Concept of fermionic pairing in condensed matter physics

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Main goal:

BCS–BEC crossover
Different pairing mechanisms
Wide applications of pairing theory

Statistics:

- The first superconductor – quark condensation in 1s after Big Bang
- Superconductivity in neutron stars
- Superconductivity in nuclei
- Superconductivity in condensed matter since 1911

Highest $T_c = 153K$ (1993) in $\mathrm{HgBa_2Ca_2Cu_3O_8}$

Newest superconductor: diamond doped with boron $T_c = 4K$ -
Nature 428, 542 (2004)

Plan of the talk:

1. What is superconductivity and superfluidity
2. BCS theory and BCS model
3. Pairing mechanisms
4. BCS–BEC crossover
5. Conclusions

Odkrycie nadprzewodnictwa

1911 - Heike Kamerlingh Onnes (Leida - Holandia)

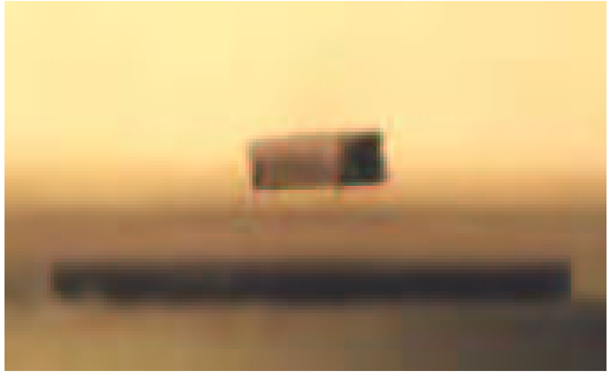
- Poniżej $T_c = 4.15$ K opór rtęci spada do zera
- Inne nadprzewodniki: Al, In, Nb, Pb, ...
- Duży prąd lub zewnętrzne pole magnetyczne (ułamka tesli) niszczą nadprzewodnictwo

Nagroda Nobla - 1913

Efekt Meissnera

1933 - Meissner i Ochsenfeld (Berlin - Niemcy)

Poniżej T_c słabe zewnętrzne pole magnetyczne jest całkowicie wypychane z próbki



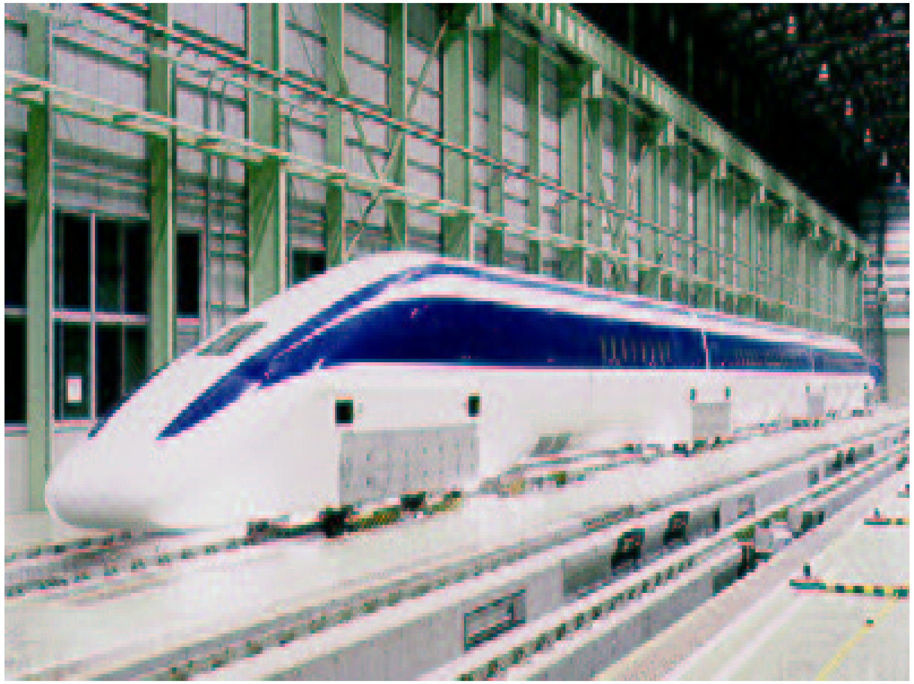
lewitujący nadprzewodnik

MAGLEV, MLX01

nadprzewodzące elektromagnesy

14 kwietnia 1999

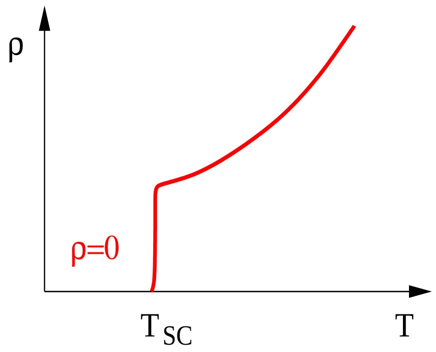
osiąga 617 km/h !



Definicja nadprzewodnictwa

nadprzewodnik $\rightarrow$ "*zerowy, nieskończony, doskonały*"

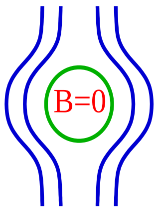
idealny przewodnik



zerowy opór: $\rho = 0$

nieskończone przewodnictwo: $\sigma = \infty$

idealny diamagnetyk

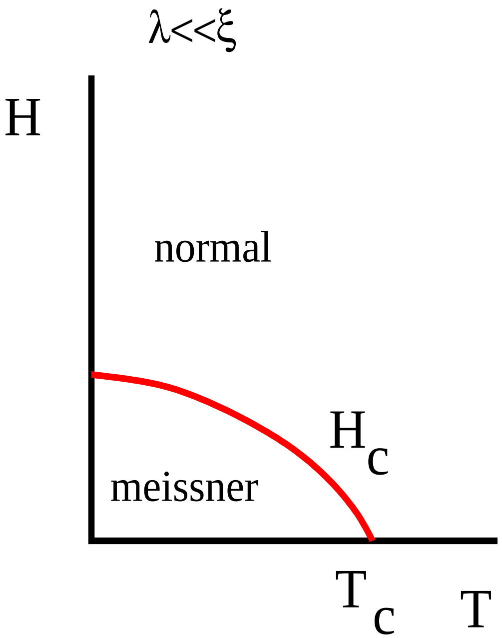


doskonały diamagnetyk: $\chi = -1$

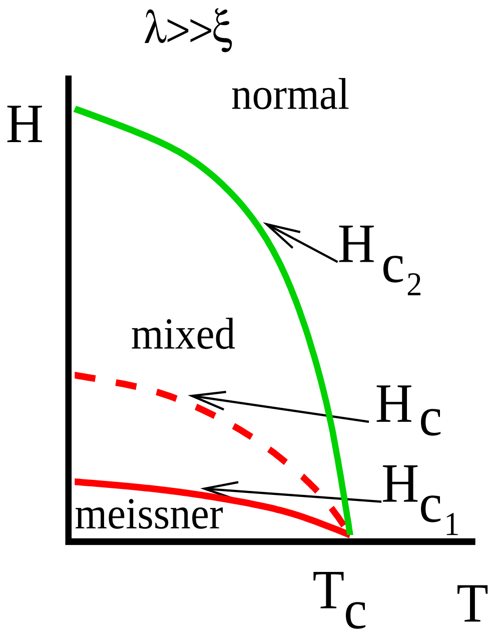
nadprzewodnik

Jeden z niewielu idealnych stanów materii we Wszechświecie

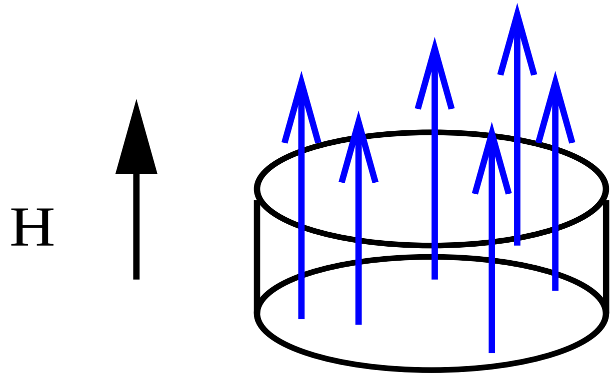
Type I and type II superconductors $\kappa = \lambda/\xi$



