What was the Nobel Price in 2003 given for?

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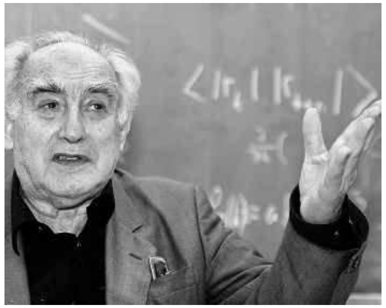
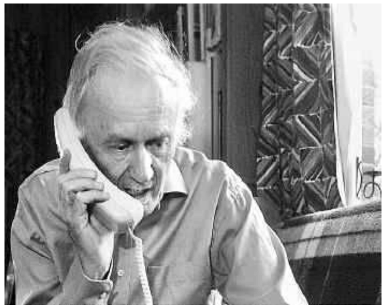
2003 Nobel Trio

Alexei A. Abrikosov, born 1928 (75 years) in Moscow, the former Soviet Union, American (and Russian) citizen.



Vitaly L. Ginzburg, born 1916 (87 years) in Moscow, Russia (Russian citizen).



Anthony J. Leggett, born 1938 (65 years) in London, England (British and American citizen).

“for pioneering contributions to the theory of superconductors and superfluids”

To remember:

- Ginzburg - effective field theory with complex order parameter
- Abrikosov - topological defects
- Leggett - spin-orbital symmetry breaking

Plan of the talk

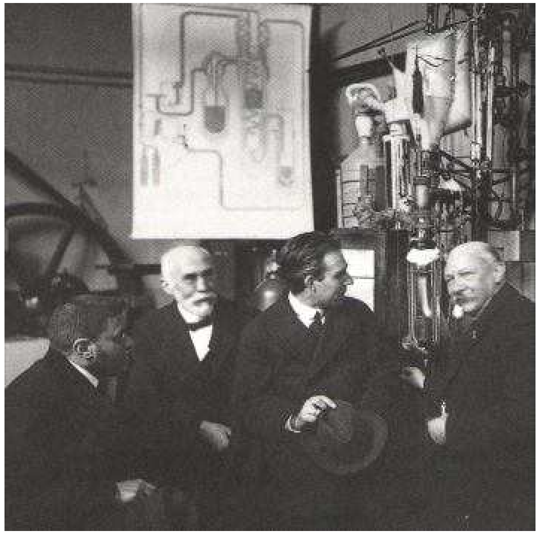
1. Definition of superconductors (SC) and superfluids (SF)
2. Properties of SC and SF
3. BCS theory for SC and SF
4. Phases of ^{3}He and Leggett's work
5. Ginzburg - Landau equations for SC
6. Type II superconductors
7. Vertex, vertex lattice, and Abrikosov's work
8. Present researches in SC and SF - selected view

Super - Conductor / Fluid

SC and SF are twin phenomena for charge and neutral particles

Discovery of superconductivity

1911 - Heike Kamerlingh Onnes (Leiden - Holland)



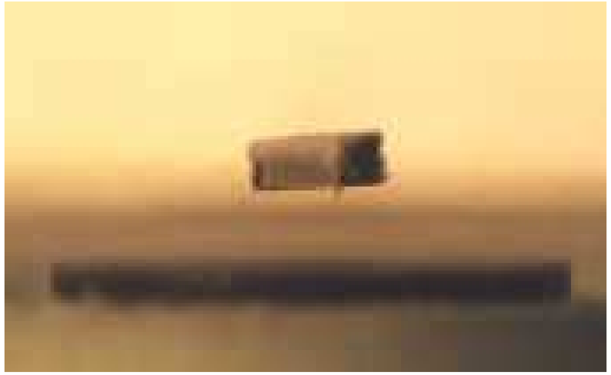
- Below $T_c = 4.15$ K resistivity of mercury drops down to zero
- Other superconductors: Al, In, Nb, Pb, ...

Nobel price - 1913

Meissner effect

1933 - Meissner and Ochsenfeld (Berlin - Germany)

Below T_c weak magnetic field is completely expelled from sample

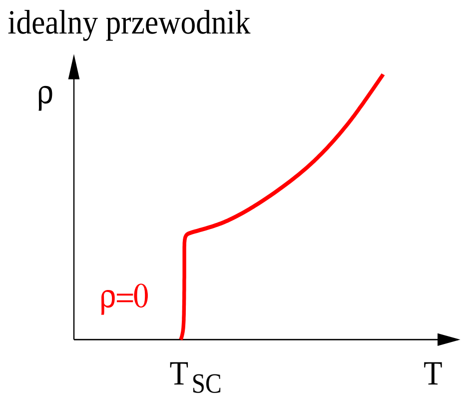


levitating superconductor

levitating magnet

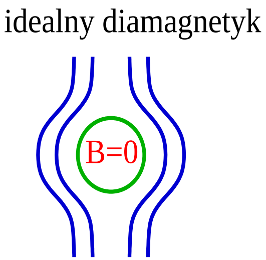
What is superconductivity

superconductor $\longrightarrow$ "*zero, infinity, ideal, perfect*"



