Supersolids

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To remember

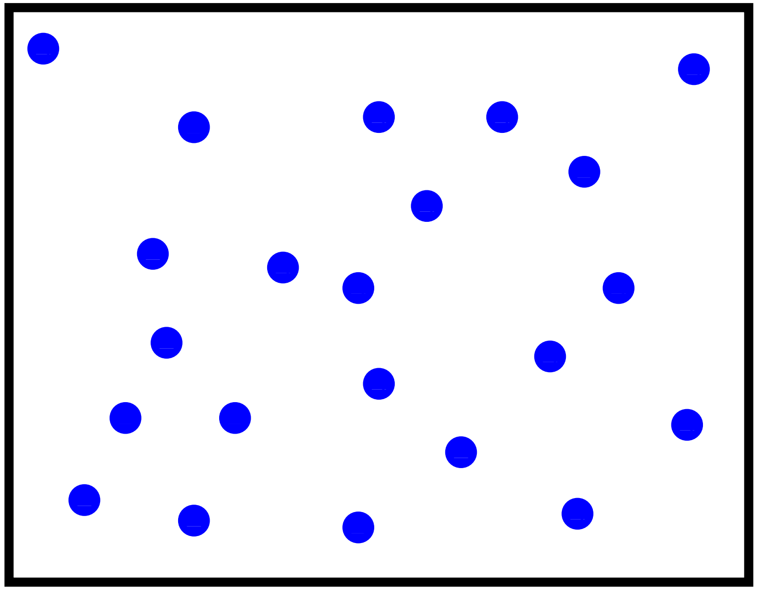
A system where its elements simultaneously