Type I



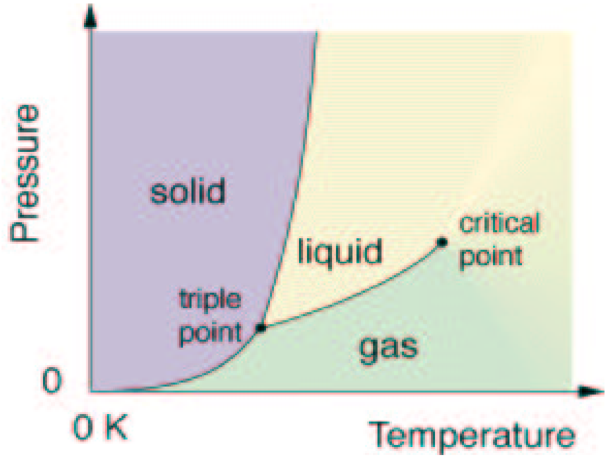
Type II



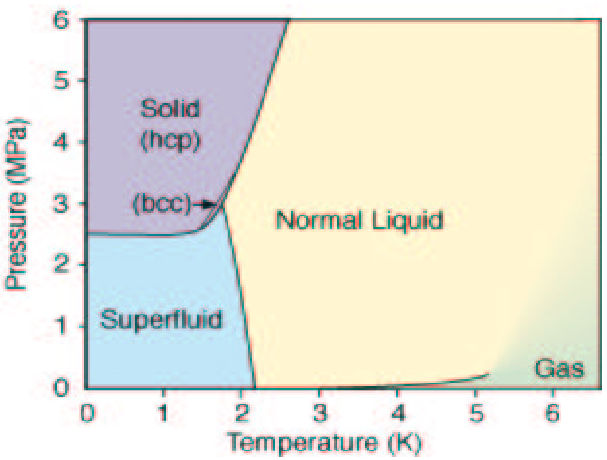
To be $\kappa < 1/\sqrt{2}$ or $\kappa > 1/\sqrt{2}$

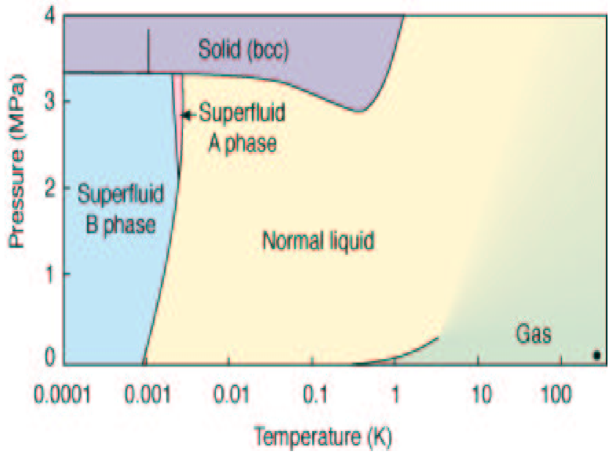
Discovery of superfluidity

Two isotopes of Helium:
^{4}He (2p2n2e - "boson") and ^{3}He (2p1n2e - "fermion")



Cooling ordinary gas / liquid





Quantum liquids up to $T = 0$K

Cooling ^{4}He and ^{3}He

Discovery of superfluidity

^{4}He

- He liquid - Kamerling Onnes 1908
- He solid under pressure - F. Simon 1934
- λ transition - W.H. Keesom 1932
- He film creeping - J.G. Daunt, K. Mendelssohn 1939
- frictionless flowing, perfect heat conductor - P. Kapitza 1938, 1941
- fountain effect - J.F. Allen, H. Jones 1938
- ...

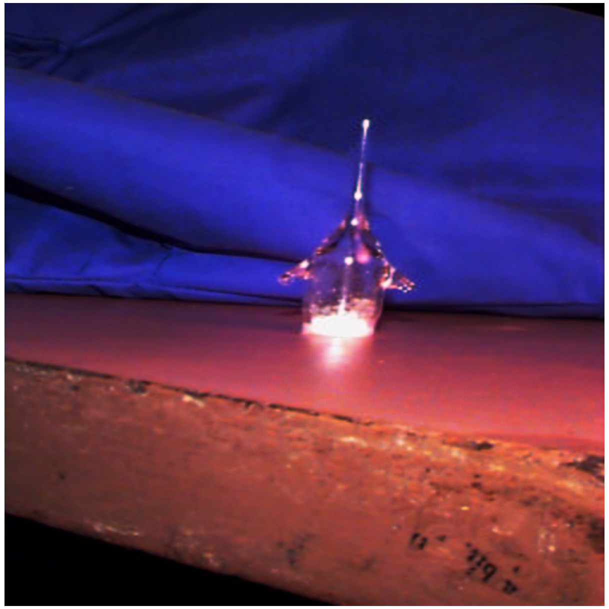
Nobel price - Kapitza 1978

^{3}He

- superfluidity - D.D. Osheroff, R.C. Richardson, D.M. Lee 1972
- theory - A.J. Leggett 1972
- ...

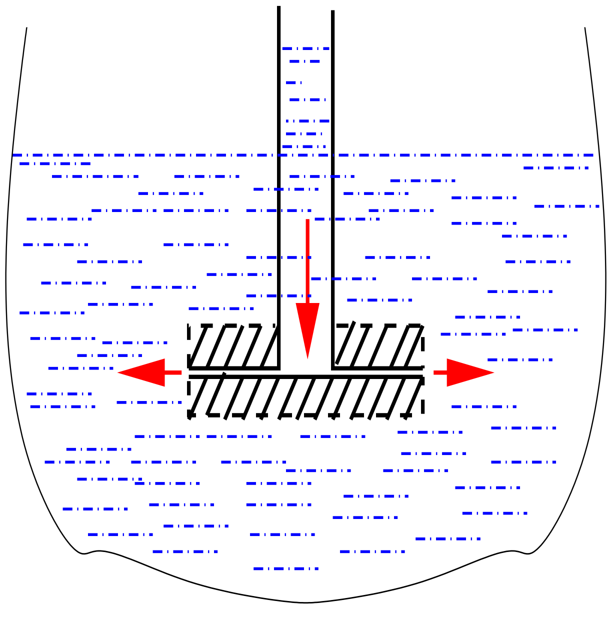
Nobel prices - ORL 1996, Legget 2003

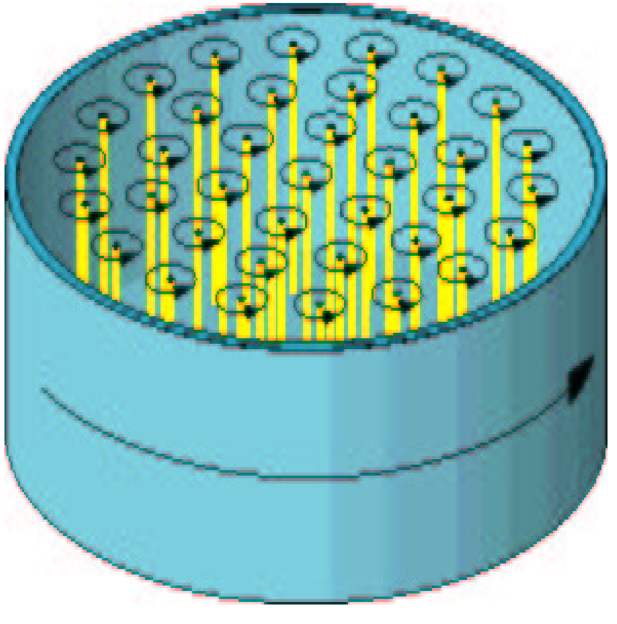
Discovery of superfluidity



What is superfluidity?

