

Three electrons in a box

$$|4\rangle = |d_1 d_2 d_3\rangle$$



$$\begin{matrix} \uparrow & \uparrow \\ a_{k_2 \uparrow}, a_{k_2 \downarrow} \\ a_{k_1 \uparrow}, a_{k_1 \downarrow} \end{matrix}$$

$$d_1 = (k_1, \uparrow)$$

$$k_1 = 0$$

$$k_2 = \frac{2\pi}{L}$$

$$d_2 = (k_2, \downarrow)$$

$$d_3 = (k_2, \uparrow)$$

$$d_4 = (k_2, \downarrow)$$

$$|4\rangle = \frac{1}{\sqrt{6}} \left[|d_1 d_2 d_3\rangle + |d_3 d_1 d_2\rangle + |d_2 d_3 d_1\rangle - |d_3 d_2 d_1\rangle - |d_1 d_3 d_2\rangle - |d_2 d_1 d_3\rangle \right]$$

$$\begin{aligned} a_{d_3}^+ |d_1 d_2 d_3\rangle &= |d_3 \underbrace{d_1 d_2}_{\downarrow} d_3\rangle = (-1)^2 |d_1 d_2 \underbrace{d_3 d_3}_{\downarrow}\rangle = \\ &= (-1)^3 |d_1 d_2 d_3 d_3\rangle = 0 \end{aligned}$$

$$a_{d_4}^+ |d_1 d_2 d_3\rangle = |d_4 \underbrace{d_1 d_2}_{\downarrow} d_3\rangle = - |d_1 d_2 d_3 d_4\rangle$$

$$a_{d_1} |d_1 d_2 d_3\rangle = |d_2 d_3\rangle$$

$$a_{d_2} |d_1 d_2 d_3\rangle = - a_{d_2} |d_2 d_1 d_3\rangle = - |d_1 d_3\rangle$$

$$a_{d_3} |d_1 d_2 d_3\rangle = |d_1 d_2\rangle$$

$$a_{d_4} |d_1 d_2 d_3\rangle = 0 \quad (\text{no } d_4 \text{ state})$$