- have crystalline order, and
- flow like a liquid with no viscosity

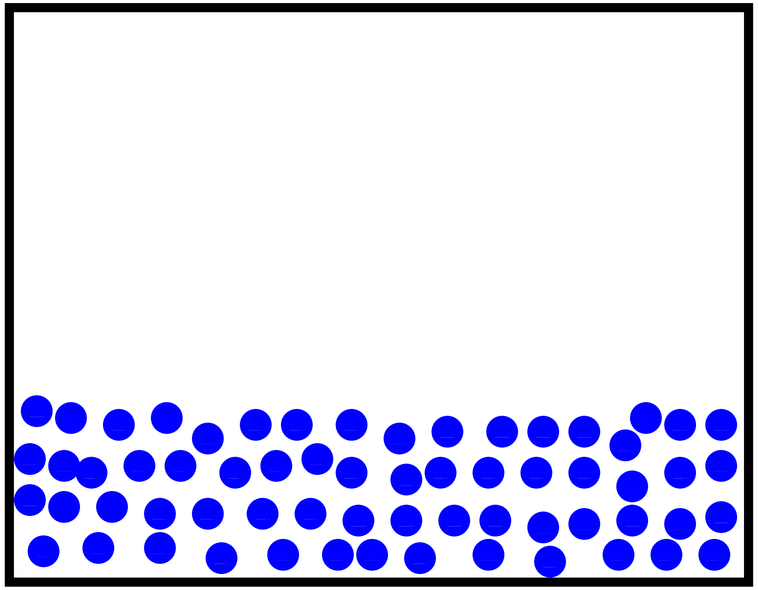
is called a supersolid

Classical states of matter

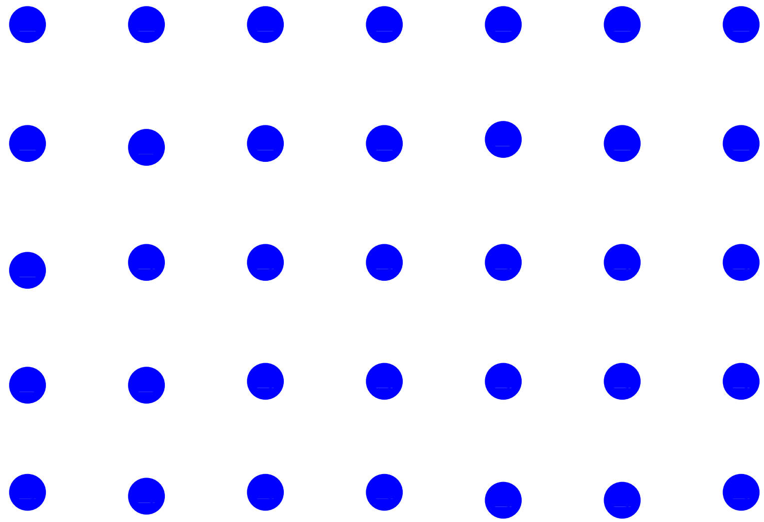
elements are basically distinguishable



gas - no ordering at all



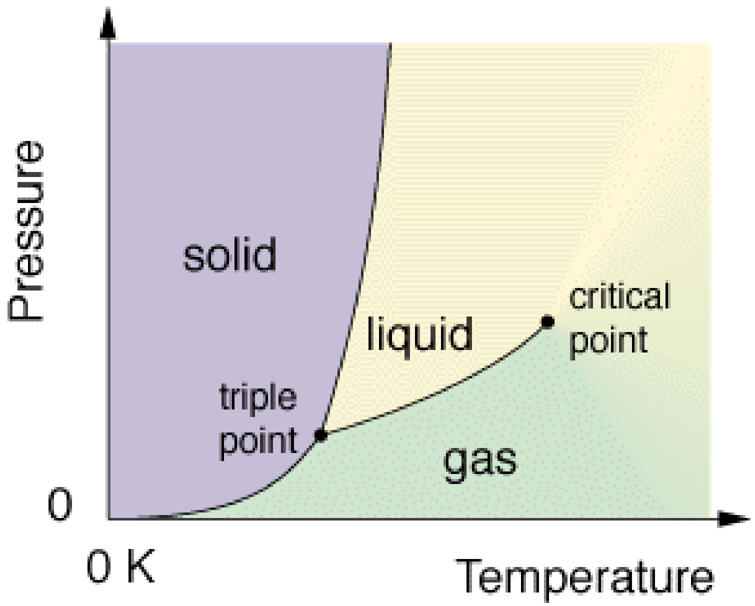
liquid - ordering on short range scale

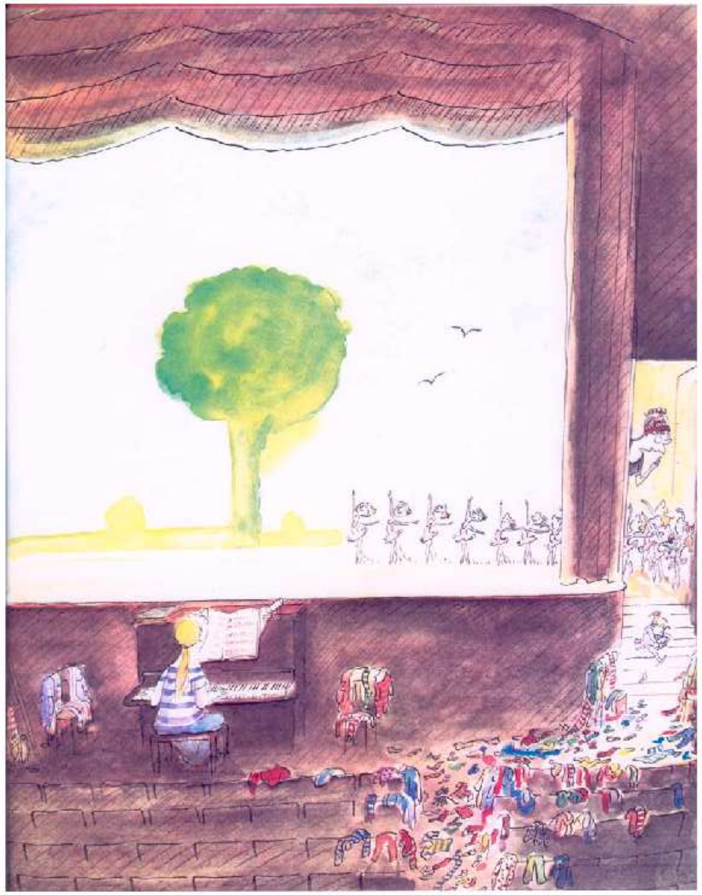


solid - ordering on any length scale

Long Range Order

Phase transitions - Order from chaos





Long Range Order (LRO)

$\langle \rho(\mathbf{r}_1)\rho(\mathbf{r}_2)\rangle \to \langle \rho(\mathbf{r}_1)\rangle\langle \rho(\mathbf{r}_2)\rangle \neq 0$ for $|\mathbf{r}_1 - \mathbf{r}_2| \to \infty$

crystal - long range correlations, rigidity

(Sempe)

Quantum states of matter

When does quantum mechanics start to play a role?