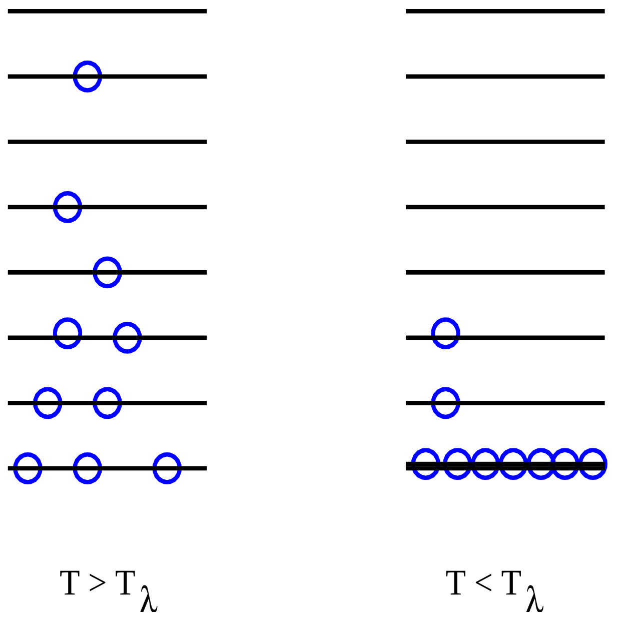
- the ability to flow through microscopic passages with no apparent friction;
- the quantization of vortices;
- the ability to support four wave modes





Microscopic theory of ^{4}He

Some kind of Bose - Einstein condensation of interacting particles



Landau, Feynman, Huang, Bogoliubov, ...,E.Lieb, J. Piasecki

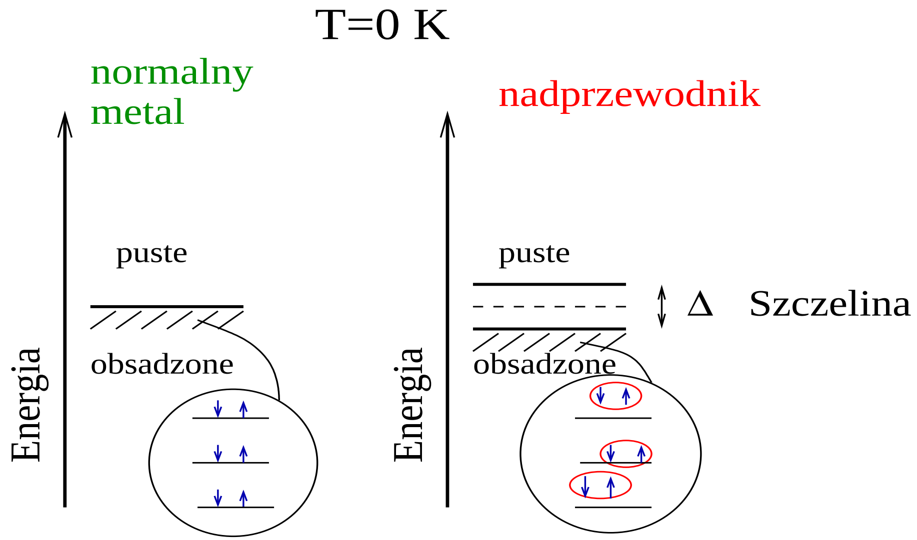
Microscopic theory of superconductors

1957 - J. Bardeen, L. Cooper, R. Schrieffer Nobel price -1972

Instability of Fermi gas/liquid

Pairs of fermions (Cooper pairs) condensate at T_c

New thermodynamic state of matter for $T \leq T_c$



BCS model of superconductivity

$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} a^\dagger_{\mathbf{k}\sigma} a_{\mathbf{k}\sigma} + \sum_{\mathbf{k}\,\mathbf{k}'} V_{\mathbf{k}\,\mathbf{k}'} a^\dagger_{\mathbf{k}\uparrow} a^\dagger_{-\mathbf{k}\downarrow} a_{-\mathbf{k}'\downarrow} a_{\mathbf{k}\uparrow}$$

with attractive potential

$$V_{\mathbf{k}\,\mathbf{k}'} < 0 \quad \text{near the Fermi level}$$

BCS used many-body variational wave - function

$$|\Psi\rangle = \prod_{\mathbf{k}} \left(u_{\mathbf{k}} + v_{\mathbf{k}} a^\dagger_{\mathbf{k}\uparrow} a^\dagger_{-\mathbf{k}\downarrow} \right) |\text{Fermi sea}\rangle$$

Singlet pairing superconductivity $S = 0$, $S_z = 0$!

Energy spectrum with a gap

$$E_{\mathbf{k}} = \sqrt{(\epsilon_{\mathbf{k}} - \mu)^2 + |\Delta_{\mathbf{k}}|^2}$$

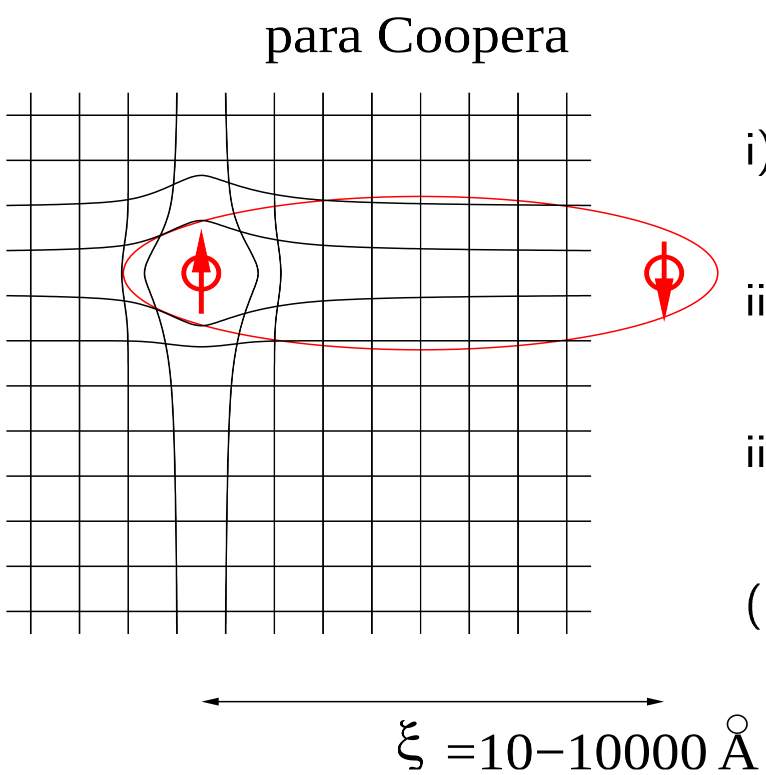
where

$$\Delta_{\mathbf{k}} = \sum_{\mathbf{l}} V_{\mathbf{kl}} \frac{\Delta_{\mathbf{l}}}{2E_{\mathbf{l}}} \tanh\left(\frac{E_{\mathbf{l}}}{2k_B T}\right)$$

$\Delta_{\mathbf{k}} \neq 0$ at $T < T_c$ is the SC order parameter!

