Supersolid - an exotic state of quantum condensted matter

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Condensed matter theory division

Wikipedia:

Condensed matter physics is the field of physics that deals with the macroscopic physical properties of matter. In particular, it is concerned with the "condensed" phases that appear whenever the number of constituents in a system is extremely large and the interactions between the constituents are strong. The most familiar examples of condensed phases are solids and liquids, which arise from the bonding and electromagnetic force between atoms. More exotic condensed phases include the superfluid and the Bose-Einstein condensate found in certain atomic systems at very low temperatures, the superconducting phase exhibited by conduction electrons in certain materials, and the ferromagnetic and antiferromagnetic phases of spins on atomic lattices.

Condensed matter physics is by far the largest field of contemporary physics. Much progress has also been made in theoretical condensed matter physics. By one estimate, one third of all US physicists identify themselves as condensed matter physicists. Historically, condensed matter physics grew out of solid-state physics, which is now considered one of its main subfields.

Condesed matter physics has a large overlap with chemistry, materials science, nanotechnology and engineering.

Principles of condensed matter physics

- Emergence principle - *More is different*
- Quasiparticles - particle like excitations
- Renormalization and universality
- Sponteneous symmetry breaking

Example: SUPERSOLIDS

To remember on supersolids

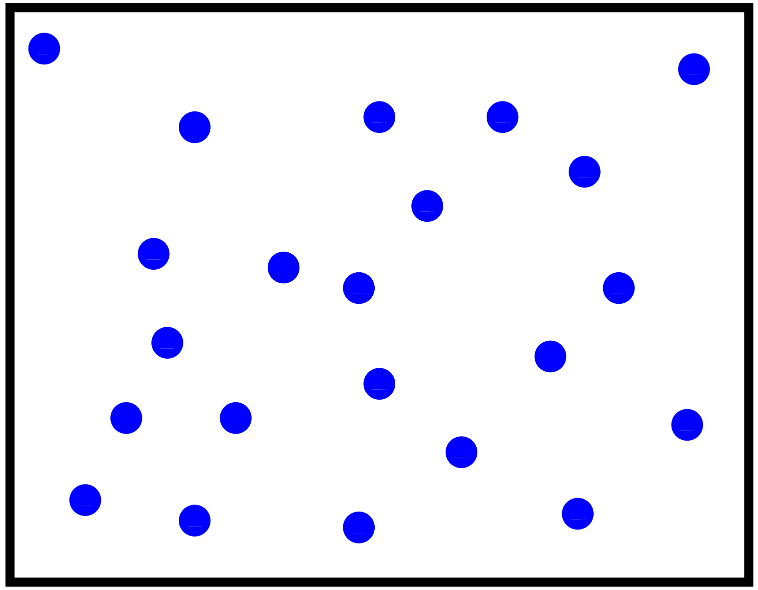
A system where its elements simultaneously

