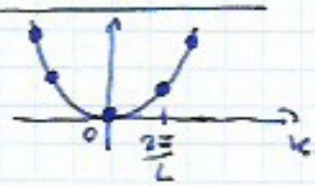
Three spinless bosons $\qquad k_1 = 0, \; k_2 = k_3 = \frac{2\pi}{L}$



(a) $|k_1\, k_2\, k_3\} = \sqrt{3!}\, \hat{S}_0\, |k_1\, k_2\, k_3) = \frac{\sqrt{3!}}{3!} \sum_P (+1)^P\, |k_{P_1}\, k_{P_2}\, k_{P_3}) =$

$= \frac{1}{\sqrt{3!}} \Big[|k_1 k_2 k_3) + |k_3 k_1 k_2) + |k_2 k_3 k_1) + |k_3 k_2 k_1) +$

$|k_1 k_3 k_2) + |k_2 k_1 k_3) \Big] \overset{k_2 = k_3}{=}$

$= \frac{1}{\sqrt{3!}} \Big[\underline{|k_1 k_2 k_2)} + |k_2 \underline{\underline{k_1}} k_2) + |\underset{\sim}{k_2 k_2} k_1) + |\underset{\sim}{k_2 k_2} k_1) +$

$\underline{|k_1 k_2 k_2)} + |k_2 \underline{\underline{k_1}} k_2) \Big] =$

$= \frac{2}{\sqrt{3!}} \Big[|k_1 k_2 k_2) + |k_2 k_1 k_2) + |k_2 k_2 k_1) \Big] = |k_1\, k_2\, k_2\}$

$|k_1\, k_2\, k_2\rangle = \frac{1}{\sqrt{2!}} \; \frac{2}{\sqrt{6}} \Big[|k_1 k_2 k_2) + |k_2 k_1 k_2) + |k_2 k_2 k_1) \Big]$

(b) $\{k_1\, k_2\, k_2 \,|\, k_1\, k_2\, k_2\} = \frac{4}{6} \Big[(k_1 k_2 k_2| + (k_2 k_1 k_2| + (k_2 k_2 k_1| \Big] \cdot$

$\Big[|k_1 k_2 k_2) + |k_2 k_1 k_2) + |k_2 k_2 k_1) \Big] =$

$= \frac{4}{6} \cdot 3 = 2$

$\langle k_1\, k_2 k_2 \,|\, k_1 k_2 k_2 \rangle = \frac{1}{2} \cdot \{k_1 k_2 k_2 \,|\, k_1 k_2 k_2\} = \frac{2}{2} = 1$

missing

(c) $a_\alpha |\alpha_1 \cdots \alpha_N\} = \sum_{i=1}^{N} \xi^{i-1}\, \delta_{\alpha \alpha_i}\, |\alpha_1 \cdots \alpha_i \cdots \alpha_N\}$

$a_{k_1} |k_1\, k_2\, k_2\} = \sum_{i=1}^{3} (+1)^{i-1}\, \delta_{k_1 k_i}\, |k_1\, k_2\, k_2\} = |k_2\, k_2\}$

$a_{k_2} |k_1 k_2 k_2\} = |k_1\, k_2\} + |k_1\, k_2\} = 2 |k_1\, k_2\}$

(d)

$$a^+_{\alpha_i} |\alpha_1 \dots \alpha_N\rangle = \sqrt{n_i+1}\, |\alpha_1 \dots \alpha_i \dots \alpha_N\rangle\, \xi^{i-1}$$

$$a_{\alpha_i} |\alpha_1 \dots \alpha_N\rangle = \frac{1}{\sqrt{n_i}} \sum_{j=1}^{N} \xi^{j-1} \delta_{\alpha_i \alpha_j} |\alpha_1 \dots \check{\alpha}_j \dots \alpha_N\rangle$$

$$a^+_{k_1} a_{k_1} |k_1 k_2 k_2\rangle = \frac{1}{\sqrt{2}} a^+_{k_1} a_{k_1} |k_1 k_2 k_2\} = \frac{1}{\sqrt{2}} a^+_{k_1} |k_2 k_2\} =$$

$$= \frac{1}{\sqrt{2}} |k_1 k_2 k_2\} = |k_1 k_2 k_2\rangle = 1 \cdot |k_1 k_2 k_2\rangle$$

$$a^+_{k_2} a_{k_2} |k_1 k_2 k_2\rangle = \frac{1}{\sqrt{2}} a^+_{k_2} a_{k_2} |k_1 k_2 k_2\} = \frac{1}{\sqrt{2}} a^+_{k_2}\, 2 |k_1 k_2\} =$$

$$= \frac{1}{\sqrt{2}}\, 2 |k_2 k_1 k_2\} = 2 \frac{1}{\sqrt{2}} |k_1 k_2 k_2\} = 2 |k_1 k_2 k_2\rangle$$