Origin of pairing

1956 - L. Cooper after H. Fröhlich (1950-54)

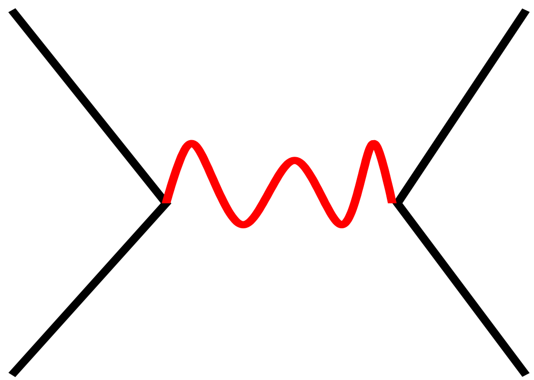


i) deformacja (polaryzacja) sieci jonów

ii) przyciąganie

iii) stabilna para

(iv) pairing induced by phonon exchange



Other mechanism: Electronic charge fluctuation pairing

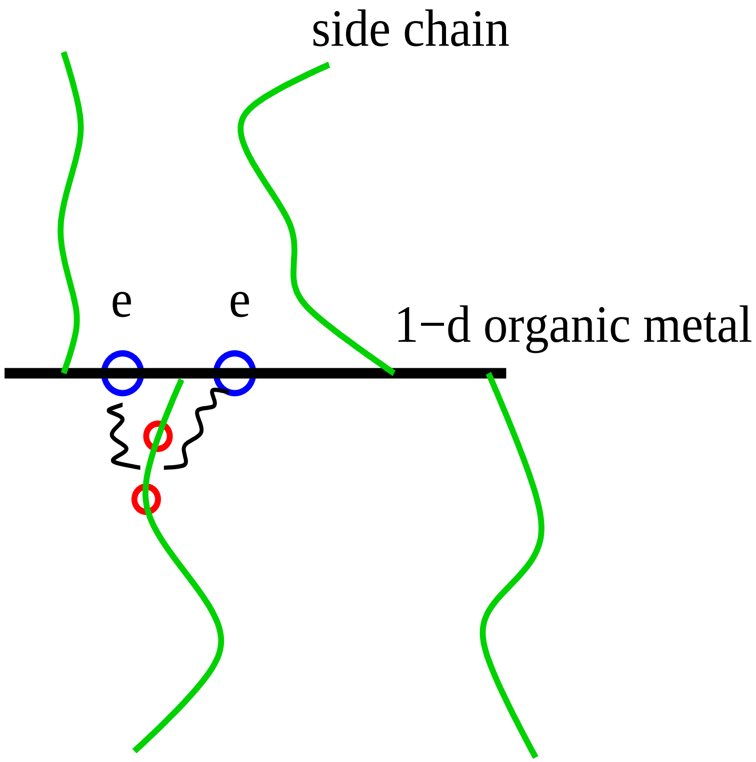
Little (1964), Ginzburg (1965), Kohn–Luttinger (1966), et al.

Idea: replace lattice polarization by other polarizable medium

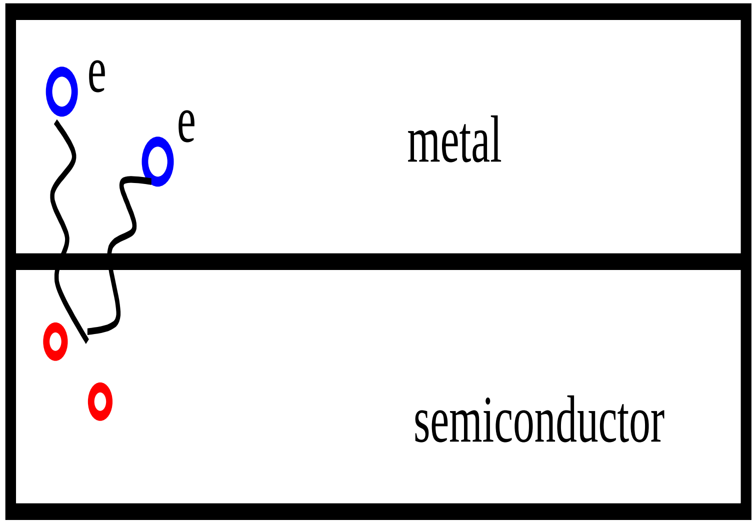
- lattice polarizes – virtual phonons
- electrons redistribute in partially filled band – virtual plasmons
- electrons redustribute in hegher unfilled subbands – virtual excitons
- spin cloud around electron polarizes – paramagnons
- ...

Two or more pairing mechanisms can act in parallel!

“Excitons”

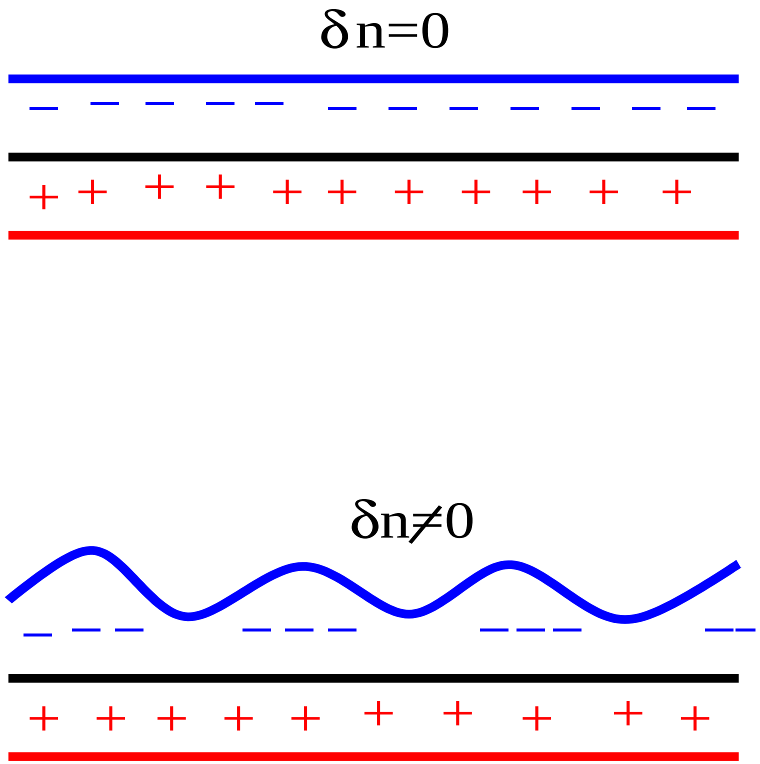


Little mechanism

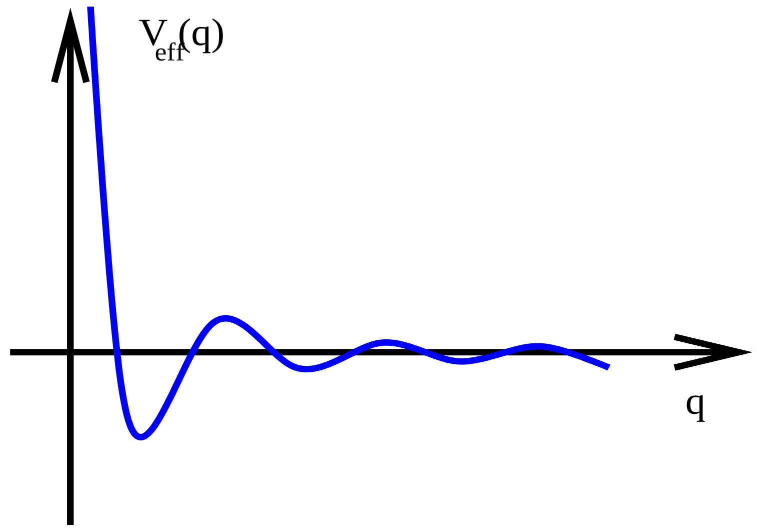


Bray–Bardeen mechanism

"Plasmons"