- have crystalline order, and
- flow like a liquid with no viscosity

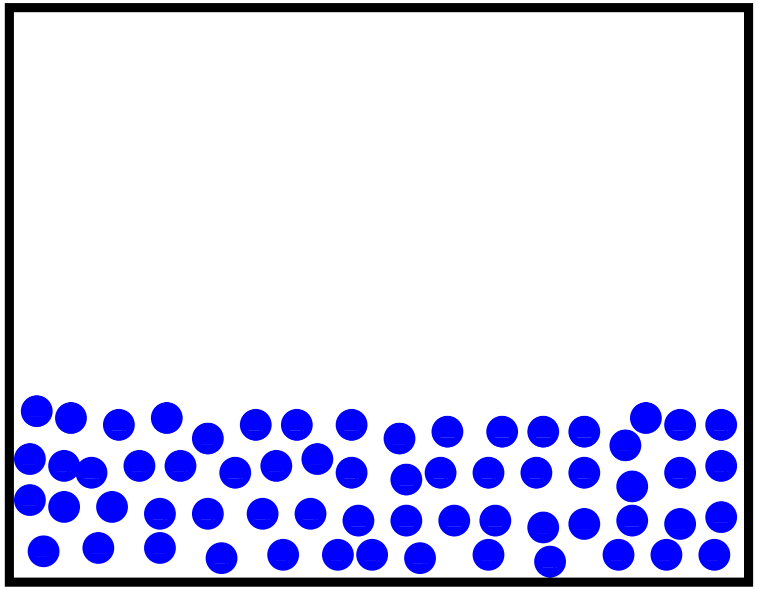
is called a supersolid

Classical states of matter

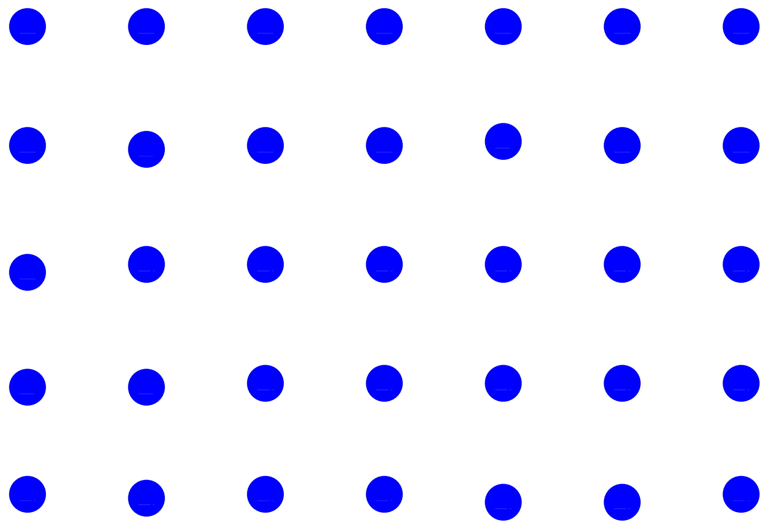
elements are basically distinguishable



gas - no ordering at all



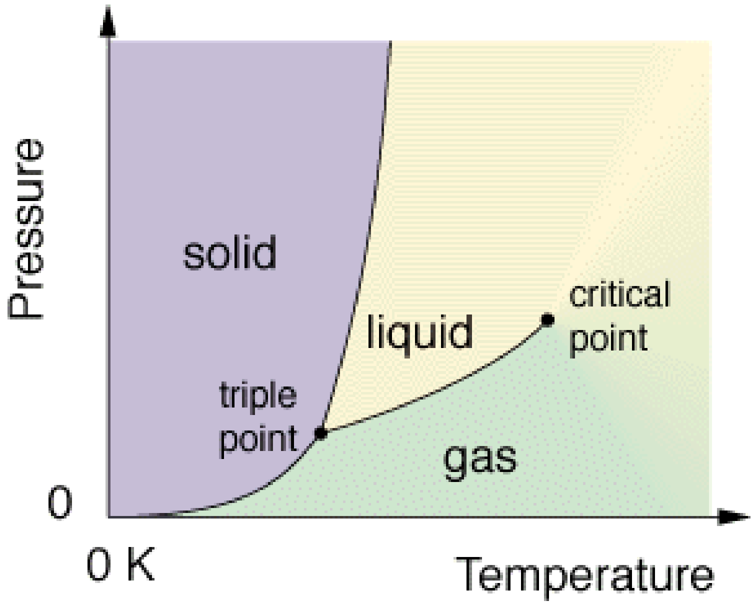
liquid - ordering on short range scale

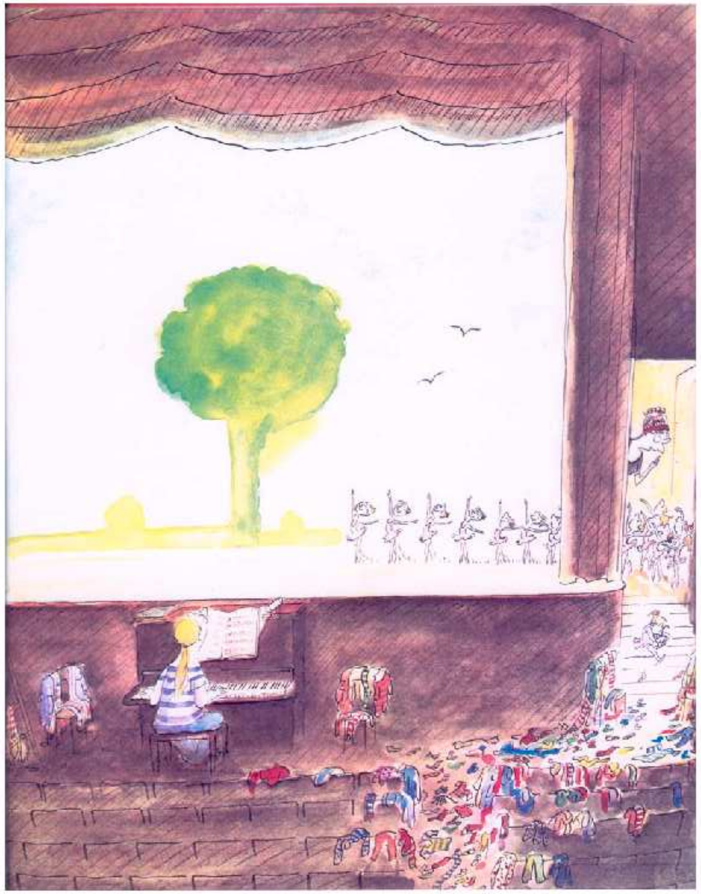


solid - ordering on any length scale

Long Range Order

Phase transitions - Order from chaos





Long Range Order (LRO)

