Dynamical mean-field theory for correlated lattice bosons and fermions in normal and condensed phases

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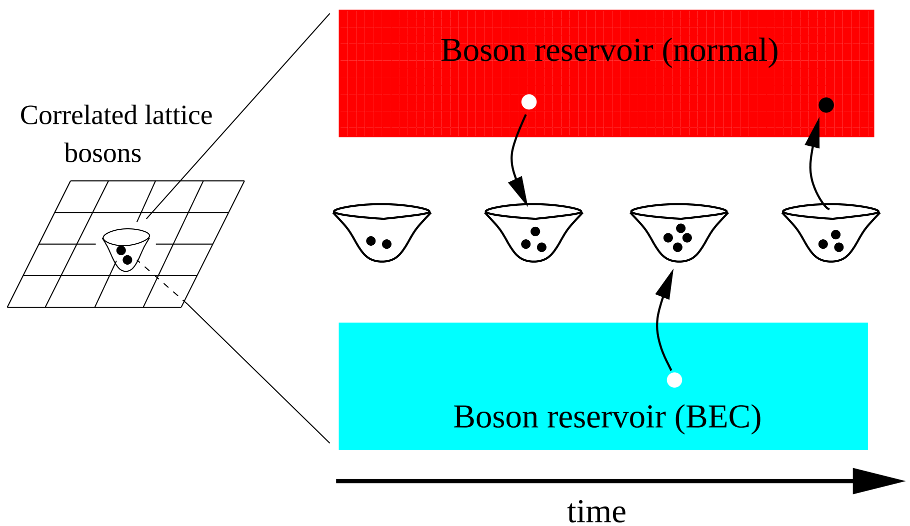
April 12th, 2007



Mean-field theory for lattice bosons and fermions

$$H = H^{\text{hopping}} + H_{\text{loc}}^{\text{interaction}}$$

- comprehensive (all input parameters, temperatures, all phases, ...)
- thermodynamically consistent and conserving
- provides exact solutions in certain non-trivial limit (large d)

$$\langle H \rangle, \quad \langle H^{\text{hopping}} \rangle, \quad \langle H_{\text{loc}}^{\text{interaction}} \rangle$$

are finite and generically non-zero, and

$$\langle \left[H^{\text{hopping}}, H_{\text{loc}}^{\text{interaction}} \right] \rangle \neq 0$$

to describe non-trivial competition

W. Metzner and D. Vollhardt (Phys. Rev. Lett. **62**, 324 (1989)) started DMFT for fermions by introducing scaling $t \to t^*/\sqrt{2d}$ and $d \to \infty$ limit

BEC and normal bosons on the lattice in $d \to \infty$ limit

Bosons can condense into a one-particle state and a singe scaling cannot yield a comprehensive mean-field theory in large d. We have introduced:

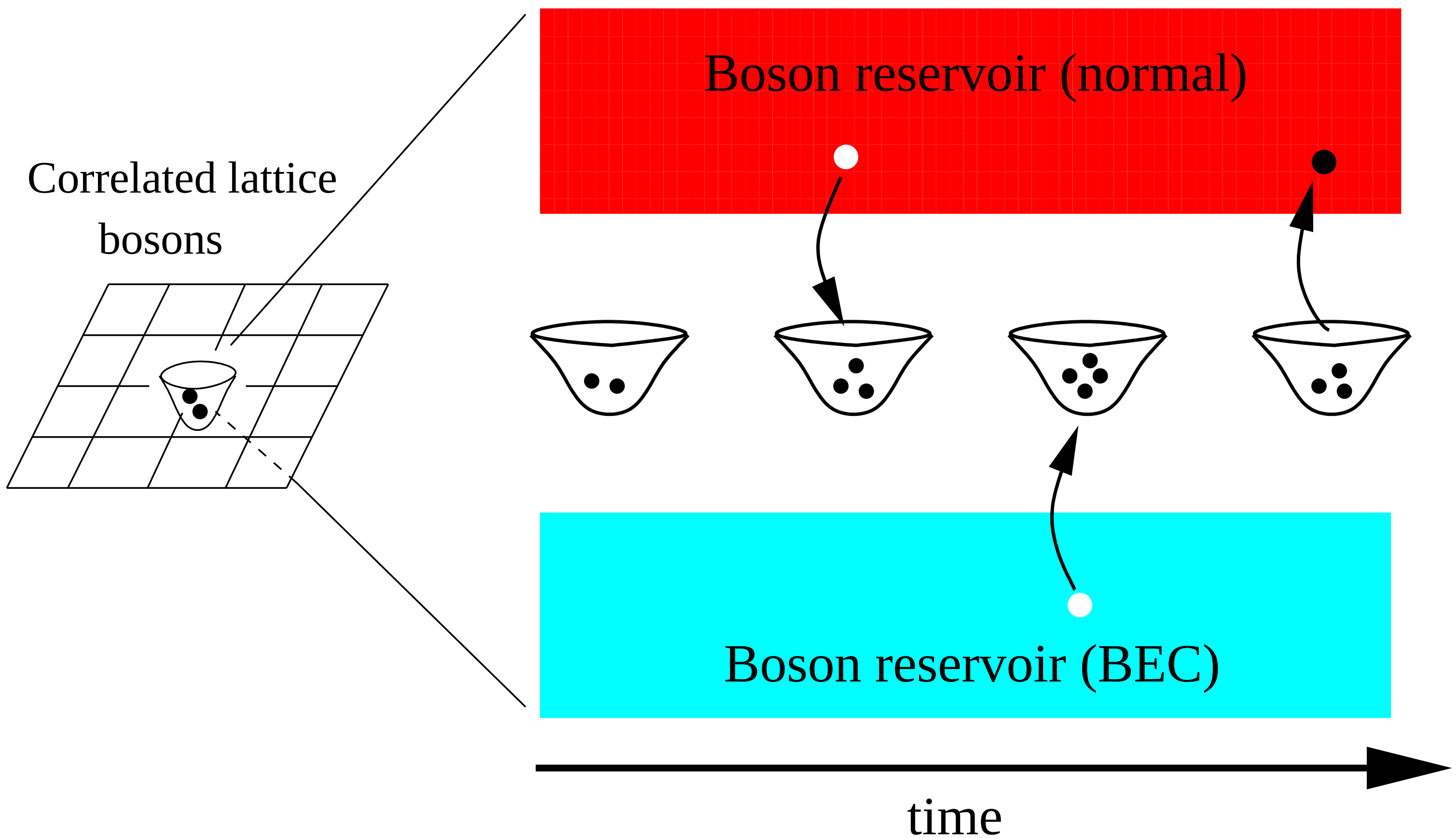
1. Scaling is made inside a thermodynamical potential (action, Lagrangian) but not at the level of the Hamiltonian operator
   - normal parts: $t_{ij} = \frac{t^*_{ij}}{(2d)^{\frac{||R_i - R_j||}{2}}}$ - fractional rescaling
   - BEC parts: $t_{ij} = \frac{t^*_{ij}}{(2d)^{||R_i - R_j||}}$ - integer rescaling
2. Limit $d \to \infty$ taken afterwards in this effective potential

Only this procedure gives consistent derivation of B-DMFT equations as exact ones in $d \to \infty$ limit for boson models with local interactions