Thermal de Broglie wavelength: $\lambda_{dB} = \sqrt{2\pi\hbar^2/mk_BT}$

Typical distance between atoms: d

Quantum behavior if $\lambda_{dB} \gtrsim d$

Neon (Ne) at $T_c = 27$K: $\lambda_{dB} \approx 0.07$nm, $d \approx 0.3$nm

Helium (He) at $T_c = 4.2$K: $\lambda_{dB} \approx 0.4$nm, $d \approx 0.27$nm

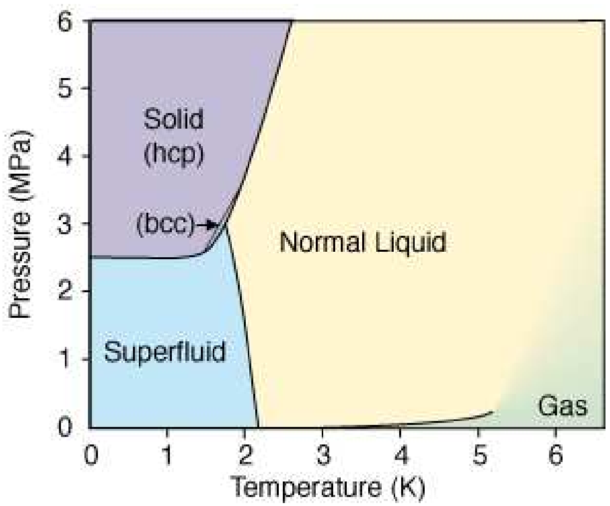
Quantum states of matter

atoms are indistinguishable - bosons and fermions

quantum gas, quantum liquid, superfluid, quantum crystal, supersolid, ...

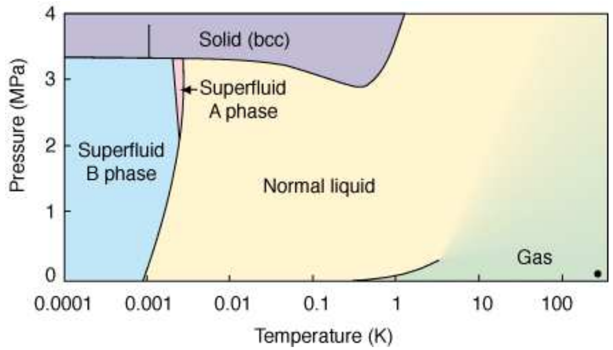
$$\lambda_{dB} \gtrsim d$$

Isotops: ^{4}He (2p2n2e - "bosons") and ^{3}He (2p1n2e - "fermions")



Quantum

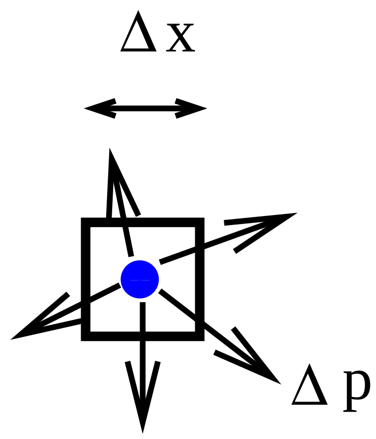
liquids at T=0K!



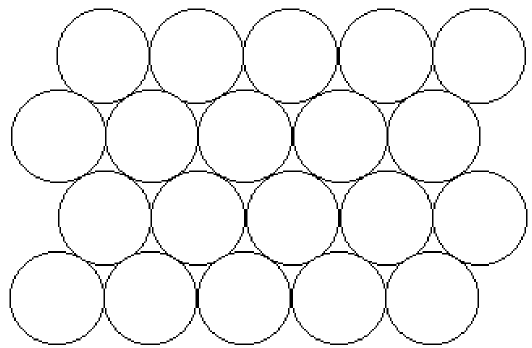
Why quantum liquids at T=OK?

- Heisenberg uncertainty principle

$$\Delta x \Delta p \gtrsim \frac{\hbar}{2}$$



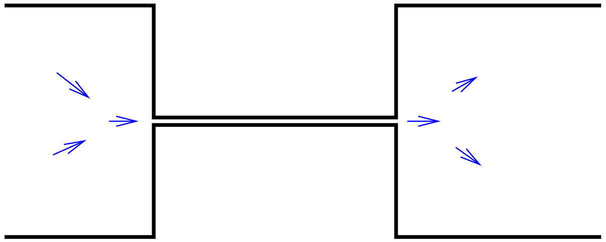
- the more localized particle in a space the fast it moves - zero point motions
- He at p_{atm} does not crystallize due to zero point motions
- kinetic energy is large as compared to $v(r) < 0$ potential
- He crystallizes at $p \gg p_{atm}$ due to hard core repulsion
- (as hard bowls form dense packed structure (hcp))