$\langle \rho(\mathbf{r}_1)\rho(\mathbf{r}_2)\rangle \rightarrow \langle \rho(\mathbf{r}_1)\rangle\langle \rho(\mathbf{r}_2)\rangle \neq 0$ for $|\mathbf{r}_1 - \mathbf{r}_2| \rightarrow \infty$

crystal - long range correlations, rigidity

(Sempe)

Quantum states of matter

When does quantum mechanics start to play a role?

Thermal de Broglie wavelength: $\lambda_{dB} = \sqrt{2\pi\hbar^2/mk_BT}$

Typical distance between atoms: d

Quantum behavior if $\lambda_{dB} \gtrsim d$

Neon (Ne) at $T_c = 27$K: $\lambda_{dB} \approx 0.07$nm, $d \approx 0.3$nm

Helium (He) at $T_c = 4.2$K: $\lambda_{dB} \approx 0.4$nm, $d \approx 0.27$nm

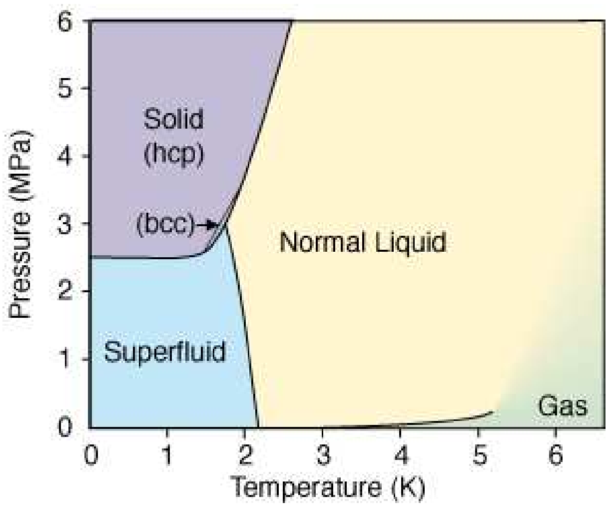
Quantum states of matter

atoms are indistinguishable - bosons and fermions

quantum gas, quantum liquid, superfluid, quantum crystal, supersolid, ...

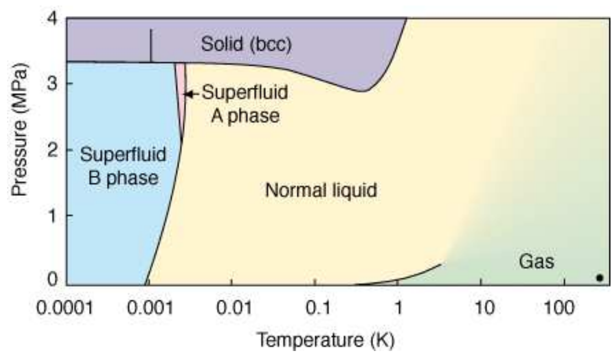
$$\lambda_{dB} \gtrsim d$$

Isotops: ^{4}He (2p2n2e - "bosons") and ^{3}He (2p1n2e - "fermions")



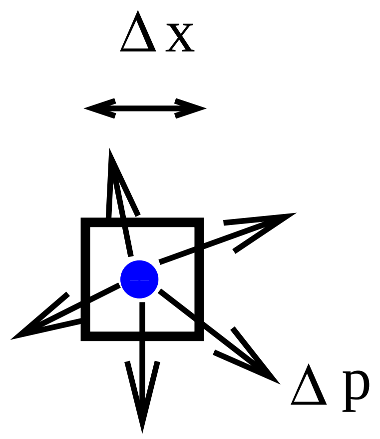
Quantum

liquids at T=0K!



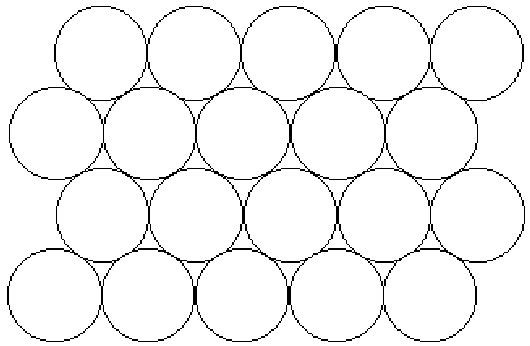
Why quantum liquids at T=OK?