$$V_{eff}(q,\omega) = \frac{v(q)}{\epsilon(q,\omega)}$$

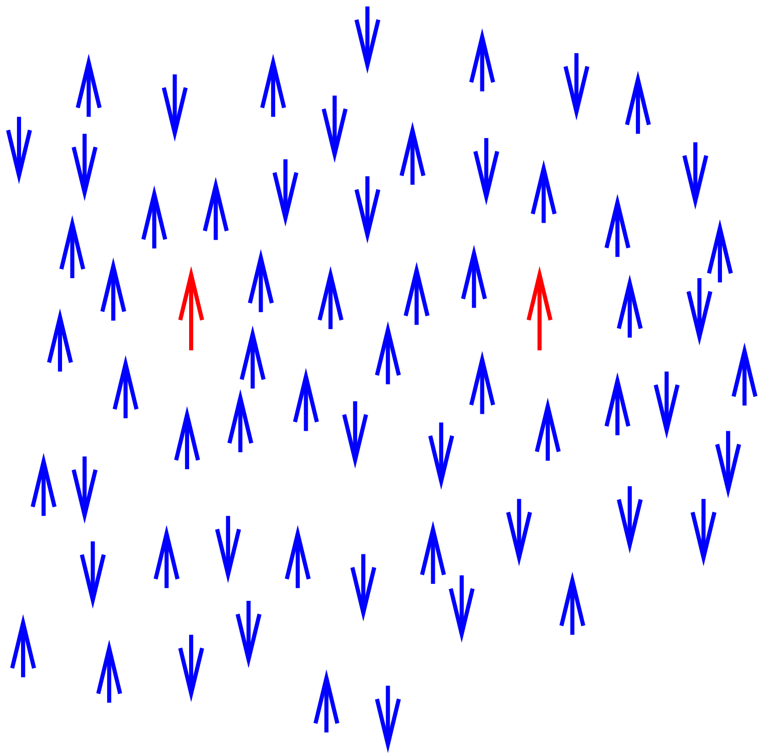


jellium model

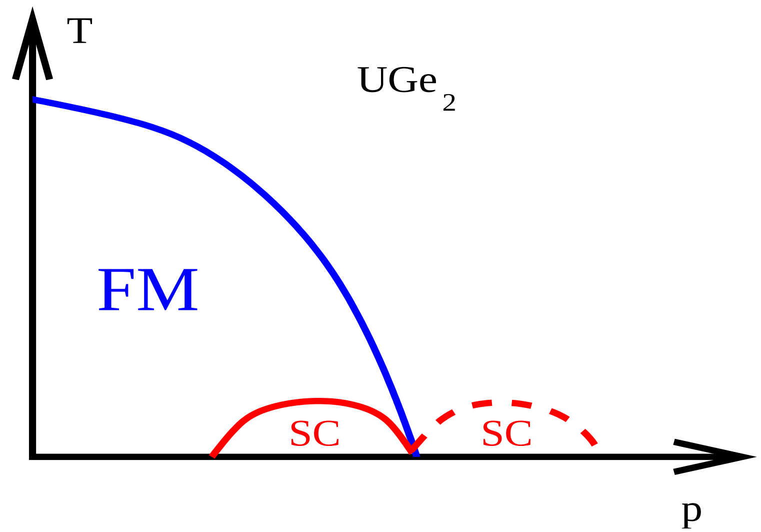
$$\Psi(r_1,\sigma_1;r_2,\sigma_2) = \phi(r_1 - r_2)\chi_{12}$$

p - wave pairing ($l = 1$)

“Ferromagnetic spin fluctuations”



$\chi_{eff} = \frac{\chi}{1-U\chi}$ – FM instability



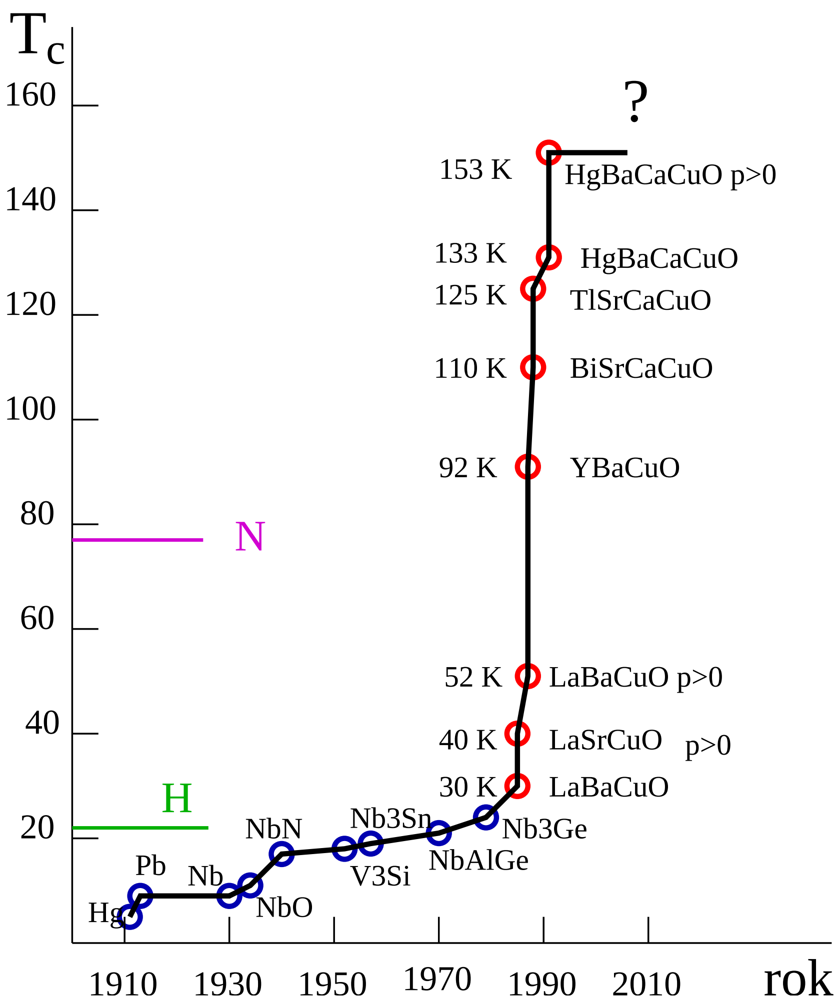
$$\Gamma_{\uparrow\uparrow} = -U^2 \frac{\chi}{1-U^2\chi^2} < 0$$

$$\Gamma_{\uparrow\downarrow} = \frac{U}{1-U^2\chi^2} + U^2 \frac{\chi}{1-U^2\chi^2} > 0$$

$$\Psi(r_1, \sigma_1; r_2, \sigma_2) = \phi(r_1 - r_2)\chi_{12}$$

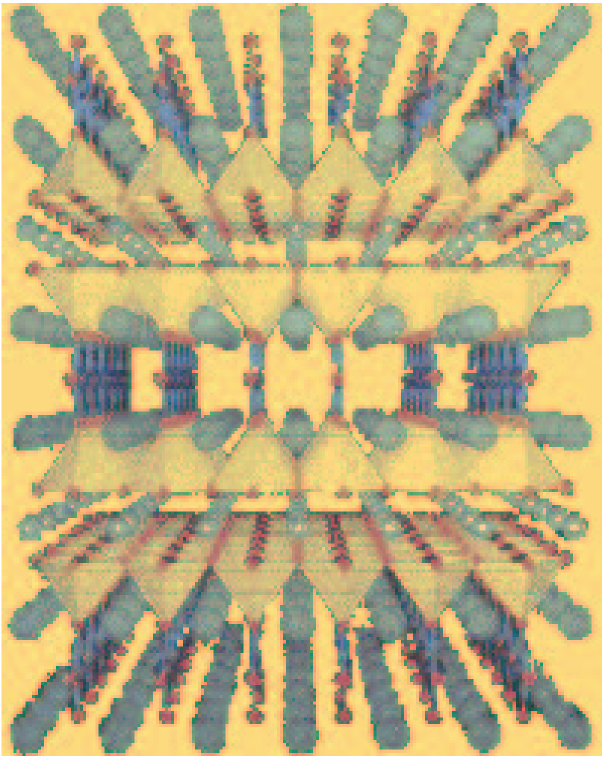
p - wave triplet pairing ($l = 1$)

High–temperature SC



Dlaczego nie 300 K ?

