Spin

$$\hat{S} = \sum_{i=1}^{N} \frac{\hbar}{2} \hat{\sigma}_i \qquad \hat{\vec{\sigma}}_i = (\hat{\sigma}_i^x, \hat{\sigma}_i^y, \hat{\sigma}_i^z)$$

$$= \left[\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right]$$

one particle case $|\alpha\rangle = |\mu\rangle \otimes |\sigma\rangle = |\mu\sigma\rangle$ [spin state

$|\uparrow\rangle$ or $|e\rangle$ or $|1\rangle$

$$\hat{\psi}(\vec{r}) = \sum_\alpha \hat{a}_\alpha \phi_\alpha(\vec{r})$$

$$\hat{S}_z = \int d^3r \, \hat{\psi}^\dagger(\vec{r}) \hat{S}_z \hat{\psi}(\vec{r}) = \sum_{\alpha\alpha'} \phi_\alpha^*(\vec{r}) \hat{S}_z \phi_{\alpha'}(\vec{r}) a_\alpha^\dagger a_{\alpha'} =$$

$$= \sum_{\alpha\alpha'} \int d^3r \, \langle \alpha|\vec{r}\rangle \hat{S}_z \langle \vec{r}|\alpha'\rangle a_\alpha^\dagger a_{\alpha'} =$$

$$= \sum_{\mu\sigma,\mu'\sigma'} \langle \mu\sigma | \underbrace{\int d^3r \, |\vec{r}\rangle\langle\vec{r}|}_{\hat{1}} \hat{S}_z |\mu'\sigma'\rangle a_{\mu\sigma}^\dagger a_{\mu'\sigma'} =$$

$$= \sum_{\mu\mu'} \underbrace{\langle \mu|\mu'\rangle}_{\delta_{\mu\mu'}} \sum_{\sigma\sigma'} \langle \sigma | \hat{S}_z | \sigma' \rangle a_{\mu\sigma}^\dagger a_{\mu'\sigma'} \qquad \sigma = \{\uparrow, \downarrow\}$$

$$= \frac{\hbar}{2} \sum_\mu \begin{pmatrix} a_{\mu\uparrow}^\dagger a_{\mu\uparrow} + a_{\mu\downarrow}^\dagger a_{\mu\downarrow} \\ i(a_{\mu\uparrow}^\dagger a_{\mu\downarrow} - a_{\mu\downarrow}^\dagger a_{\mu\uparrow}) \\ a_{\mu\uparrow}^\dagger a_{\mu\uparrow} - a_{\mu\downarrow}^\dagger a_{\mu\downarrow} \end{pmatrix} = \frac{\hbar}{2} \sum_\mu \begin{pmatrix} S_\mu^x \\ S_\mu^y \\ S_\mu^z \end{pmatrix}$$

$$(a_{\mu\uparrow}^\dagger \; a_{\mu\downarrow}^\dagger) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_{\mu\uparrow} \\ a_{\mu\downarrow} \end{pmatrix} = (a_{\mu\uparrow}^\dagger \; a_{\mu\downarrow}^\dagger) \begin{pmatrix} a_{\mu\downarrow} \\ a_{\mu\uparrow} \end{pmatrix} =$$

$$= a_{\mu\uparrow}^\dagger a_{\mu\downarrow} + a_{\mu\downarrow}^\dagger a_{\mu\uparrow}$$

etc.