Superfluid quantum liquid $T \leqslant T_\lambda$

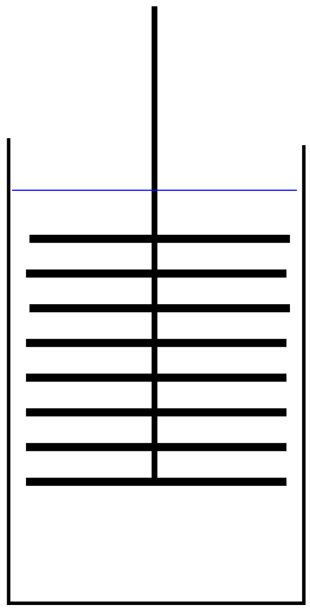
- frictionless flow through capillaries (superflow), creeping, fountain effect, ...

P. Kapitsa, Nature **141**, *74 (1938); J.F. Allen and D. Meisener, ibid., 75 (1938); E.L. Andronikashvili 1946*



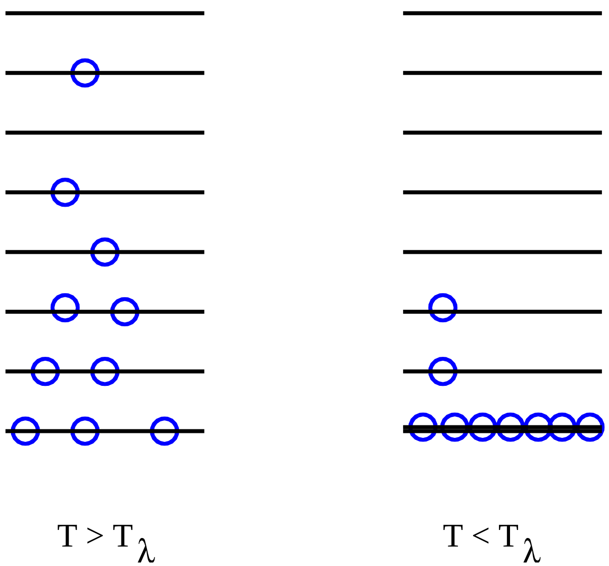
$$\frac{\Delta p}{L} \sim \eta \frac{v}{R^2}$$

$\Delta p = 0$ and $v > 0$ means zero viscosity $\eta = 0$



Microscopic theory of superfluid ^{4}He

Type of Bose - Einstein condensation of interacting bosons



- macroscopic number N_0 of bosons are in the lowest energy state
- N_0 bosons have the same wave function
- one-particle density matrix

$$\rho(\mathbf{r}-\mathbf{r}') = \langle \psi^\dagger(\mathbf{r})\psi(\mathbf{r}')\rangle$$

describes long-range order

$$\langle \psi^\dagger(\mathbf{r})\psi(\mathbf{r}')\rangle \rightarrow \langle \psi^\dagger(\mathbf{r})\rangle\langle \psi(\mathbf{r}')\rangle = N_0/V = n_0$$

gdy $\mathbf{r}-\mathbf{r}' \rightarrow \infty$

Off-diagonal long-range order (ODLRO)

Macroscopic wave function of the condensate

- there exists a wave function of the condensate

$$\Psi(\mathbf{r};T) = \sqrt{\rho_s(\mathbf{r})}e^{i\theta(\mathbf{r})}$$

order parameter

- flow velocity of the condensate ($\rho_s = \mathrm{const.}$)

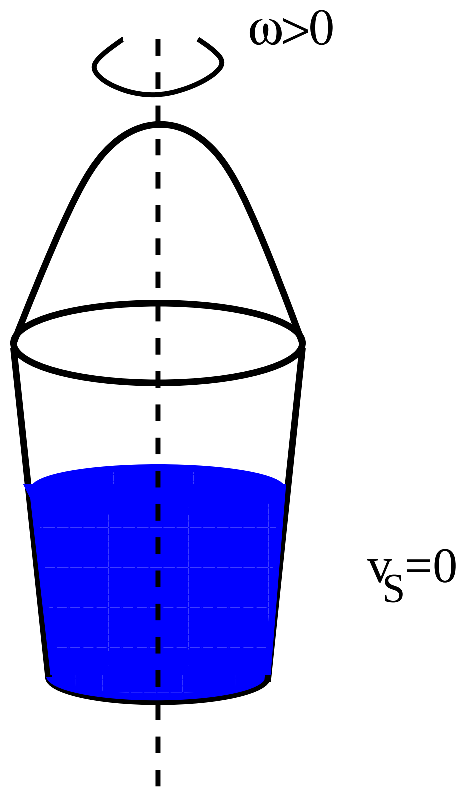
$$\mathbf{v}_s = \frac{\hbar}{m}\nabla\theta(\mathbf{r})$$