- Heisenberg uncertainty principle



$$\Delta x \Delta p \gtrsim \frac{\hbar}{2}$$

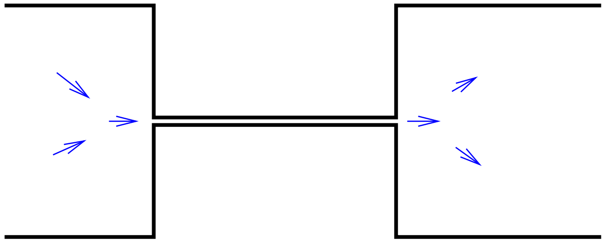
- the more localized particle in a space the fast it moves - zero point motions
- He at p_{atm} does not crystallize due to zero point motions
- kinetic energy is large as compared to $v(r) < 0$ potential
- He crystallizes at $p \gg p_{atm}$ due to hard core repulsion
- (as hard bowls form dense packed structure (hcp))



Superfluid quantum liquid $T \leqslant T_\lambda$

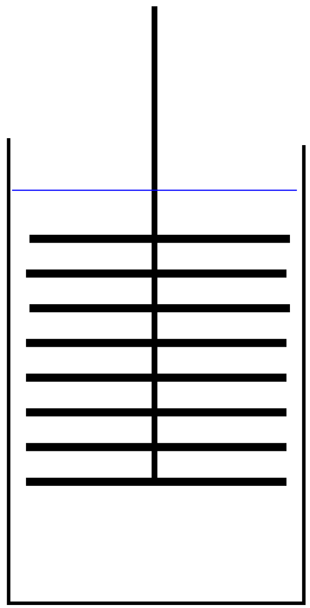
- frictionless flow through capillaries (superflow), creeping, fountain effect, ...

P. Kapitsa, Nature **141**, *74 (1938); J.F. Allen and D. Meisener, ibid., 75 (1938); E.L. Andronikashvili 1946*



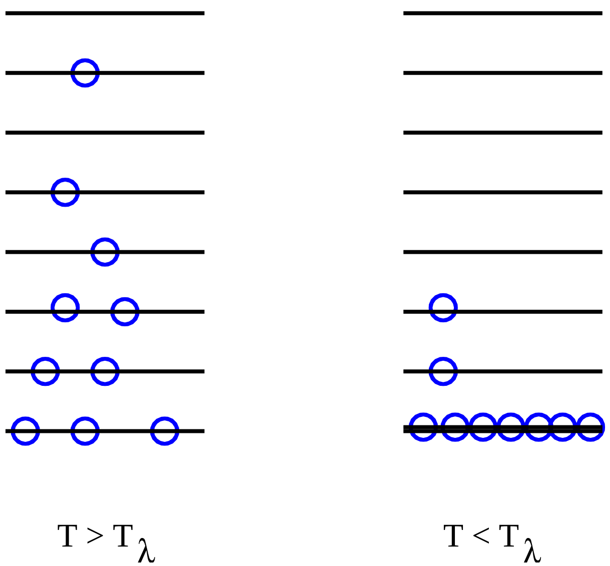
$$\frac{\Delta p}{L} \sim \eta \frac{v}{R^2}$$

$\Delta p = 0$ and $v > 0$ means zero viscosity $\eta = 0$



Microscopic theory of superfluid ^{4}He

Type of Bose - Einstein condensation of interacting bosons



- macroscopic number N_0 of bosons are in the lowest energy state
- N_0 bosons have the same wave function
- one-particle density matrix

$$\rho(\mathbf{r}-\mathbf{r}') = \langle \psi^\dagger(\mathbf{r})\psi(\mathbf{r}')\rangle$$

describes long-range order

$$\langle \psi^\dagger(\mathbf{r})\psi(\mathbf{r}')\rangle \rightarrow \langle \psi^\dagger(\mathbf{r})\rangle\langle \psi(\mathbf{r}')\rangle = N_0/V = n_0$$

gdy $\mathbf{r}-\mathbf{r}' \rightarrow \infty$

Off-diagonal long-range order (ODLRO)