zero resistivity: $\rho = 0$

infinit conductivity: $\sigma = \infty$



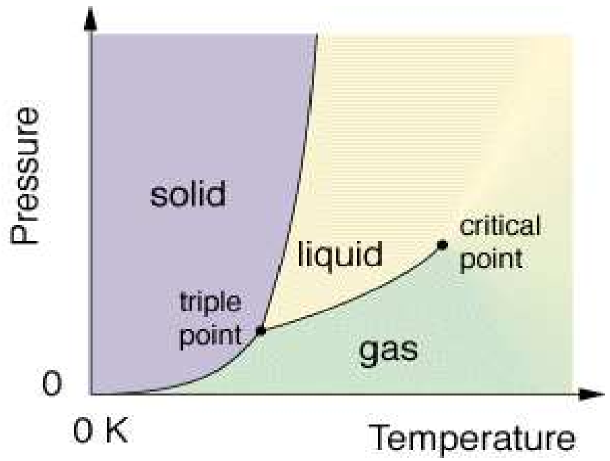
ideal diamagnet: $\chi = -1$

nadprzewodnik

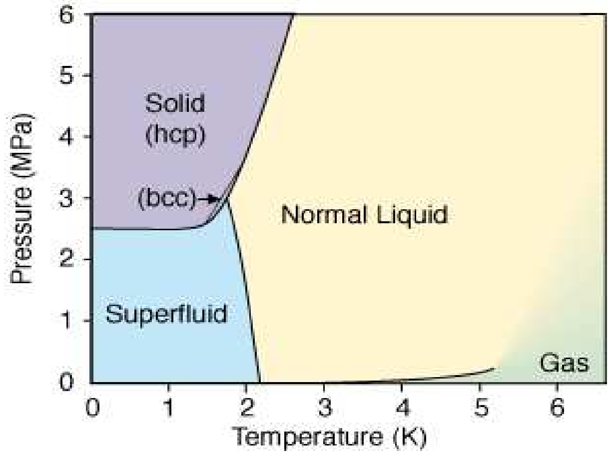
One of the perfect state of matter in the Universe!

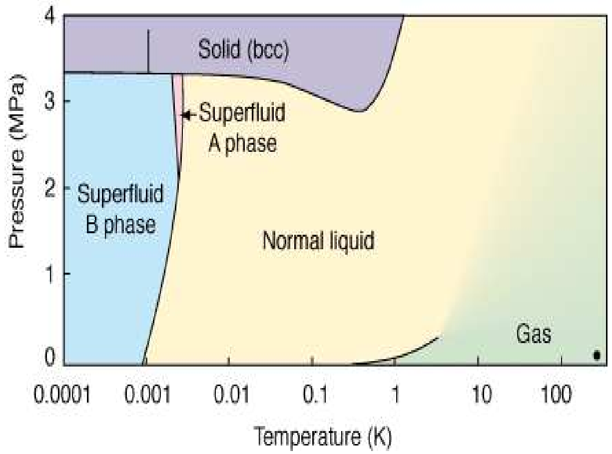
Discovery of superfluidity

Two isotopes of Helium:
^{4}He (2p2n2e - "boson") and ^{3}He (2p1n2e - "fermion")



Cooling ordinary gas / liquid





Quantum liquids up to $T = 0$K

Cooling ^{4}He and ^{3}He

Discovery of superfluidity

^{4}He

- He liquid - Kamerling Onnes 1908
- He solid under pressure - F. Simon 1934
- λ transition - W.H. Keesom 1932
- He film creeping - J.G. Daunt, K. Mendelssohn 1939
- frictionless flowing, perfect heat conductor - P. Kapitza 1938, 1941
- fountain effect - J.F. Allen, H. Jones 1938
- ...

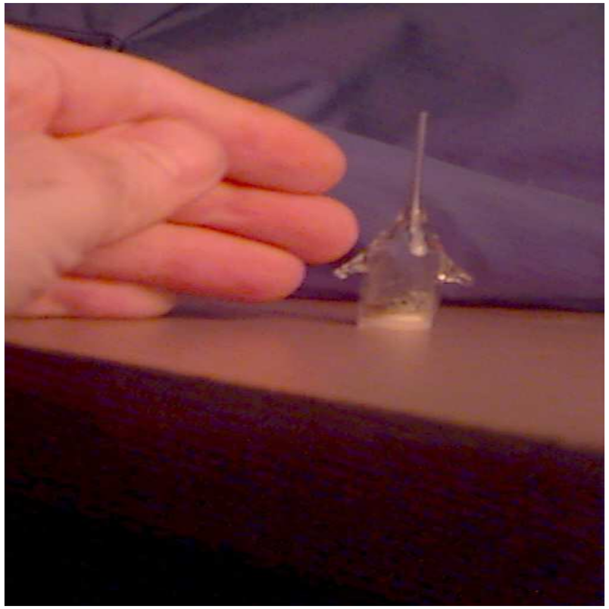
Nobel price - Kapitza 1978

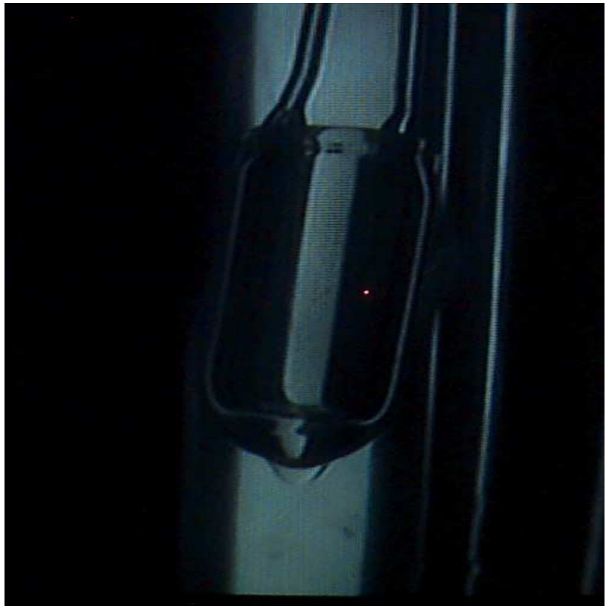
^{3}He

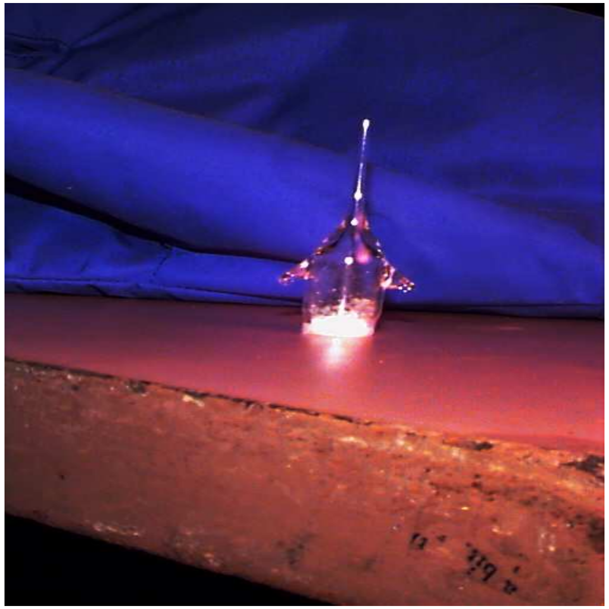
- superfluidity - D.D. Osheroff, R.C. Richardson, D.M. Lee 1972
- theory - A.J. Leggett 1972
- ...