- in 'no holes space' flow is rotationless

$$\nabla \times \mathbf{v}_s = 0$$

- in rotating vessel with ^{4}He, superfluid does not move

- non-classical moment of inertia (NCRI)



London 1954, Hess i Fairbank 1967

Non-classical moment of inertia (NCRI)

Free energy

$$F(\Omega) = F_0 + \frac{1}{2} I_{\text{class}} \Omega^2$$

with F_0 free energy at $\Omega = 0$ and moment of inertia $I_{\text{class}} = NmR^2$

NCRI means that at small Ω new term appears

$$\Delta F(\Omega) = -\frac{1}{2} \frac{\rho_s}{\rho} I_{\text{class}} \Omega^2$$

Two fluid system (normal and superfluid) has smaller moment of inertia

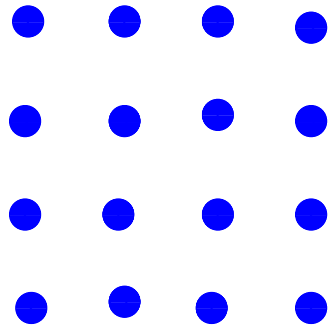
$$I_{\text{NCRI}} = \left(1 - \frac{\rho_s}{\rho}\right) I_{\text{class}}$$

Quantum crystals

- solid He at high pressure (25atm for ^{4}He and 30atm for ^{3}He)

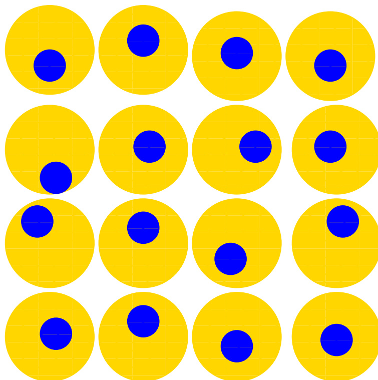
- amplitude of zero motions is 30% of a lattice constant

classical crystal - small zero motions $\Delta x \ll a$



quantum crystal - large zero motions $\Delta x \gg a$

- in quantum crystals atoms are indistinguishable



Quantum crystals - Supersolids

Can a quantum crystal be superfluid?

M. Wolfke, Ann. Acad. Sci. Techn. Varsovie **6**, *14 (1939)*

diagonal long-range order

$$\langle \rho(\mathbf{r}_1)\rho(\mathbf{r}_2)\rangle \rightarrow \langle \rho(\mathbf{r}_1)\rangle\langle \rho(\mathbf{r}_2)\rangle \neq 0$$

off-diagonal long-range order

$$\langle \psi^\dagger(\mathbf{r})\psi(\mathbf{r}')\rangle \rightarrow \langle \psi^\dagger(\mathbf{r})\rangle\langle \psi(\mathbf{r}')\rangle \neq 0$$

for $|\mathbf{r}_1 - \mathbf{r}_2| \rightarrow \infty$

Solid-state rigidity and fluidity with zero viscosity

Supersolid

Quantum crystals - Commensurate insulators

Insulating variational wave function (*O. Penrose, L. Onsager 1951, 1956; C.N. Yang 1962*)

$$\Psi_G^{PO}(\mathbf{r}_1, ..., \mathbf{r}_N) = \sqrt{\frac{1}{N!}} \sum_P \prod_{j=1}^{N} W(\mathbf{R}_j - \mathbf{r}_{Pj})$$

with $W(\mathbf{R}_j - \mathbf{r}_{Pj})$ localized Wanner wave functions, $N_L = N$

$$\langle \psi^\dagger(\mathbf{r})\psi(\mathbf{r}')\rangle = \frac{1}{N} \sum_{j=1}^{N} W^*(\mathbf{R}_j - \mathbf{r}) W(\mathbf{R}_j - \mathbf{r}') \to 0$$

no ODLRO (some delocalization, exchange is needed)

Quantum crystals - Commensurate supersolids

Bose-Einstein condensate variational wave function with exchange
(*L. Reatto 1969, G.V. Chester 1970, A. Leggett 1970*)

$$\Psi_G^{BEC}(\mathbf{r}_1, ..., \mathbf{r}_N) = \prod_{i=1}^{N} \left(\frac{1}{\sqrt{N}} \sum_{j=1}^{N} W(\mathbf{R}_j - \mathbf{r}_i) \right)$$

density wave modulation with macroscopic condensation of all N bosons in one-particle state $\phi(\mathbf{r}) = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} W(\mathbf{R}_j - \mathbf{r})$

$$\langle \psi^\dagger(\mathbf{r})\psi(\mathbf{r}')\rangle \rightarrow \rho_s \phi^*(\mathbf{r})\phi(\mathbf{r}')$$