Macroscopic wave function of the condensate

- there exists a wave function of the condensate

$$\Psi(\mathbf{r};T) = \sqrt{\rho_s(\mathbf{r})}e^{i\theta(\mathbf{r})}$$

order parameter

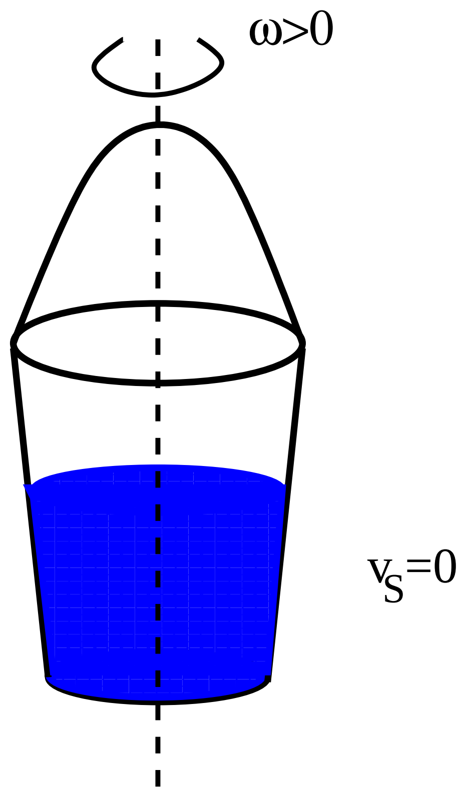
- flow velocity of the condensate ($\rho_s = \mathrm{const.}$)

$$\mathbf{v}_s = \frac{\hbar}{m}\nabla\theta(\mathbf{r})$$

- in 'no holes space' flow is rotationless

$$\nabla \times \mathbf{v}_s = 0$$

- in rotating vessel with ^{4}He, superfluid does not move



London 1954, Hess i Fairbank 1967

- non-classical moment of inertia (NCRI)

Non-classical moment of inertia (NCRI)

Free energy

$$F(\Omega) = F_0 + \frac{1}{2} I_{\rm class} \Omega^2$$

with F_0 free energy at $\Omega = 0$ and moment of inertia $I_{\rm class} = NmR^2$

NCRI means that at small Ω new term appears

$$\Delta F(\Omega) = -\frac{1}{2} \frac{\rho_s}{\rho} I_{\rm class} \Omega^2$$

Two fluid system (normal and superfluid) has smaller moment of inertia

$$I_{\rm NCRI} = \left(1 - \frac{\rho_s}{\rho}\right) I_{\rm class}$$

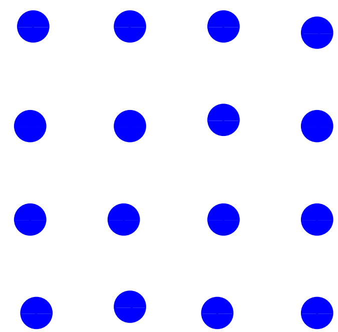
Quantum crystals

- solid He at high pressure (25atm for ^{4}He and 30atm for ^{3}He)

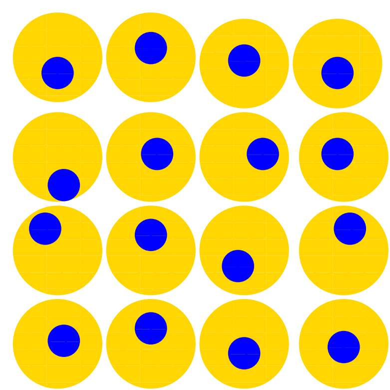
- amplitude of zero motions is 30% of a lattice constant

classical crystal - small zero motions $\Delta x \ll a$

quantum crystal - large zero motions $\Delta x \gg a$



- in quantum crystals atoms are indistinguishable



Quantum crystals - Supersolids

Can a quantum crystal be superfluid?

M. Wolfke, Ann. Acad. Sci. Techn. Varsovie **6**, *14 (1939)*

diagonal long-range order

$$\langle \rho(\mathbf{r}_1)\rho(\mathbf{r}_2)\rangle \rightarrow \langle \rho(\mathbf{r}_1)\rangle\langle \rho(\mathbf{r}_2)\rangle \neq 0$$

off-diagonal long-range order

$$\langle \psi^\dagger(\mathbf{r})\psi(\mathbf{r}')\rangle \rightarrow \langle \psi^\dagger(\mathbf{r})\rangle\langle \psi(\mathbf{r}')\rangle \neq 0$$

for $|\mathbf{r}_1 - \mathbf{r}_2| \rightarrow \infty$

Solid-state rigidity and fluidity with zero viscosity

Supersolid

Quantum crystals - Commensurate insulators

Insulating variational wave function (*O. Penrose, L. Onsager 1951, 1956; C.N. Yang 1962*)

$$\Psi_G^{PO}(\mathbf{r}_1, ..., \mathbf{r}_N) = \sqrt{\frac{1}{N!}} \sum_P \prod_{j=1}^{N} W(\mathbf{R}_j - \mathbf{r}_{Pj})$$

with $W(\mathbf{R}_j - \mathbf{r}_{Pj})$ localized Wanner wave functions, $N_L = N$

$$\langle \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}') \rangle = \frac{1}{N} \sum_{j=1}^{N} W^*(\mathbf{R}_j - \mathbf{r}) W(\mathbf{R}_j - \mathbf{r}') \to 0$$

no ODLRO (some delocalization, exchange is needed)

