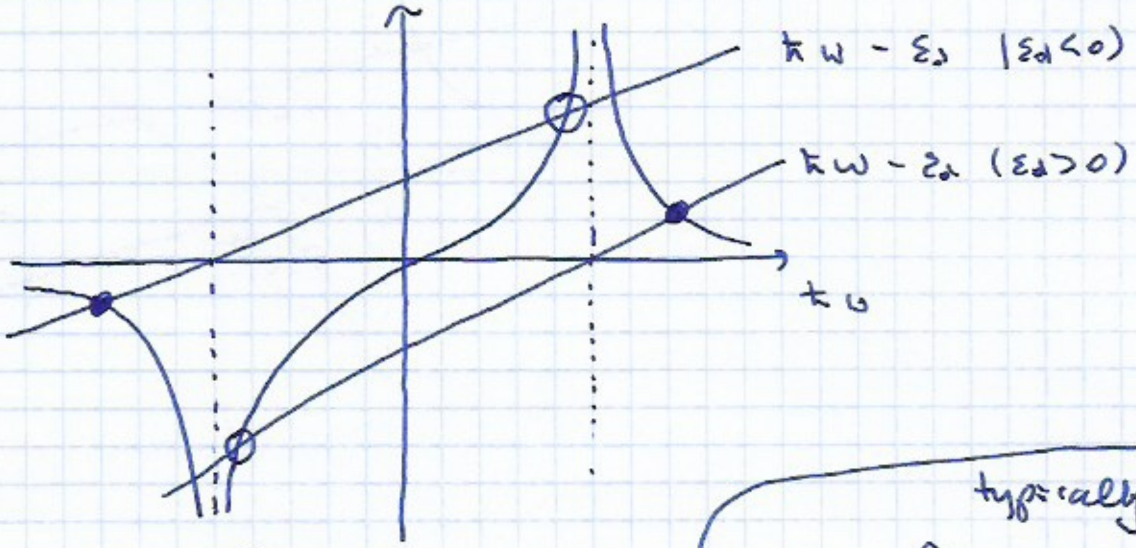
$$\Delta(\omega) = \frac{V^2}{2D}\int_{-D}^{D} d\varepsilon \, \frac{1}{\hbar\omega - \varepsilon + i\eta} =$$

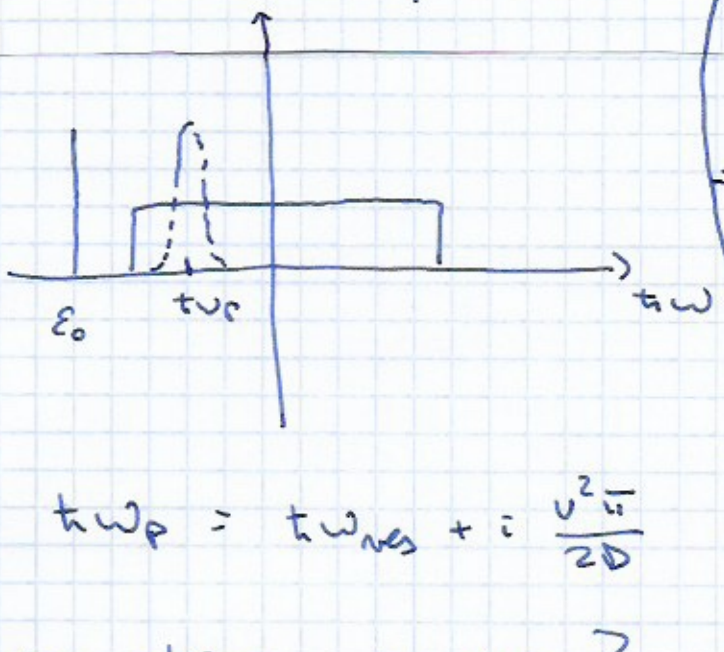
$$\boxed{\frac{1}{x \pm i\eta} = \mathcal{P}\frac{1}{x} \mp i\pi\delta(x)}$$

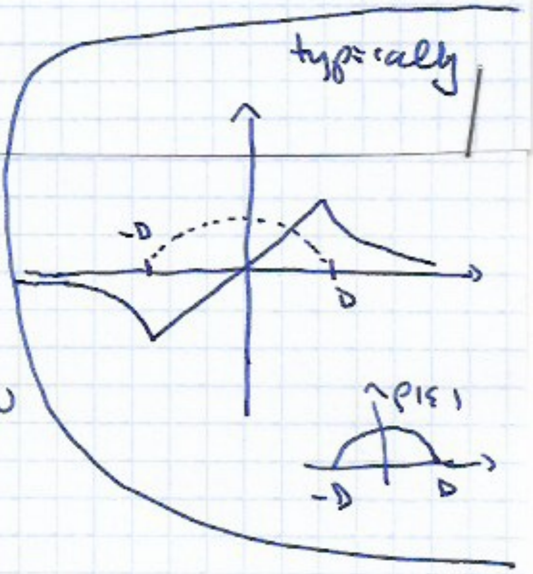
$$= -\frac{V^2}{2D}\int_{-D}^{D} d\varepsilon \, \frac{1}{\varepsilon - \hbar\omega - i\eta} =$$