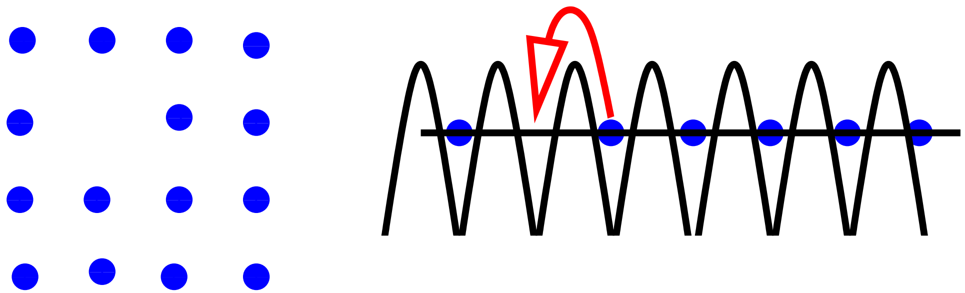
Zero probability to occur in Nature a commensurate supersolid in continuous space
(*N. Prokof'ev, B. Svistunov 2005*)

$$N_L = N \times k$$

condition can happen by accident or be designed in laboratory (cold atoms)

Incommensurate quantum crystals - vacancies

a vacancy moves backward as compared with movements of atoms

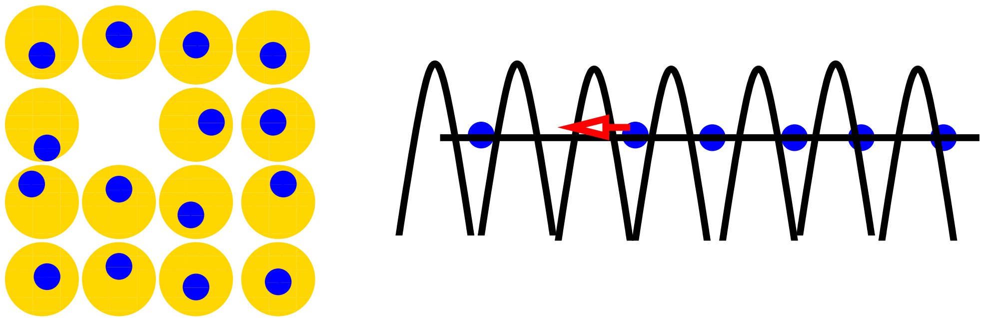


classical crystal

to overcome energy barier

activation energy Δ needed

$$p \sim \exp(-\Delta/k_B T)$$



quantum crystal

atoms tunnel

vacancy is not static

vacancy becomes a quasiparticle

vacanson, defecton

$$|\mathbf{k}\rangle = \sum_i e^{i\mathbf{k}\mathbf{R}_i}|\mathbf{R}_i\rangle$$

coherent quasiparticles with dispersion $\epsilon(\mathbf{k})$

Incommensurate quantum crystals - Supersolid

Andreev, Lifshitz (1969), Chester (1970), Leggett (1970), ...

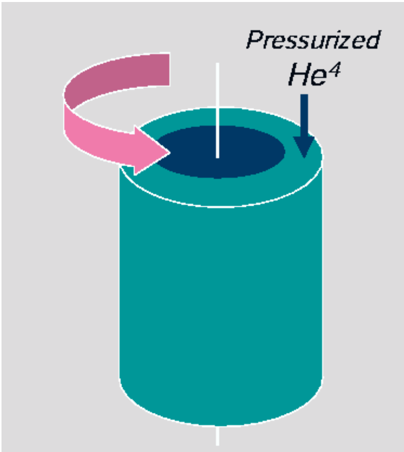
- defectons in solid ^{4}He are bosons
- BEC condensation of defectons below T_c
- BEC defectons are superfluid
- BEC defectons do not move with the system
- moment of inertia should be reduced