Quantum crystals - Commensurate supersolids

Bose-Einstein condensate variational wave function with exchange
(*L. Reatto 1969, G.V. Chester 1970, A. Leggett 1970*)

$$\Psi_G^{BEC}(\mathbf{r}_1, ..., \mathbf{r}_N) = \prod_{i=1}^{N} \left(\frac{1}{\sqrt{N}} \sum_{j=1}^{N} W(\mathbf{R}_j - \mathbf{r}_i) \right)$$

density wave modulation with macroscopic condensation of all N bosons in one-particle state $\phi(\mathbf{r}) = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} W(\mathbf{R}_j - \mathbf{r})$

$$\langle \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}') \rangle \to \rho_s \phi^*(\mathbf{r}) \phi(\mathbf{r}')$$

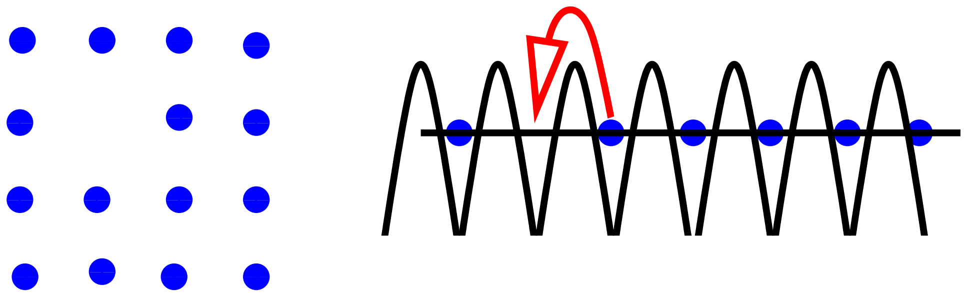
Zero probability to occur in Nature a commensurate supersolid in continuous space
(*N. Prokof'ev, B. Svistunov 2005*)

$$N_L = N \times k$$

condition can happen by accident or be designed in laboratory (cold atoms)

Incommensurate quantum crystals - vacancies

a vacancy moves backward as compared with movements of atoms

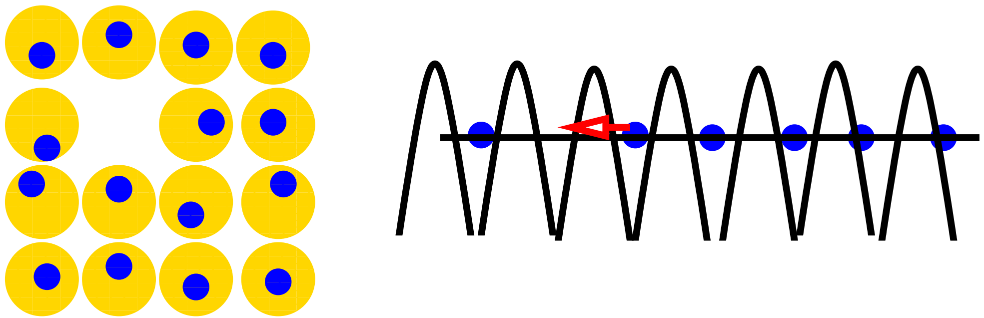


classical crystal

to overcome energy barier

activation energy Δ needed

$$p \sim \exp(-\Delta/k_B T)$$



quantum crystal

atoms tunnel

vacancy is not static

vacancy becomes a quasiparticle

vacanson, defecton

$$|\mathbf{k}\rangle = \sum_i e^{i\mathbf{k}\mathbf{R}_i}|\mathbf{R}_i\rangle$$

coherent quasiparticles with dispersion $\epsilon(\mathbf{k})$

Incommensurate quantum crystals - Supersolid

Andreev, Lifshitz (1969), Chester (1970), Leggett (1970), ...

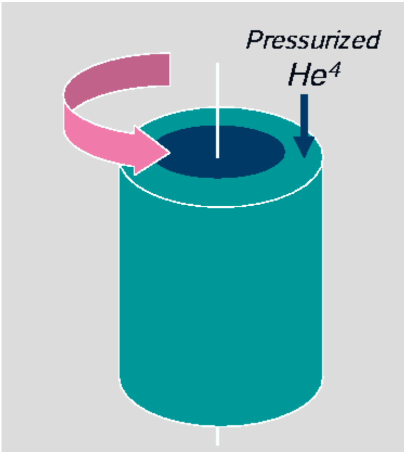
- defectons in solid ^{4}He are bosons
- BEC condensation of defectons below T_c
- BEC defectons are superfluid
- BEC defectons do not move with the system
- moment of inertia should be reduced