Płaszczyzny miedziowo-tlenowe



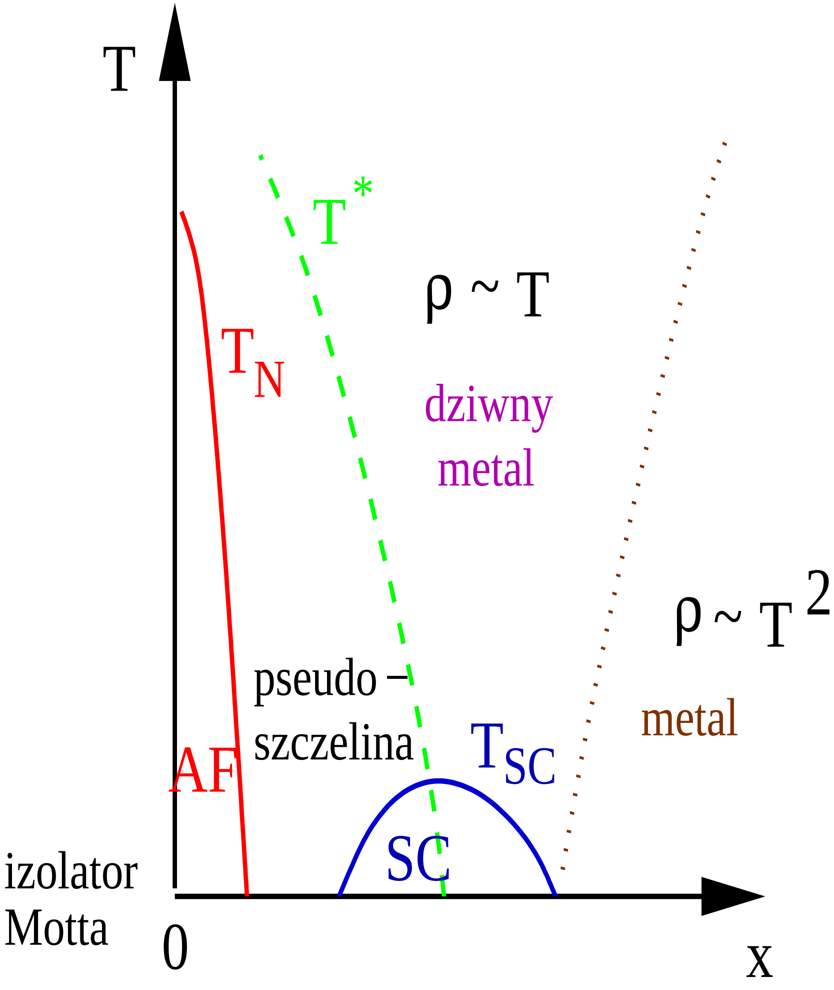
wspólna cecha wszystkich nadprzewodników wysokotemperaturowych

$$T_c = T_c^0 + T_c' \cos\left(\frac{\pi}{p+1}\right)$$

p - liczba sąsiednich płaszczyzn

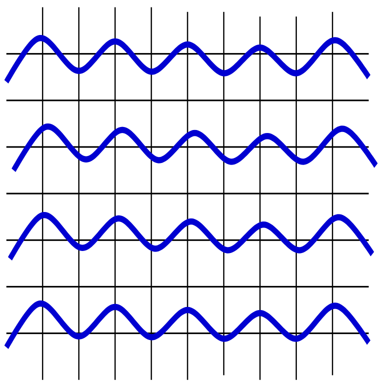
Uniwersalny diagram fazowy

$La_{1-x}Sr_xCuO_4$ - gdzie La^{3+}, Sr^{2+}
zmiana x odpowiada zmianie elektronóna Cu

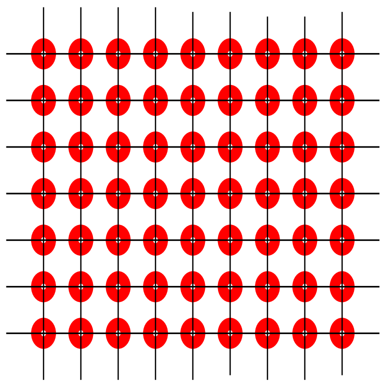


Metal a izolator Motta

Orbitale walencyjne elektronów miedzi typu d^9 mają mały promień, dlatego energia oddziaływania elektronów na tym samym atomie jest duża

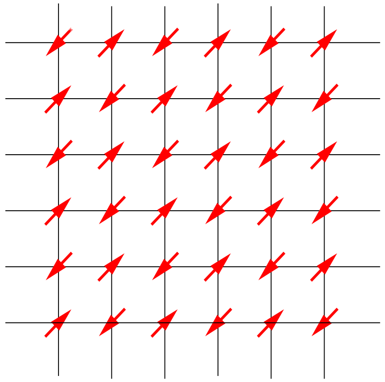


metal



izolator Motta

Antyferromagnetyczny izolator Motta

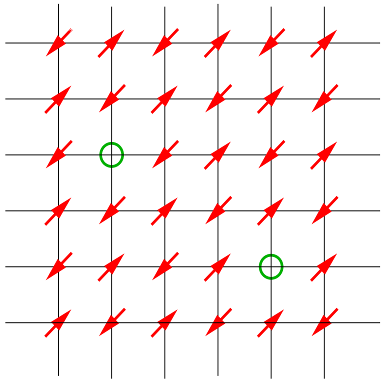


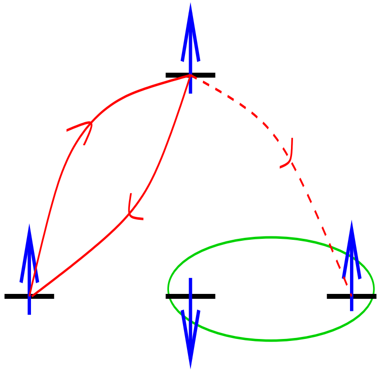
Energia oddziaływania >

energia kinetyczna !

$$H = t\sum_{ij\sigma} a^{\dagger}_{i\sigma} a_{j\sigma} + \frac{2t^2}{U}\sum_{ij}(\mathbf{S}_i \cdot \mathbf{S}_j - \tfrac{1}{4} n_i n_j)$$

