Statistical Physics A
Colloquium I

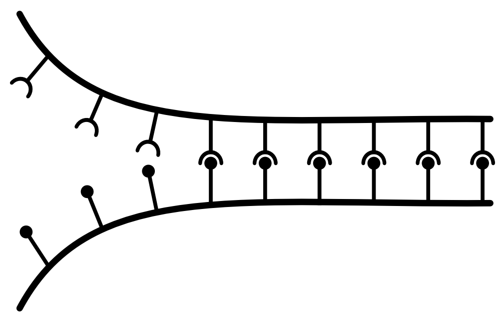
30 November 2020

Please solve problems and send scans with your own solutions via e-mail to: byczuk@fuw.edu.pl before 12:45 pm. Write in the subject: SPA TEST

Each problem is worth 10 points. Explanations and comments in writing, alongside mathematical formulation, are necessary and will count to final points. All problems must be solved by yourself and without any help from any other persons. In case of similar solutions both or more persons will have to be interrogated. It is allowed to use your own notes, lecture notes, and books. If you have questions or comments during the test time, please send them via e-mail to either maciej.lisicki@fuw.edu.pl or marta.waclawczyk@fuw.edu.pl. The will reply with answers, which will be send to all students via USOS e-mail. Therefore you are recommended to check your e-mails regularly during the test time.
Good luck!

Problem 1. Gas fluctuations in a spherical bowl A spherical container of radius R with thermally isolating (adiabatic) walls encloses N particles of an ideal gas. Find the dispersion of the centre of mass of the system in equilibrium.

Problem 2. Zipper model of the DNA A zipper consists of N pairs of teeth, of which each pair can either be connected or disconnected. The energy of a connected pair is 0, and a disconnected pair has the energy E. We also require the zipper to open on one side only (e.g. from the left; see figure). This means that the pair k can be open only if the pairs $1, 2, \ldots, k-1$ are open. Find the partition function and the average number of open pairs $\langle N_{\mathrm{op}} \rangle$ as a function of temperature T. In the limit of low temperatures ($kT \ll E$), how does the average number of open pairs depend on the total length N of the zipper? What is the value of $\langle N_{\mathrm{op}} \rangle$ for high temperatures ($kT \gg E$)?



Problem 3. Entropy of mixing Consider two ideal gases (treated as independent) with N_1 and N_2 particles, respectively, filling a container of volume V and having the same temperature T. Using the classical canonical ensemble, derive formulas for the entropy of mixing. Show that the Helmholz free energy and internal energy of the system are sums of, respectively free energies and internal energies of the gases. Write the results in terms of gas molar fractions $x_{1,2} = N_{1,2}/N$, where $N = N_1 + N_2$ is the total number of particles.