$$= -\frac{V^2}{2D}\left[\ln\left|\frac{D - \hbar\omega}{-D - \hbar\omega}\right| + i\pi \, \Theta\left(D - |\hbar\omega|\right)\right]$$

$$\boxed{\hbar\omega_p - \varepsilon_d + \frac{V^2}{2D}\ln\left|\frac{D - \hbar\omega}{D + \hbar\omega}\right| + i\frac{V^2\pi}{2D}\Theta(D - |\hbar\omega|) = 0}$$







$$\hbar\omega_p = \hbar\omega_{res} + i\frac{V^2\pi}{2D} \in \mathbb{C}$$

Why complex states?

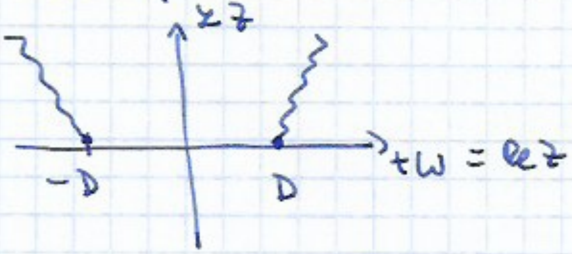
$$\Delta(\omega) = \frac{V^2}{2D}\int_{-D}^{D} d\varepsilon \frac{1}{\hbar\omega - \varepsilon + i\eta} = \qquad \eta \to 0^+$$

$$= -\frac{V^2}{2D}\int_{-D}^{D} d\varepsilon \frac{1}{\varepsilon - \hbar\omega - i\eta} =$$

$$= -\frac{V^2}{2D}\Big(\ln(D - \hbar\omega - i\eta) - \ln(-D - \hbar\omega - i\eta)\Big)$$

two branch-cut points , $z \in \mathbb{C}$

$$\Delta(z) = -\frac{V^2}{2D}\Big(\ln(D - z) - \ln(-D - z)\Big)$$



The pole moves away from the real axis down into the lower half-plane

Let $\underline{z = x + iy}$ $|x| <$

$$\begin{cases} x - \varepsilon_d - \operatorname{Re}\Delta(x+iy) \overset{!}{=} 0 \\ y - \operatorname{Im}\Delta(x+iy) \overset{!}{=} 0 \end{cases}$$

<u>for $|x| < D$</u>

$$\Delta(z = x+iy) = -\frac{V^2}{2D}\Big(\ln(D - x - iy) - \ln(-D - x - iy)\Big) =$$

$$= -\frac{V^2}{4D}\Big(\ln\sqrt{(D-x)^2 + y^2} - \ln\sqrt{(D+x)^2+y^2} +$$

$$+ i\arctan\left(\frac{-y}{D-x}\right) + i\arctan\left(\frac{-y}{-D-x}\right)\Big)$$

$\arg z = \phi \in [-\pi, \pi]$

$$\begin{cases} x - \varepsilon_d + \dfrac{v^2}{2D} \ln\sqrt{\dfrac{(D-x)^2+y^2}{(D+x)^2+y^2}} = 0 \\[2ex] y + \dfrac{v^2}{2D}\left(\arctan\left(\dfrac{y}{x-D}\right) + \arctan\left(\dfrac{y}{x+D}\right)\right) = 0 \end{cases}$$

<u>There is no pole at $V \neq 0$!</u>

•) the complex pole leaves the first Riemann sheet

••) $G_{dd}(z)$ is multivalued and not defined along a branch cut $[-D, D]$

Find discontinuity along the branch cut

$$\delta\Delta(\omega) = \Delta(\omega + i\eta) - \Delta(\omega - i\eta) =$$

$$= \frac{v^2}{2D}\left(\int_{-D}^{D} d\varepsilon \left(\frac{1}{\hbar\omega - \varepsilon + i\eta} - \frac{1}{\hbar\omega - \varepsilon - i\eta}\right)\right)$$

$$= \frac{v^2}{2D}\int_{-D}^{D} d\varepsilon \frac{-2\eta i}{(\hbar\omega-\varepsilon)^2+\eta^2} \xrightarrow[\eta\to 0^+]{} -\frac{v^2}{D}\pi i \int_{-D}^{D} d\varepsilon\, \delta(\hbar\omega - \varepsilon) =$$

$$= -\frac{v^2\pi}{D} i$$

On the second Riemann sheet

$$\Delta_{II}(z) = \Delta(z) - \frac{v^2\pi}{D} i$$

$$\begin{cases} x - \varepsilon_d - \operatorname{Re}\Delta(x+iy) = 0 \\[1ex] y - \operatorname{Im}\Delta(x+iy) - \dfrac{v^2\pi}{D} = 0 \end{cases}$$

There are complex solutions - <u>resonances</u>