Nobel prices - ORL 1996, Legget 2003

Discovery of superfluidity

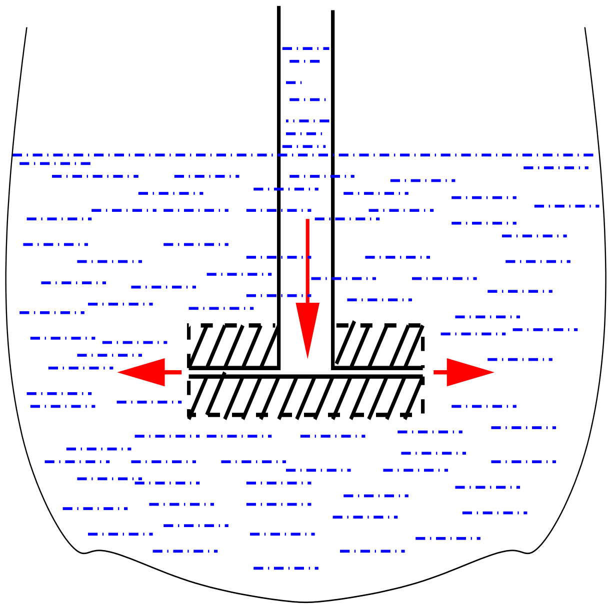


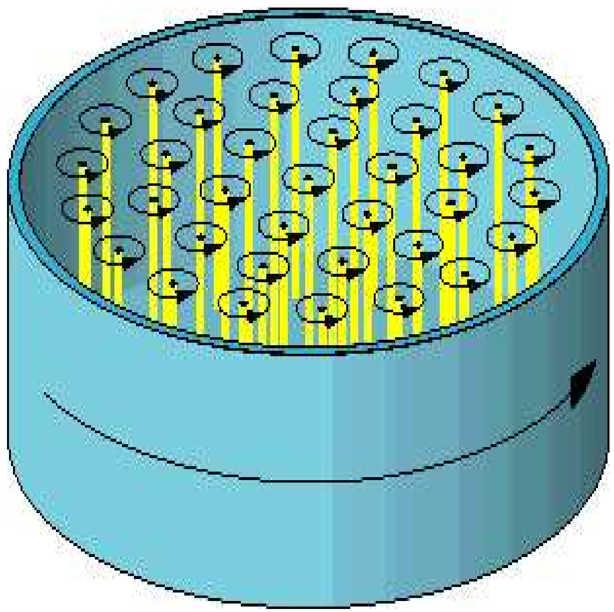




What is superfluidity?

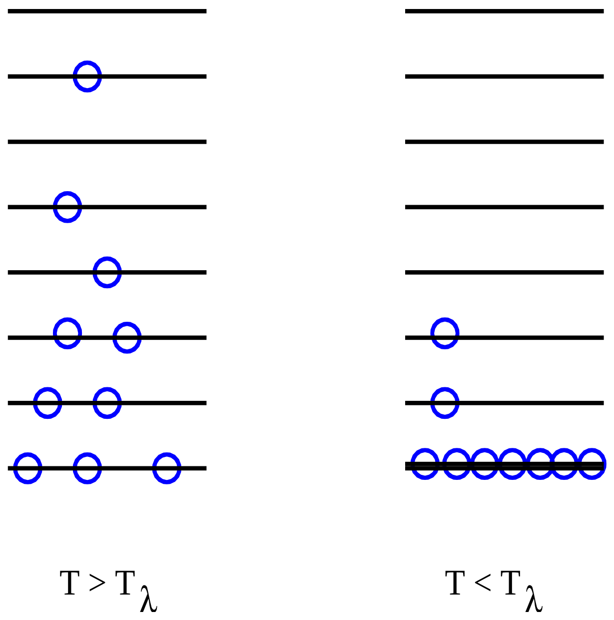
- the ability to flow through microscopic passages with no apparent friction;
- the quantization of vortices;
- the ability to support four wave modes





