Week 1

1. $$p = \left(\frac{nT^3}{27AV}\right)^{1/2}$$

2. Hint: use the formula $c_V = T(\partial S/\partial T)_V$ and the Maxwell relation
$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V$$

3. $c_p - c_V = R$

4. $U = 2ApV^2/n$

Week 2

1. enthalpy: $H^*(S,H) = U - \mu_0 MH$, Helmholz free energy: $F(M,T) = U - TS$, Gibbs free energy: $G(H,T) = U - TS - \mu_0 HM$.
$$c_M = \left(\frac{\partial U}{\partial T}\right)_M \qquad c_H = \left(\frac{\partial U}{\partial T}\right)_H - \mu_0 H\left(\frac{\partial M}{\partial T}\right)_H$$

2. enthalpy: $H^*(S,E) = U - EP$, Helmholz free energy: $F(P,T) = U - TS$, Gibbs free energy: $G(E,T) = U - TS - EP$.
$$c_P = \left(\frac{\partial U}{\partial T}\right)_P \qquad c_E = \left(\frac{\partial U}{\partial T}\right)_E - E\left(\frac{\partial P}{\partial T}\right)_E$$

3. Hint: Calculate the total differential dm and substitute it to Eq. (4). Rearrange to $đQ = C_H dT - \frac{CH}{T} dH$.

4. Hint: In case of a simple fluid of a single component $G(T,P,N) = \mu N$. Use relations:
$$V = \left(\frac{\partial G}{\partial p}\right)_{T,N}, \qquad -S = \left(\frac{\partial G}{\partial T}\right)_{p,N}$$
to show that $pV = const$ if $T = const$ and to calculate the entropy. Use $F = G - PV$ to calculate Helmholz free energy.

Week 3

1. $$\Delta S = nc_V \ln\frac{T_2}{T_1} + nR\ln\left(\frac{V_2 - nb}{V_1 - nb}\right)$$

2. Solution:
$$\frac{\pi n}{2n+1} \xrightarrow{n\to\infty} \frac{\pi}{2}$$
Hint: Notice that
$$(2n)! = (1\cdot 3\cdot \dots \cdot (2n-1))2^n n!$$

3. Hint: Denote $\xi_n = V_n/r^n$. Check that $\xi_1 < \xi_2 < \xi_3 < \xi_4 < \xi_5$. Show by induction that $\xi_{n+1} < \xi_n$ for $n > 5$.
4. $I = \Gamma(4)/81$.

Week 4

1. $p(S = 6) = 5/36$, $\langle S \rangle = 7$, $\sigma^2 = \langle S^2 \rangle - \langle S \rangle^2 = 5.833$
2. If L denotes number of balls in the left half of the container

$$\Omega_L = \binom{N}{L}.$$

Total number of microstates $\Omega = 2^N$. If the balls change the position every second then the average time after which we can find all balls in the left half is $t = 2^N$s.

Week 5

1. $\Omega(N, M) = \binom{N}{N_+}$, $\Omega(N) = \sum_M \Omega(N, M) = 2^N$. Use the Stirling approximation to express $\ln \Omega(N, M) = N \ln 2 - N_+ \ln(1 + M/N) - N_- \ln(1 - M/N)$. Treat M/N as a small parameter and develop the logarithms in the Taylor series.
2. $\Omega = N!/(n_1!n_2! \dots n_k!)$. To find the most probable distribution maximize the function $\ln \Omega(n_1, n_2, \dots, n_{k-1}, n_k)$ with the constraint $n_1 + n_2 + \dots + n_k = N$.
3. Solution steps are already provided

Week 6

1. A hint is already given
2. A hint is already given
3. $c_V = a\alpha k_B(\alpha + 1)T^\alpha V$
4. $$\sigma_M/M = \pm \frac{1}{\sqrt{N} \sinh(\beta\mu B)}.$$

Week 7

1. Statistical sum of the mixture is a product of statistical sums of the components
2. $$U = \frac{3\epsilon \exp(-\beta\epsilon) + 2\epsilon \exp(-2\beta\epsilon)}{1 + 3\exp(-\beta\epsilon) + \exp(-2\beta\epsilon)}$$

3. $$Z = \left(\frac{mk_BT}{2\pi\hbar^2}\right)^{N/2} \left(L - N\sigma\right)/N!$$

4. Hint: Calculate first the normalisation factor
$$\int E^a \exp(-\beta E)dE = \Gamma(a+1)/\beta^{a+1}$$

Week 8

1. Should be moved to Week 9

2. $$p(N_I) = \frac{Z_I Z_{II}}{\sum_{N_I=0}^{N} Z_I Z_{II}}$$

3.

4.