Izolator Motta domieszkowany

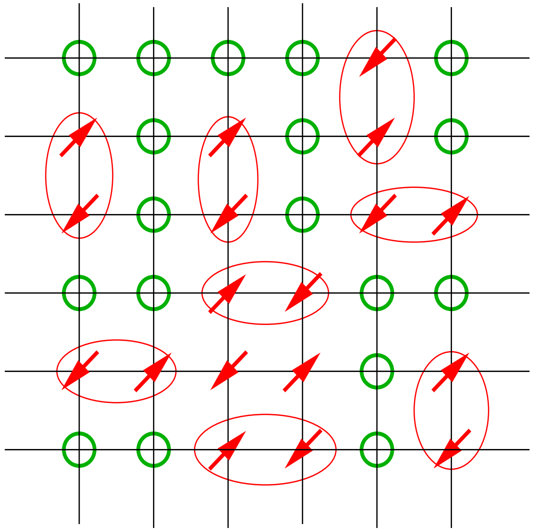




real-space pairing

Real–space pairing

Teoria RVB rezonujące wiązanie walencyjne

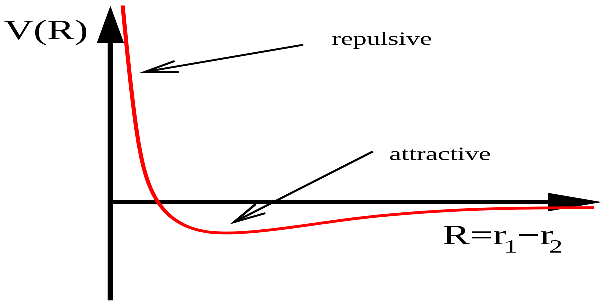


small coherence length

$$H = t \sum_{ij\sigma} a_{i\sigma}^{\dagger} a_{j\sigma} - \frac{2t^2}{U} \sum_{ijk} B_{ij}^{\dagger} B_{jk}$$

Microscopic theory of superfluid ^{3}He

He-He interaction of van der Waals type



Wave function of a pair with $2 \cdot (2L+1) \cdot (2S+1)$ amplitudes

$$\Psi \sim e^{i\mathbf{k}(\mathbf{r}_1+\mathbf{r}_2)}\Phi(\mathbf{r}_1-\mathbf{r}_2)\chi(\alpha,\beta)$$

Predictions for ^{3}He

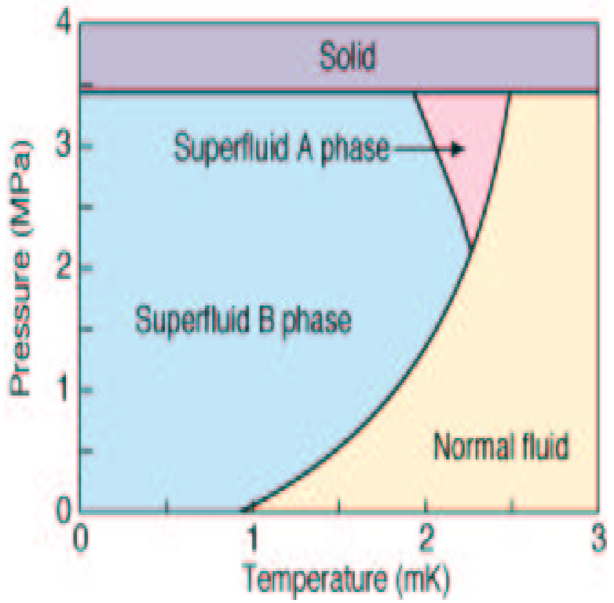
$$L = 1,\ 3,\ 5, \ldots \quad S = 1$$

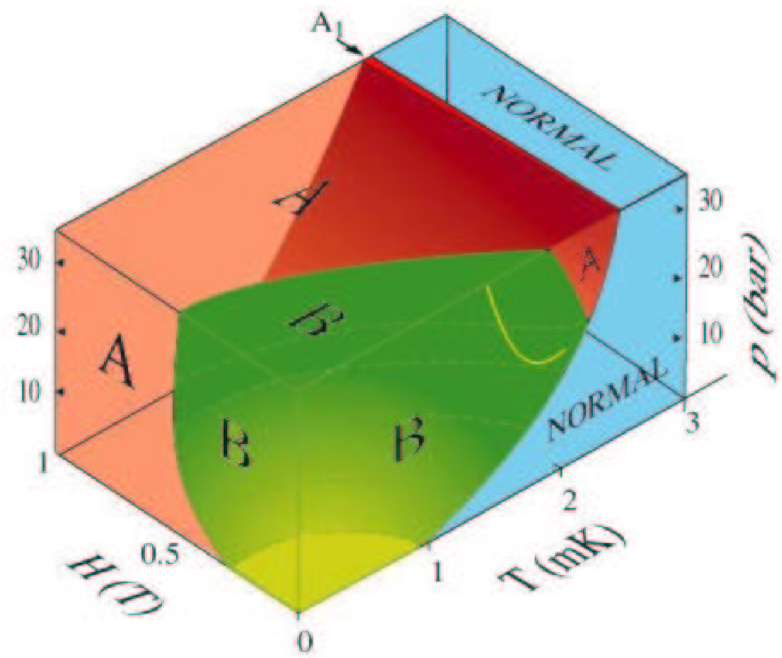
$$L = 2,\ 4\ , .., \quad S = 0$$

Microscopic theory of superfluid ^{3}He

Only experiment AND theory could resolve the problem

$$T_c^{\rm th} \sim T_c^{\rm exp}$$





Microscopic theory of superfluid ^{3}He

In all three phase pairing is with $S = 1$ and $L = 1$, so called p-pairing

orbital base: $p_x, \; p_y, \; p_z$
spin base: $s_x, \; s_y, \; s_z$

Order parameter with 18 components:

$$\hat{\Delta} = \begin{pmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{yx} & A_{yy} & A_{yz} \\ A_{zx} & A_{zy} & A_{zz} \end{pmatrix}$$

The symmetry group:

$$G = SO(3)_L \times SO(3)_S \times U(1)_\phi$$

Which symmetry breaking occurs in liquid ^{3}He?

Microscopic theory of superfluid ^{3}He

Introduced the concept of spin-orbit symmetry breaking

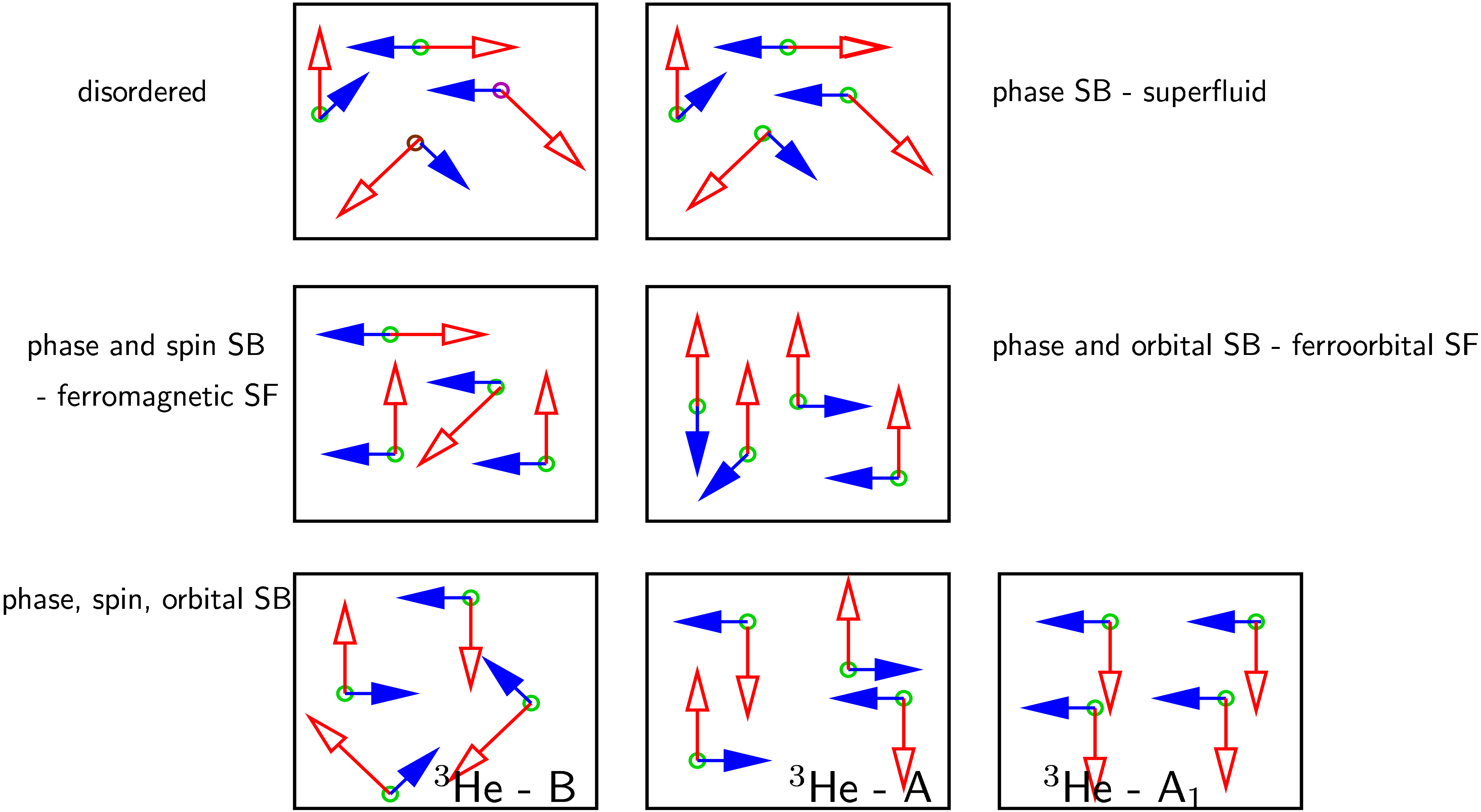
Developed microscopic theory combining BCS and NMR