Microscopic theory of ^{4}He

Some kind of Bose - Einstein condensation of interacting particles



Landau, Feynman, Huang, Bogoliubov, ...,E.Lieb, J. Piasecki

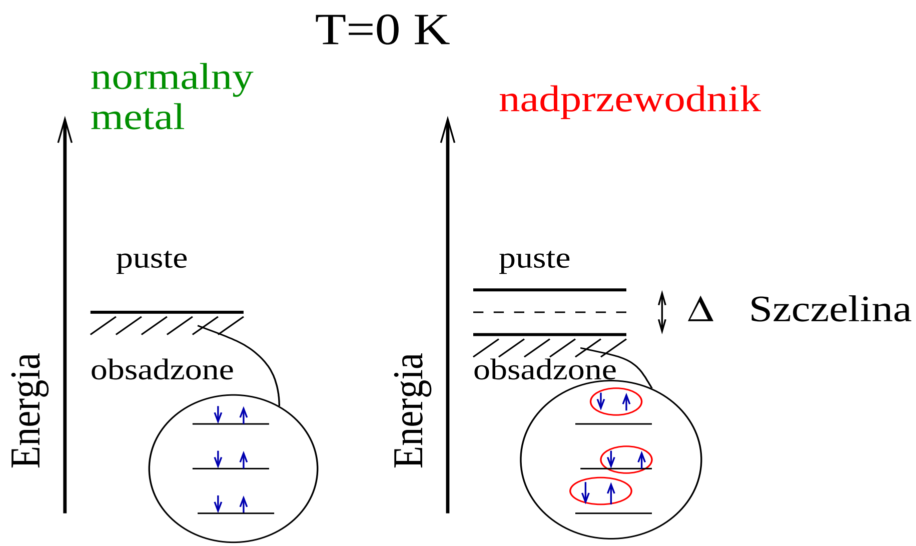
Microscopic theory of superconductors

1957 - J. Bardeen, L. Cooper, R. Schrieffer Nobel price -1972

Instability of Fermi gas/liquid

Pairs of fermions (Cooper pairs) condensate at T_c

New thermodynamic state of matter for $T \leq T_c$



BCS model of superconductivity

$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} a^{\dagger}_{\mathbf{k}\sigma} a_{\mathbf{k}\sigma} + \sum_{\mathbf{k}\,\mathbf{k}'} V_{\mathbf{k}\,\mathbf{k}'} a^{\dagger}_{\mathbf{k}\uparrow} a^{\dagger}_{-\mathbf{k}\downarrow} a_{-\mathbf{k}'\downarrow} a_{\mathbf{k}\uparrow}$$

with attractive potential

$$V_{\mathbf{k}\,\mathbf{k}'} < 0 \quad \text{near the Fermi level}$$

BCS used many-body variational wave - function

$$|\Psi\rangle = \prod_{\mathbf{k}} \left(u_{\mathbf{k}} + v_{\mathbf{k}} a^{\dagger}_{\mathbf{k}\uparrow} a^{\dagger}_{-\mathbf{k}\downarrow} \right) |\text{Fermi sea}\rangle$$

Singlet pairing superconductivity $S = 0$, $S_z = 0$!

Energy spectrum with a gap

$$E_{\mathbf{k}} = \sqrt{(\epsilon_{\mathbf{k}} - \mu)^2 + |\Delta_{\mathbf{k}}|^2}$$

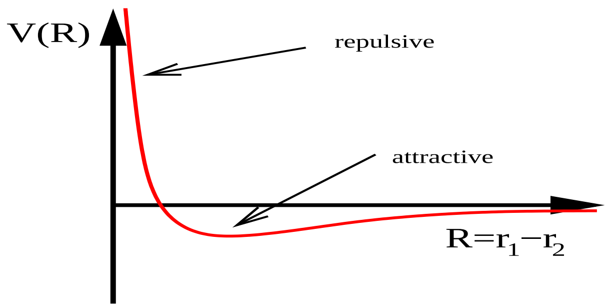
where

$$\Delta_{\mathbf{k}} = \sum_{\mathbf{l}} V_{\mathbf{kl}} \frac{\Delta_{\mathbf{l}}}{2E_{\mathbf{l}}} \tanh\left(\frac{E_{\mathbf{l}}}{2k_B T}\right)$$

$\Delta_{\mathbf{k}} \neq 0$ at $T < T_c$ is the SC order parameter!

Microscopic theory of superfluid ^{3}He

He-He interaction of van der Waals type



Wave function of a pair with $2 \cdot (2L+1) \cdot (2S+1)$ amplitudes

$$\Psi \sim e^{i\mathbf{k}(\mathbf{r}_1+\mathbf{r}_2)}\Phi(\mathbf{r}_1 - \mathbf{r}_2)\chi(\alpha, \beta)$$