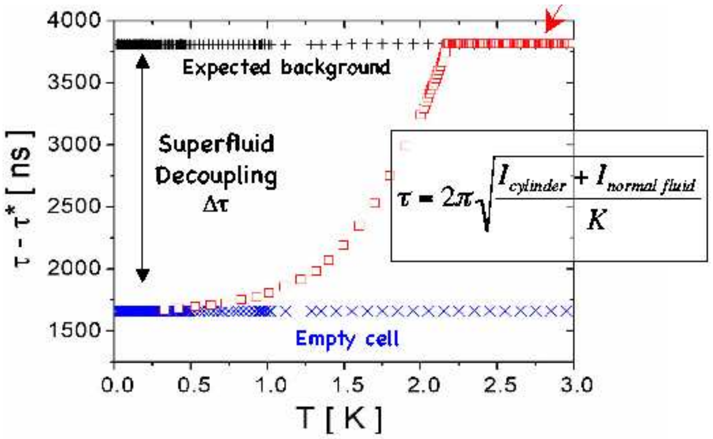
$$I(T) = I_{\text{class}} \cdot \left(1 - \frac{\rho_s(T)}{\rho}\right)$$

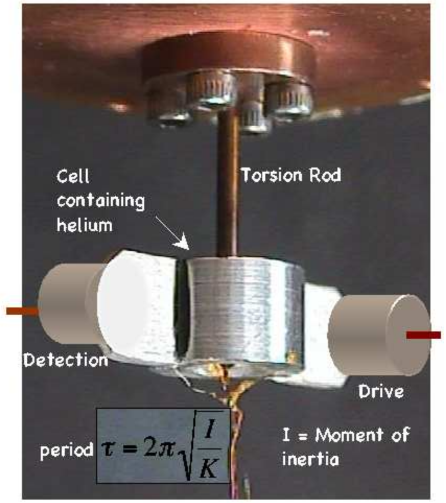
supersolid = LRO + ODLRO

“Mary goes round” experiments

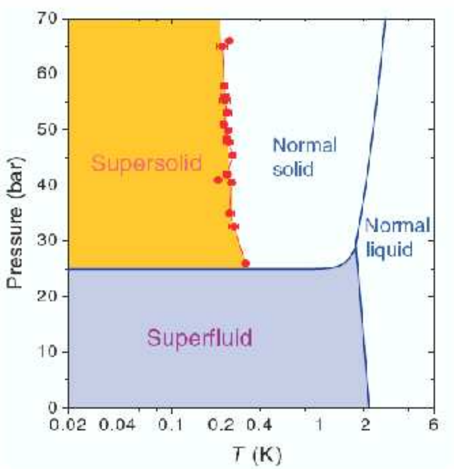
E. Kim, M.H. Chan, Nature (2004), Science (2004), J. Low Temp. Phys. (2005)