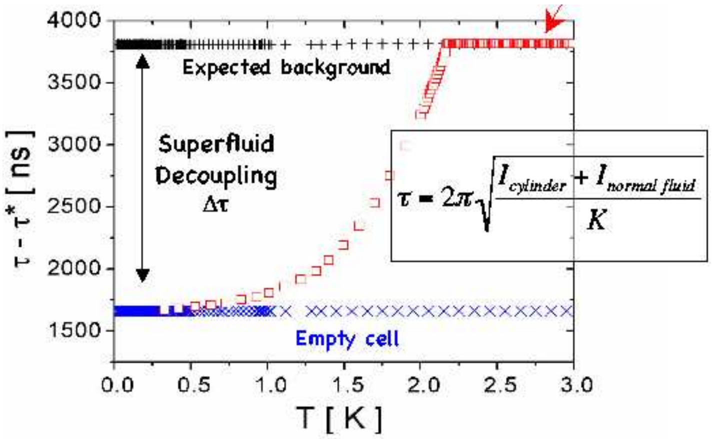
$$I(T) = I_{\rm class} \cdot \left(1 - \frac{\rho_s(T)}{\rho}\right)$$

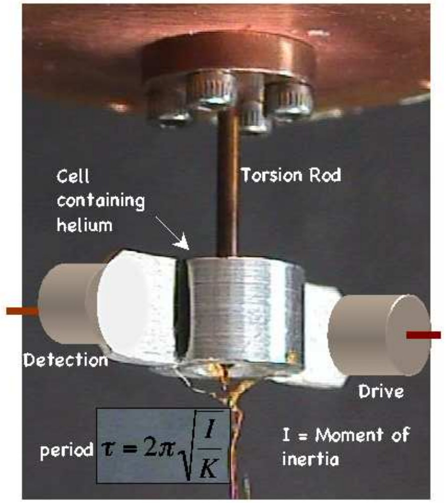
supersolid = LRO + ODLRO

“Mary goes round” experiments

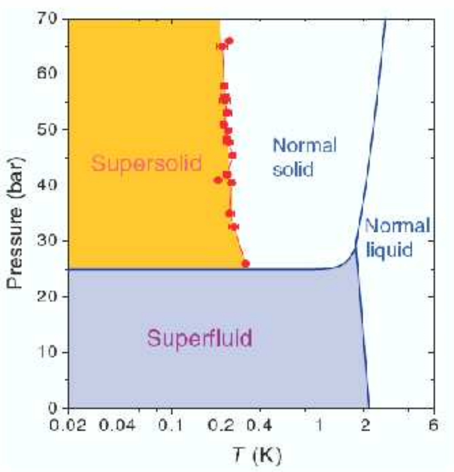
E. Kim, M.H. Chan, Nature (2004), Science (2004), J. Low Temp. Phys. (2005)