Predictions for ^{3}He

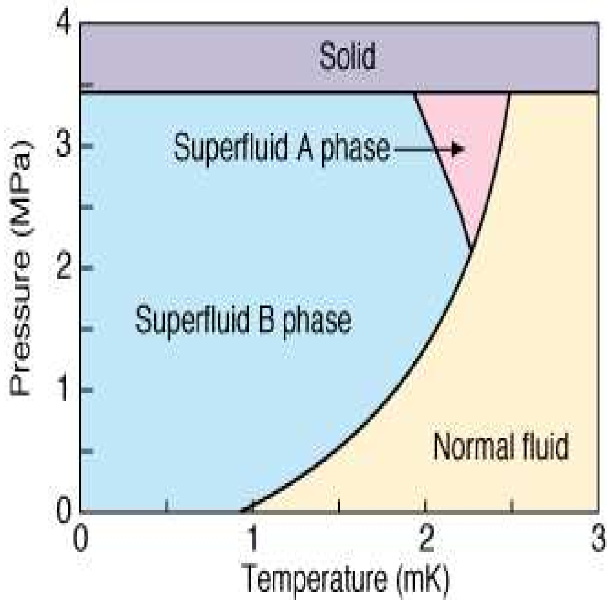
$$L = 1,\ 3,\ 5, \ldots \quad S = 1$$

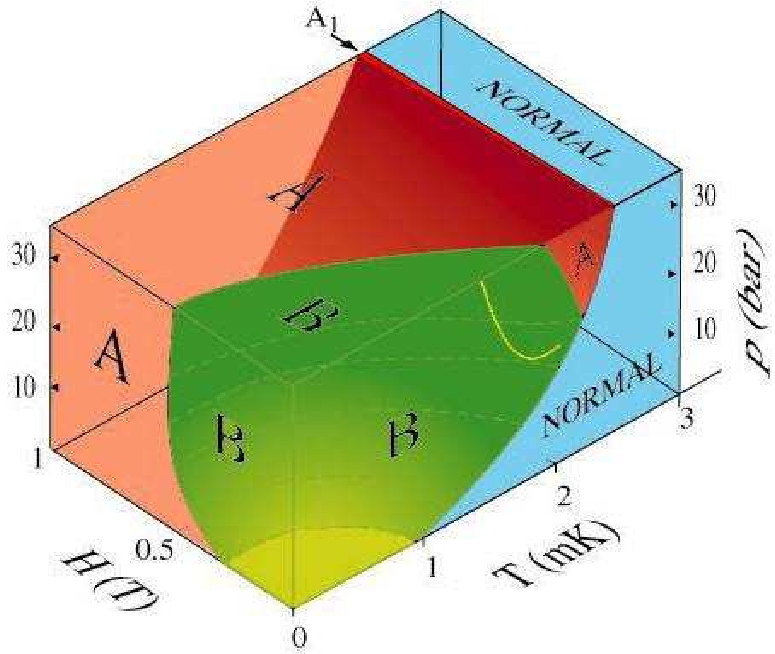
$$L = 2,\ 4\ , .., \quad S = 0$$

Microscopic theory of superfluid ^{3}He

Only experiment AND theory could resolve the problem

$$T_c^{\rm th} \sim T_c^{\rm exp}$$





Microscopic theory of superfluid ^{3}He

In all three phase pairing is with $S = 1$ and $L = 1$, so called p-pairing

orbital base: $p_x,\ p_y,\ p_z$
spin base: $s_x,\ s_y,\ s_z$

Order parameter with 18 components:

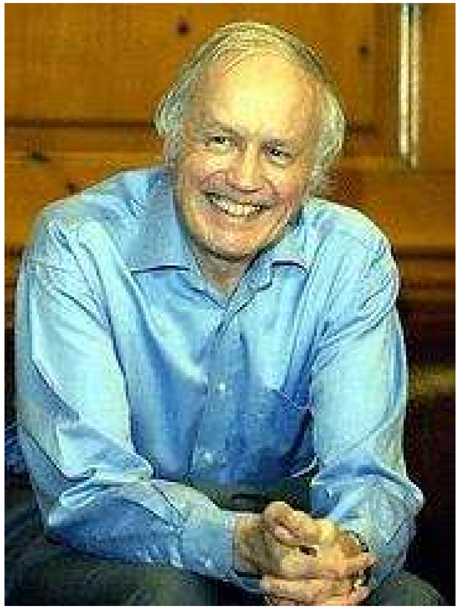
$$\hat{\Delta} = \begin{pmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{yx} & A_{yy} & A_{yz} \\ A_{zx} & A_{zy} & A_{zz} \end{pmatrix}$$

The symmetry group:

$$G = SO(3)_L \times SO(3)_S \times U(1)_\phi$$

Which symmetry breaking occurs in liquid ^{3}He?

Microscopic theory of superfluid ^{3}He



Introduced the concept of spin-orbit symmetry breaking

Developed microscopic theory combining BCS and NMR

Identified A, B, and A_1 phases with proper order parameter

Explained NMR spectra

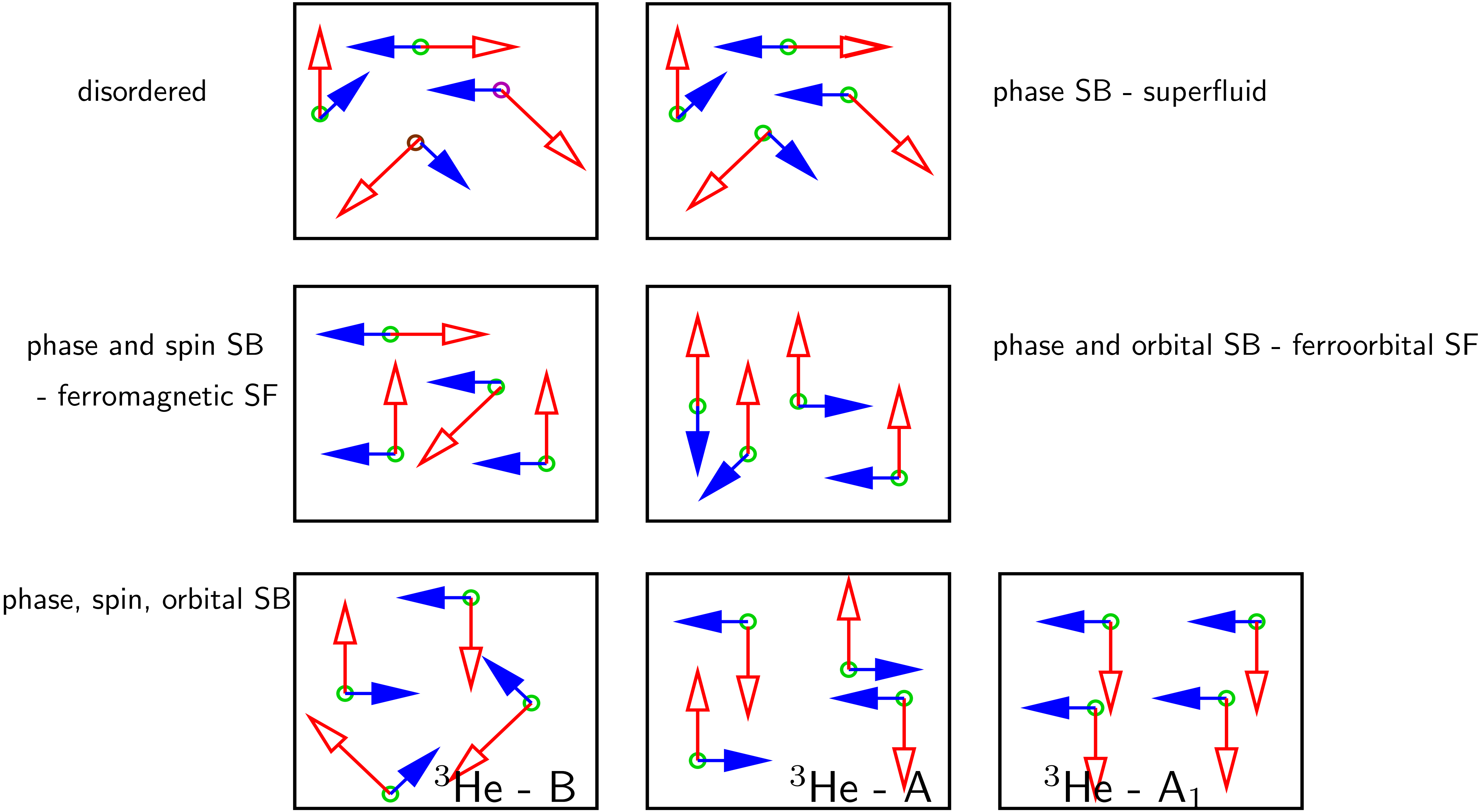
Leggett's equations of motion with dipole-dipole interactions