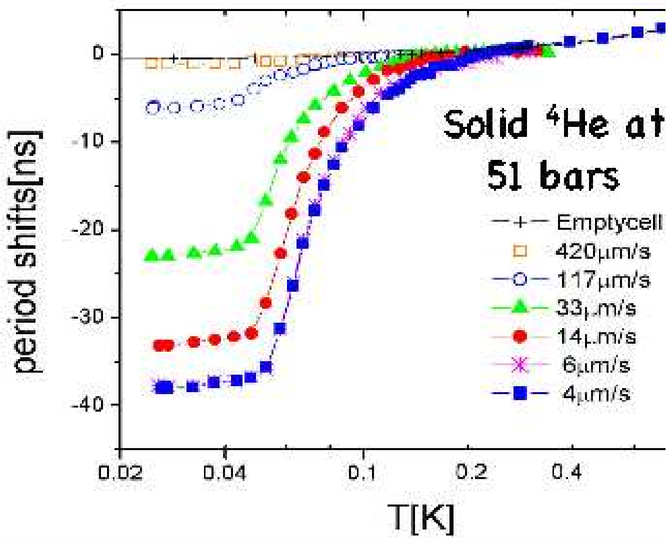


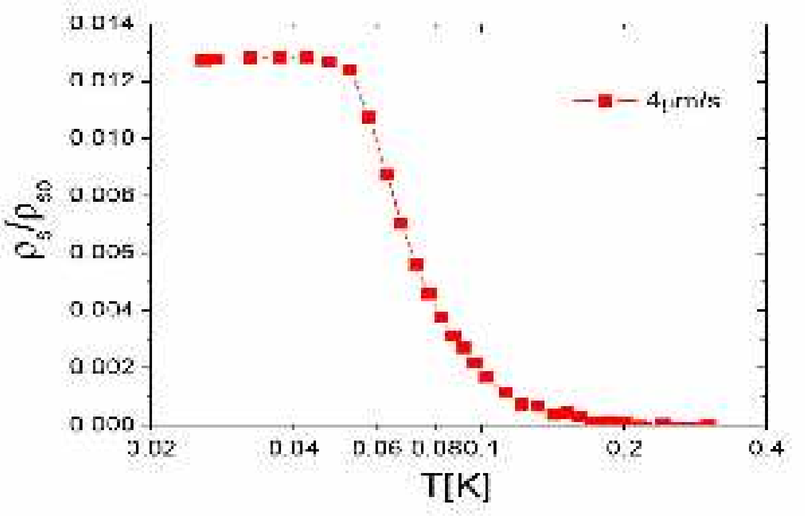




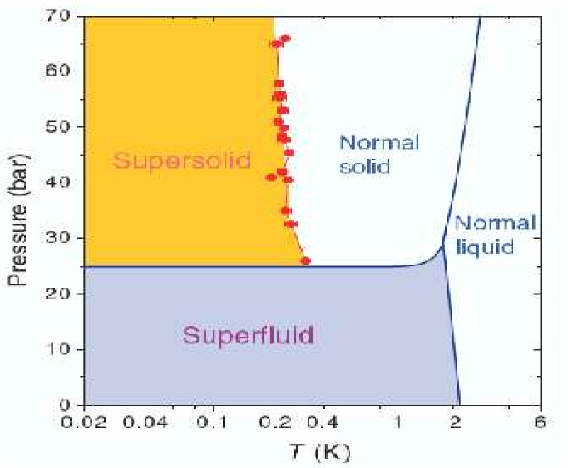
in solid ^{4}He below 0.2K decrease of oscilation period - moment of inertia decreases

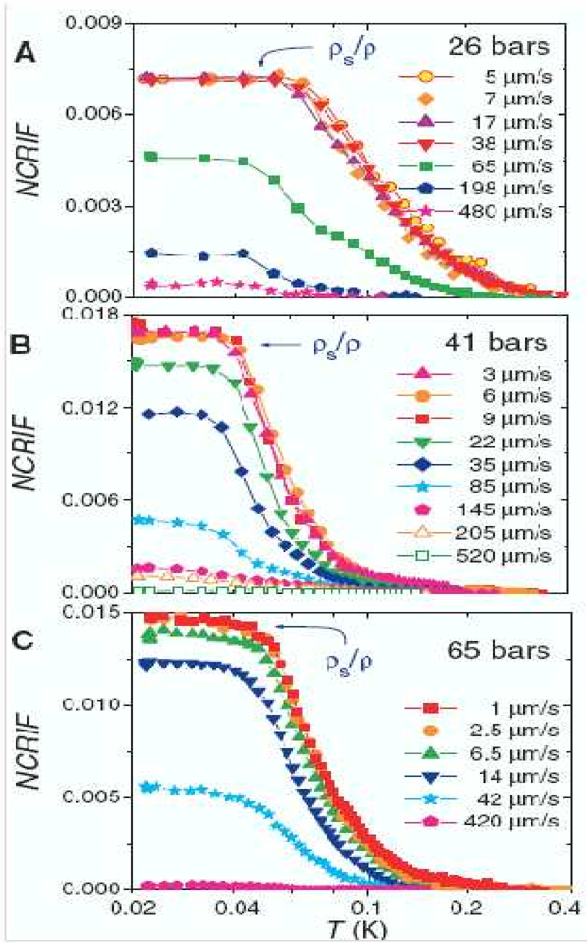






Supersolid?





in Kim and Chan's experiments - ca. 1,3% of a crystal is a supersolid

problems: in solid helium number of defects is orders of magnitudes smaller!

commensurate supersolid, phase separation, defects, microcrystals, vortices, glass ...???

Experiment positively reproduced in 3 other laboratories but with history depend results!

Conclusions and outlooks

- supersolid combines crystal rigidity and superfluidity

- idea applies to H, He, cold atoms in MOT and optical lattices, or atom impurities in crystal hosts, and superconducting electrons

- Kim and Chan's experiments still call for a correct interpretation ...

- ... but they have certainly renewed an interest in supersolids

Reading:

1. E. Kim, M.H. Chan, Nature **425**, 227 (2004); Science **305**, 1941 (2004).
2. Physics Today, November 2004, p. 23.
3. Adv. in Physics **56**, 381 (2007); Contemporary Phys. **48**, 31 (2007); Science **319**, 120 (2008).
4. A. Leggett, *Quantum liquids* (Oxford Univ. Press 2006).
5. K. Byczuk, Postepy Fizyki **58**, 194 (2007).