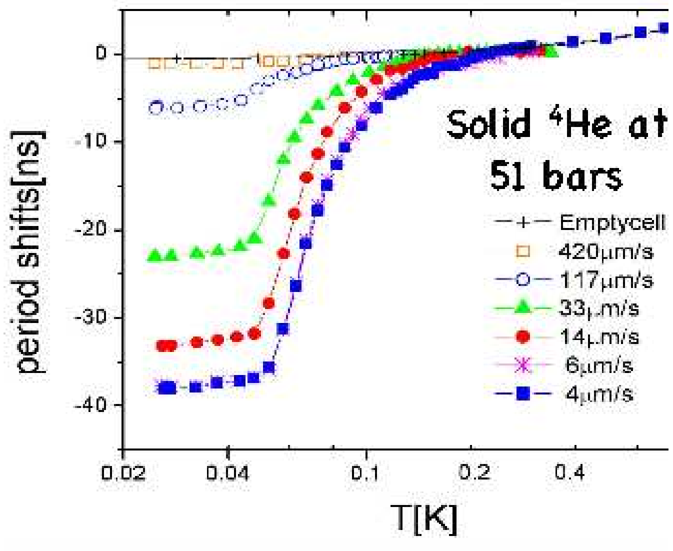


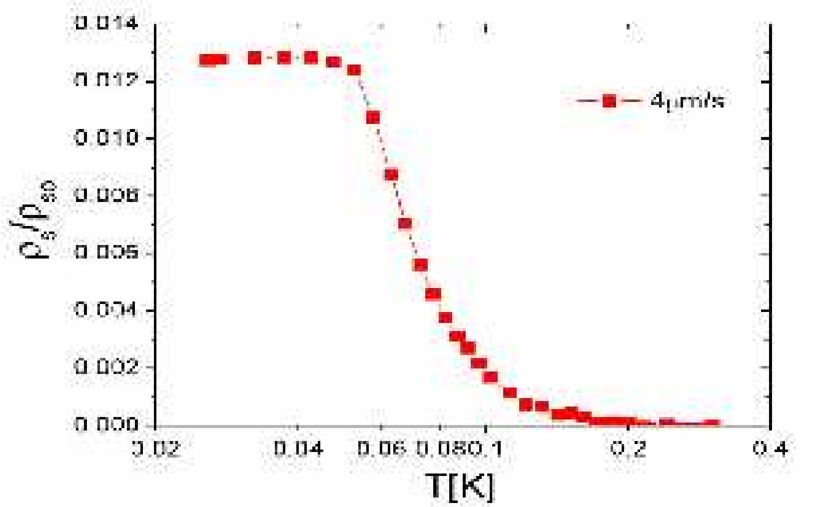




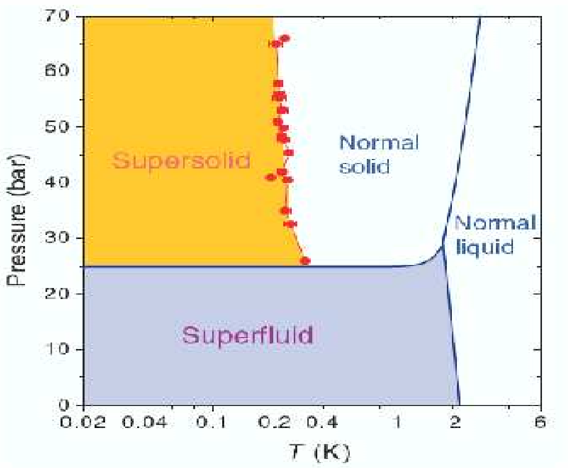
in solid ^{4}He below 0.2K decrease of oscilation period - moment of inertia decreases

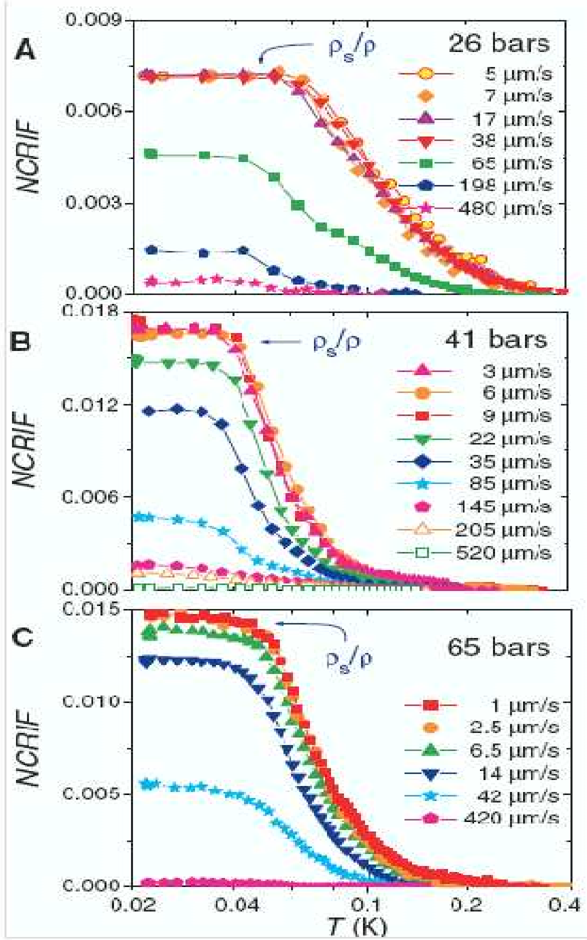






Supersolid?





in Kim and Chan's experiments - ca. 1,3% of a crystal is a supersolid

problems: in solid helium number of defects is orders of magnitudes smaller!

commensurate supersolid, phase separation, defects, microcrystals, vortices, glass ...???

Experiment positively reproduced in 3 other laboratories but with history depend results!

Conclusions and outlooks

- supersolid combines crystal rigidity and superfluidity
- idea applies to H, He, cold atoms in MOT and optical lattices, or atom impurities in crystal hosts, and superconducting electrons
- Kim and Chan's experiments still call for a correct interpretation ...
- ... but they have certainly renewed an interest in supersolids

Reading:

1. E. Kim, M.H. Chan, Nature **425**, 227 (2004); Science **305**, 1941 (2004).
2. Physics Today, November 2004, p. 23.
3. Adv. in Physics **56**, 381 (2007); Contemporary Phys. **48**, 31 (2007); Science **319**, 120 (2008).
4. A. Leggett, *Quantum liquids* (Oxford Univ. Press 2006).
5. K. Byczuk, Postepy Fizyki **58**, 194 (2007).