$$\dot{\vec{S}} = \gamma \vec{S} \times \vec{H} + \frac{6}{5} g_D(T)(\vec{d} \times \vec{l})(\vec{d} \cdot \vec{l})$$

$$\dot{\vec{d}} = \vec{d} \times \gamma \left(\vec{H} - \frac{\gamma \vec{S}}{\chi} \right)$$

Microscopic theory of superfluid ^{3}He

$$G = SO(3)_L \times SO(3)_S \times U(1)_\phi$$



Microscopic theory of superfluid ^{3}He

B - phase

Balian, Werthamer (weak coupling) 1963

Spin-orbit group $SO(3)_L \times SO(3)_S$ breaks down to $SO(3)_{L+S}$

All spin states equally populated

$$|\uparrow\uparrow\rangle, \; |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle, \; |\downarrow\downarrow\rangle$$

Isotropic gap function $\Delta(\mathbf{k}) = \Delta_0$ (but anisotropic condensate!)

A - phase

Anderson, Morel 1961, Brinkman 1973 (strong coupling)

Gauge-orbit group $SO(3)_L \times U(1)_\phi$ breaks down to $U(1)_{L_z+\phi}$ and spin group $SO(3)_S$ to $U(1)_{s_z}$

Only $s_z = \pm 1$ spin states populated

$$|\uparrow\uparrow\rangle, \; |\downarrow\downarrow\rangle$$

Anisotropic gap function $\Delta(\mathbf{k}) = \Delta_0 \sin \hat{k} \cdot \hat{l}$ in anisotropic condensate

A$_1$ - phase

Gauge-orbit group $SO(3)_L \times U(1)_\phi$ breaks down to $U(1)_{L_z+\phi}$ and spin group $SO(3)_S$ to $U(1)_{s_z+\phi}$, magnetic superfluid

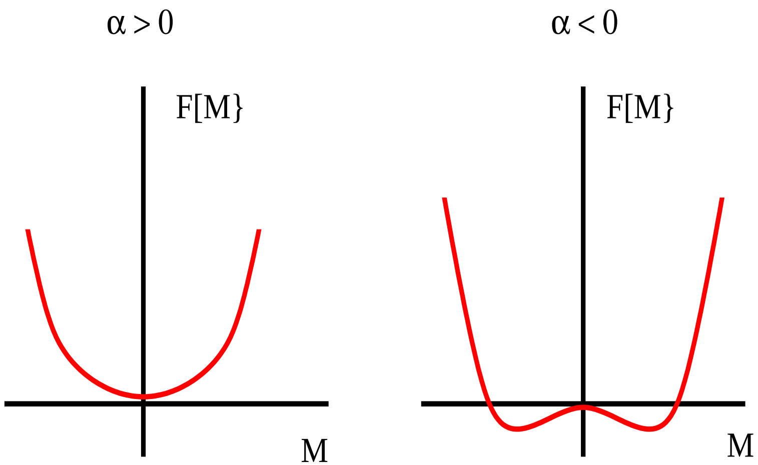
Only $s_z = +1$ spin states populated due to external magnetic field

$$|\uparrow\uparrow\rangle$$

Phenomenological approach to phase transitions - L.D. Landau 1932

Introduced the concept of the order parameter in phase transitions

Landau functional $F[M] = F_0 + \alpha(T)M(T)^2 + \frac{1}{2}\beta M(T)^4 + \ldots$

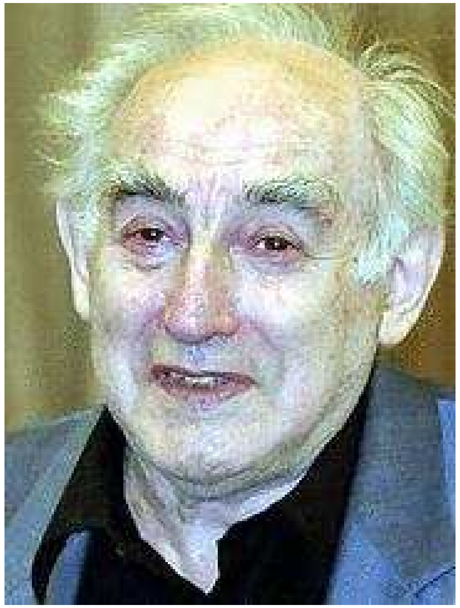


First ϕ^4 field theory

$$f[m(r)] = f_0 + \alpha(T)m(r,T)^2 + \frac{1}{2}\beta m(r,T)^4 + \gamma[\nabla m(r,T)]^2$$

$\alpha = \alpha_0(T - T_c)$, $m \in \mathbb{R}$

Phenomenological approach to SC phase transition - Ginzburg, Landau 1950



Introduced the complex order parameter in SC phase transition

$\Psi(r) = \sqrt{n_s(r)} e^{i\phi(r)}$ "wave function" of the whole SC

First ϕ^4 complex field theory

Microscopically derivable from BCS, $\Psi(r) = \langle a_\downarrow(r) a_\uparrow(0) \rangle$