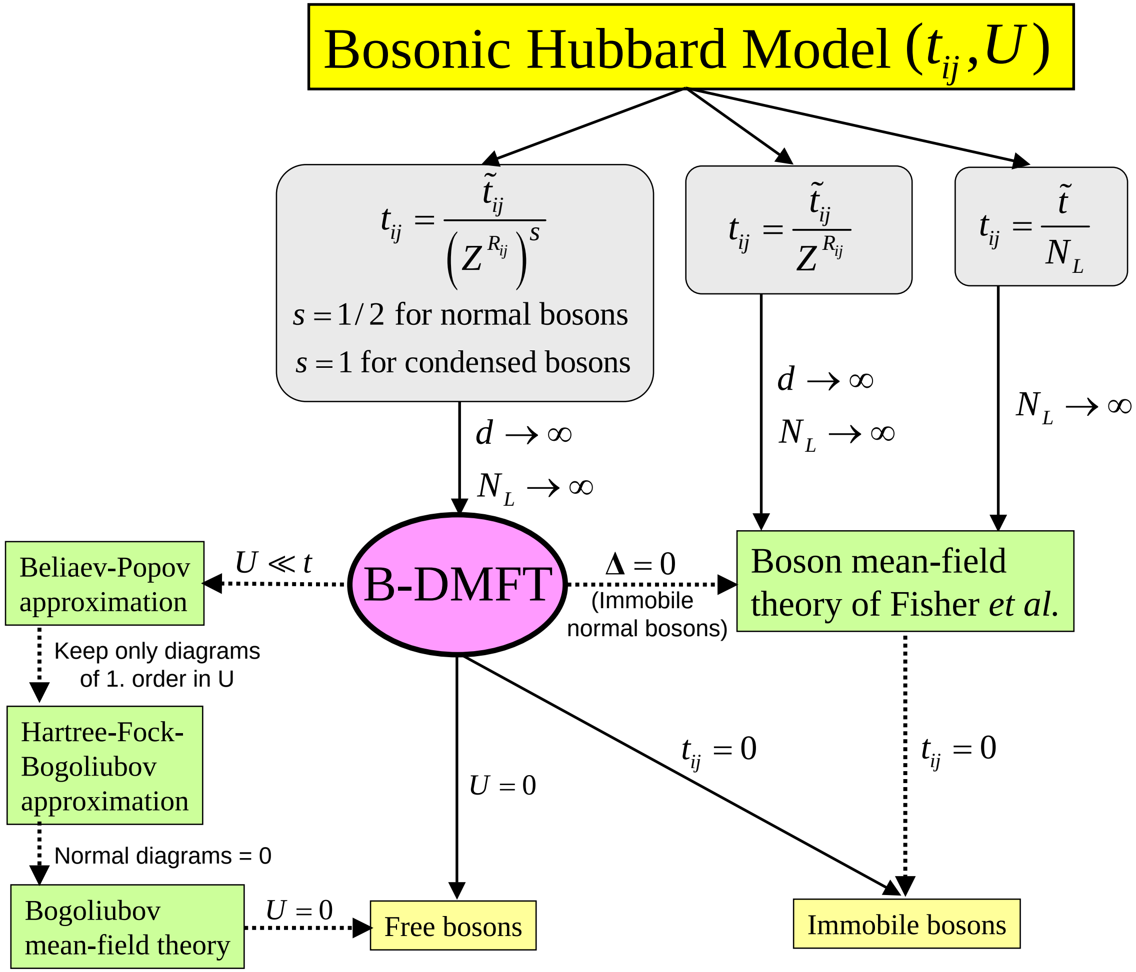
K. Byczuk and D. Vollhardt, arXiv:0706.0839

Bosonic-Dynamical Mean-Field Theory

- Exact mapping of the lattice bosons in infinite dimension onto a single site
- Single site coupled to two reservoirs: normal bosons and bosons in the condensate
- Reservoirs properties are determined self-consistently, local correlations kept



B-DMFT in well-known limits



Outlook

- Develop a bosonic impurity solver
  - Hyungjung Lee - bosonic numerical renormalization group
  - Philipp Werner - continuous time quantum monte-carlo
  - KB - linked cluster (cummulants) expansion
  - ...

- Extensive investigation of lattice bosons within B-DMFT

- DMFT for mixtures of bosons (^{87}Rb) and fermions (^{40}K) in (optical) lattices
  - Equations have been derived
  - Pairing and instabilities of boson due to fermions and vice versa
  - Impurity solvers to be developed