Identified A, B, and A_1 phases with proper order parameter

Explained NMR spectra

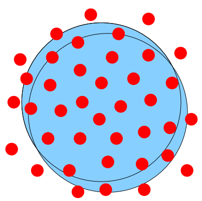
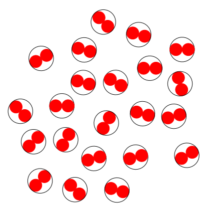
Microscopic theory of superfluid ^{3}He

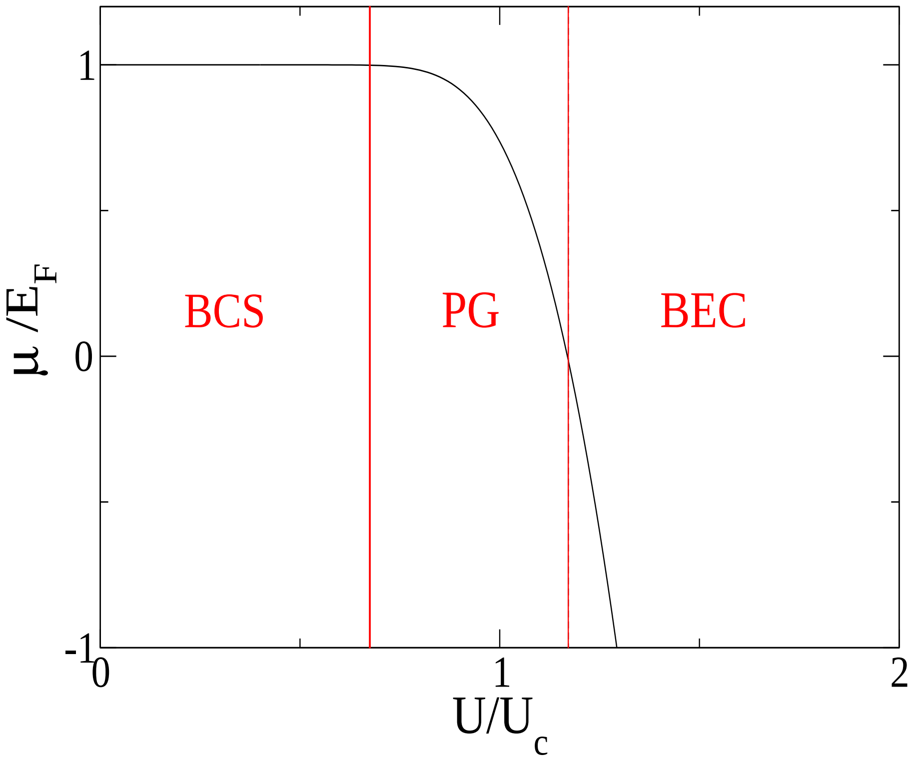
$$G = SO(3)_L \times SO(3)_S \times U(1)_\phi$$

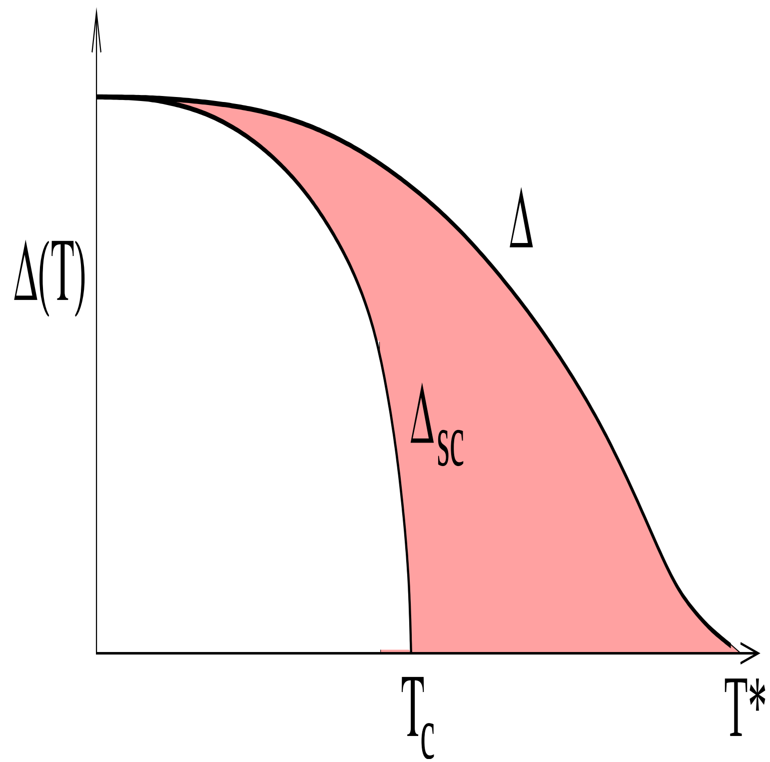


BEC–BCS crossover physics

BCS *vs* BOSE-EINSTEIN CONDENSATION

BCS	BEC
weak coupling ($\lvert U\rvert \ll \lvert U_c\rvert$)	strong coupling ($\lvert U\rvert \gg \lvert U_c\rvert$)
large pair size **k**-space pairing	small pair size **r**-space pairing
strongly overlapping Cooper pairs	ideal gas of preformed pairs
	
$T^* = T_c$	$T^* \gg T_c$





$$\Delta_{\mathbf{k}} = \frac{1}{N}\sum_{\mathbf{l}} V_{\mathbf{kl}} \frac{\Delta_{\mathbf{l}}}{2E_{\mathbf{l}}} \tanh\left(\frac{E_{\mathbf{l}}}{2k_B T}\right) \qquad E_{\mathbf{k}} = \sqrt{(\epsilon_{\mathbf{k}} - \mu)^2 + |\Delta_{\mathbf{k}}|^2}$$

$$1 - n = \frac{1}{N}\sum_{\mathbf{l}} \frac{\epsilon_{\mathbf{l}} - \mu}{E_{\mathbf{l}}} \tanh\left(\frac{E_{\mathbf{l}}}{2k_B T}\right)$$

Conclusions

BCS–BEC crossover
Different pairing mechanisms
Wide applications of pairing theory