Gauge invariant form

Mathematical description of inhomogeneous superconductors (superfluids)

$$f[\Psi(r,T)] = f_0 + \alpha(T)|\Psi(r,T)|^2 + \frac{1}{2}\beta|\Psi(r,T)|^4 + \frac{1}{2m^*}|(-i\hbar\nabla - e^*\mathbf{A})\Psi(r,T)|^2 + \frac{|B-\mu_0 H|^2}{2\mu_0}$$

$\alpha = \alpha_0(T - T_c)$, $\Psi \in \mathbb{C}$, $m^* = 2m_e$, $e^* = 2e$

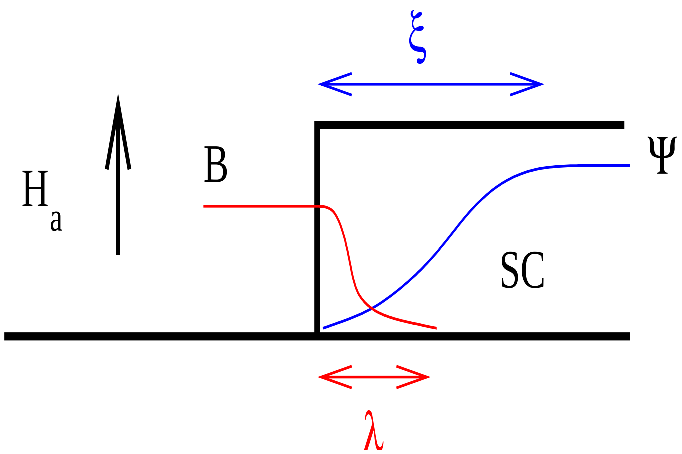
Ginzburg - Landau equations

$$\frac{\delta F[\Psi]}{\delta \Psi} = 0 \quad \text{and} \quad \frac{\delta \mathrm{F}[\Psi]}{\delta \mathbf{A}} = \mathbf{J}$$

leads to the equations of motion known as Ginzburg - Landau equations

$$\alpha\Psi + \beta|\Psi|^2\Psi + \frac{1}{2m^*}(-i\hbar\nabla - e^*\mathbf{A})^2\Psi = 0$$

$$\mathbf{J} = \frac{e^*}{m^*}\left[\Psi^*(-i\hbar\nabla - e^*\mathbf{A})\Psi + h.c.\right]$$



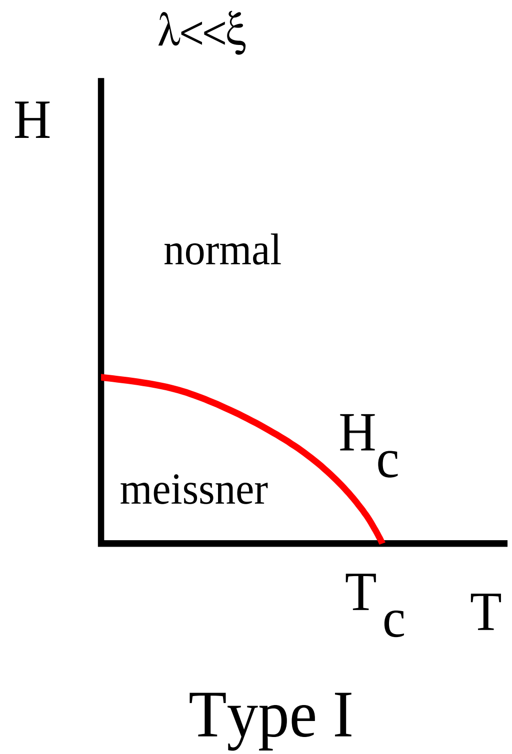
Meissner effect: $\lambda = \sqrt{\frac{m^*}{\mu_0 n_s e^{*2}}}$ - London penetration depth

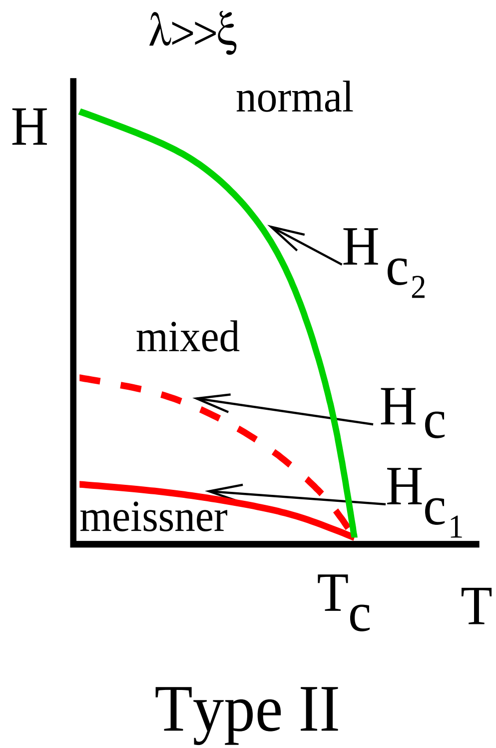
Coherence length: $\xi = \hbar\sqrt{\frac{1}{2m^*|\alpha|}}$

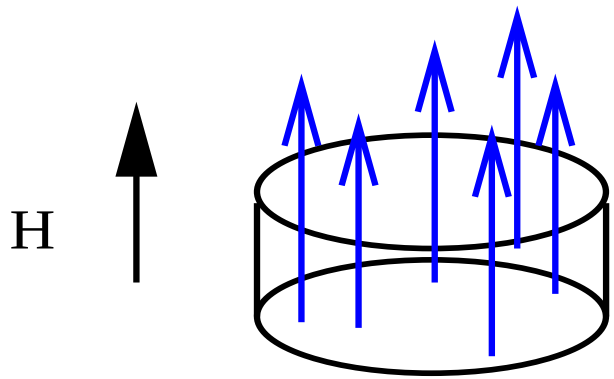
Ginzburg-Landau parameter: $\kappa = \frac{\lambda}{\xi}$

Flux quantization: $\Phi = \oint_C d\mathbf{l} \cdot \mathbf{A} = n \cdot \frac{hc}{2e} = n\Phi_0$

Type I and type II superconductors

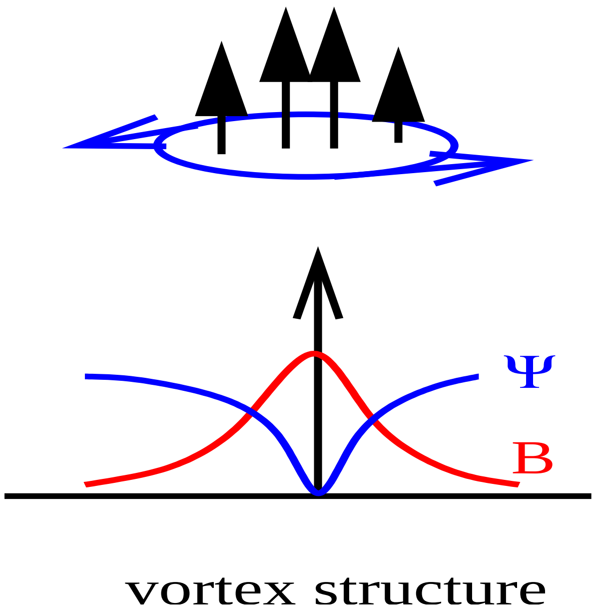


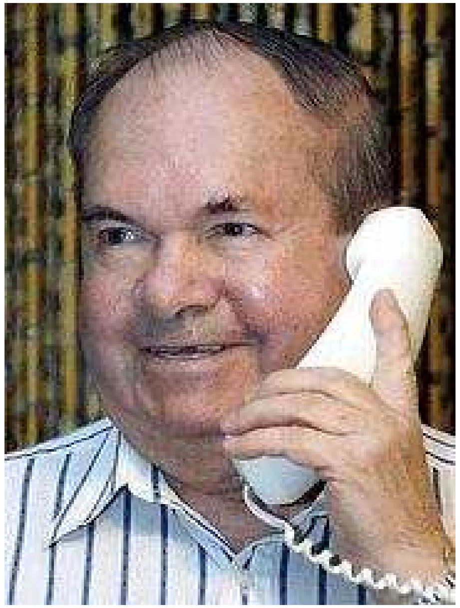




To be $\kappa < 1/\sqrt{2}$ or $\kappa > 1/\sqrt{2}$

Vortices in type II SC





Abrikosov solved GL equation in the limit $\kappa > 1/\sqrt{2}$ pioneering the theory of type II SC

Abrikosov lattice of vortices

GL eq. without Ψ^4 in a magnetic field (Landau gauge) are solved by

$$\Psi \sim e^{ik_x x + ik_z z} e^{-\frac{(y+k_x l_M)^2}{2l_M^2}}$$

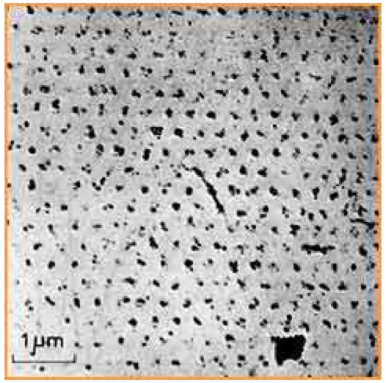
Large degeneracy of ground state - k_x arbitrary since many orbit centers

With Ψ^4 term the energy F is minimized by periodic solution which removes the degeneracy

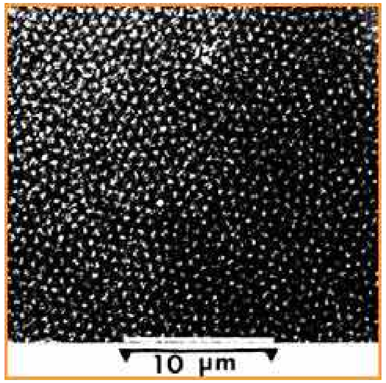
$$\Psi \sim \sum_{n=-\infty}^{\infty} e^{-\frac{2\pi}{\sqrt{3}}\left(\frac{y}{a_0}+\frac{\sqrt{3}}{2}n\right)^2 + 2\pi i n\left(\frac{x}{a_0}+\frac{1}{4}n\right)}$$

corresponds to hexagonal lattice of vortices

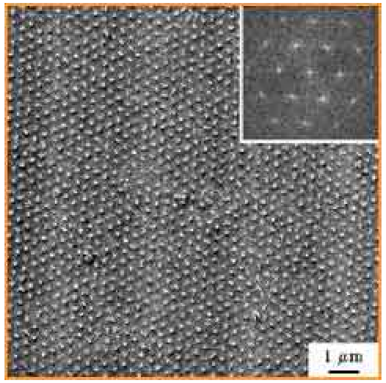
Abrikosov lattices gallery



Pb SC first observation 1967



BISCO high temperature SC discovered 1995



MgB_2 SC discovered 2001

And today ...

- ferromagnetic (triplet) solid state superconductors $SrRuO_4$, UGe_2, URhGe
- proton-neutron superconductivity in nucleus or neutron stars, color superconductivity in dense baryon matter
- type II SC in industry, medicine, and military- vortex flow problems
- GL theory as a prototype of other effective field theories
  - order parameter $\longrightarrow$ vacuum manifold
  - order parameter field $\longrightarrow$ higgs field
  - vortex $\longrightarrow$ string, etc.
- ^{3}He as a laboratory to test cosmological hypothesis (Kibble, Zurek)
- ... many others

2003 Physics Nobel Trio contributions:

- Ginzburg - effective field theory with complex order parameter
- Abrikosov - topological defects
- Leggett - spin-orbital symmetry breaking

Happy New Year!