Levitron© or about Levitation without Superconductor

Physicist's Christmas Story

by

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Levitation

Internet Wikipedia Encyclopedia:

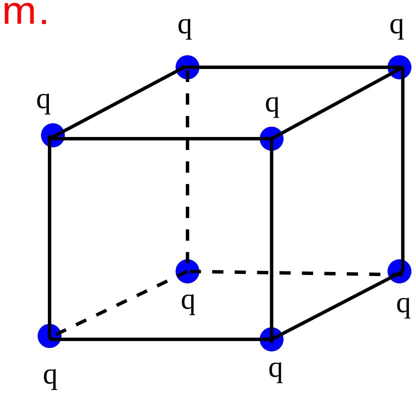
Levitation (*from Latin levo, to raise*) is the process by which an object is suspended against gravity, in a stable position, by a force without physical contact.

- human dream for ages
- 2,220,000 WWW pages with 'levitation' in Google
- miscellaneous advertisements: *At the Central Florida School of Levitation, we teach you to overcome gravity! Despite what scientists say, gravity is not an inescapable force: it is a frame of mind. Control your mind, and you control your weight.*
  ... but there's a warning:
  Don't be without Gravity Insurance!
- idea to use magnets or charges
  (but already kids know that it does not work with magnets)

Earnshaw's theorem (1842)

it is impossible to reach static levitation ...

There is no static configuration of electric, magnetic and gravity fields in empty space such that the potential energy possesses a local minimum.



Example: (*Electrodynamics*, D.J. Griffiths - Problem 3.2)

As an example, consider a cubical arrangments of fixed charges.

It looks, off hand, as though a positive charge at the center

would be suspended in midair, since it is repelled away from each corner.

Where is the leak in this 'electrostatic bottle'?

[To harness nuclear fussion as a practical energy source it is necessary to heat a plasma (soup of charges particles) to fantastic temperatures - so hot that contact would vaporize any ordinary pot. Earnshaw's theorem says that electrostatic confinement is out of the question.]

A **TOKAMAK** (*toroidal'naya kamera v magnitnykh katushkakh - toroidal chamber in magnetic coilsis*) is a machine producing a toroidal magnetic field for confining a moving plasma.

Earnshaw's theorem - proof

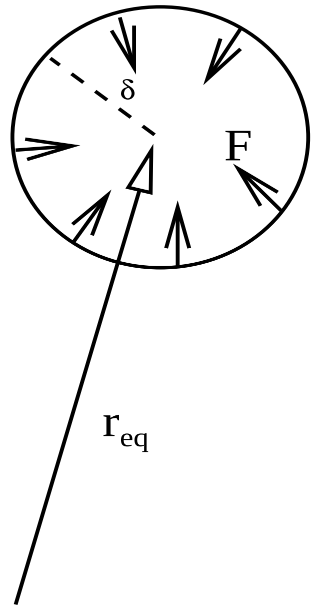
Earnshaw's theorem is a direct result of Gauss law for sourceless and rotationless fields

$$\nabla \cdot \mathbf{F}(\mathbf{r}) = 0, \qquad \nabla \times \mathbf{F}(\mathbf{r}) = 0, \quad \text{where eg. } \mathbf{F}(\mathbf{r}) = \mathrm{e}\mathbf{E}(\mathbf{r})$$

Proof:
a.a.
Let there is a point of stable equilibrium. Then

- at equilibrium point $\mathbf{r}_{eq}$ we have $\mathbf{F}(\mathbf{r}_{eq}) = 0$
- if $\mathbf{r}_{eq}$ is stable equilibrium point then in its vicinity $\mathbf{F}(\mathbf{r}_{eq} + \delta)$ has to be turned inward to $\mathbf{r}_{eq}$
- but from Gauss law we have that

$$\int_{S=\{|\mathbf{r}-\mathbf{r}_{eq}|=\delta\}} \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r} = \int_V \nabla \cdot \mathbf{F}(\mathbf{r})\ d_3 r = 0$$



and there is a contradiction.
□

Seeing means believing or levitation in practice

diamagnet ($\chi < 0$) in magnetic field (HFML, Nijmingen, Holand)

superconductor in magnetic field (Oslo University, Norway)

levitron (my lecture in Ustron, Poland 2006)

they are not static levitation

How the levitron looks like

History:

- discovery/invention: Roy Harring, U.S. Patent 4,382,245 (1983)
- theory: M.V. Berry, Proc. Roy. Soc. London A **425**, 1207 (1996)

Commercial Levitron:

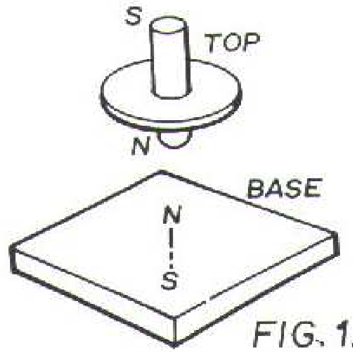
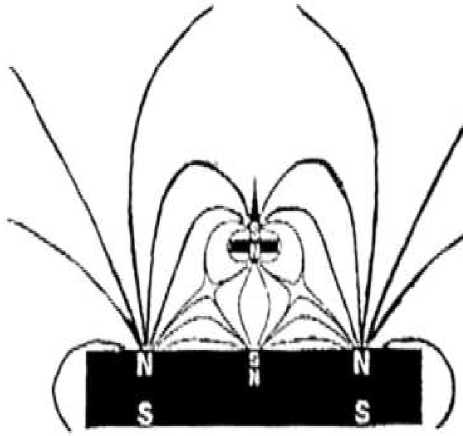


FIG. 1.

- stable base magnet (ceramic)
- magnetic top (ca. 18g)
- plate to lift
- weights (2 with 3g, 3 with 1g, 2 with 0,4g, 1 with 0,2g i 2 with 0,1g)
- starter (electric engine)
- manual (written by M. Berry)

How does it (not) work?



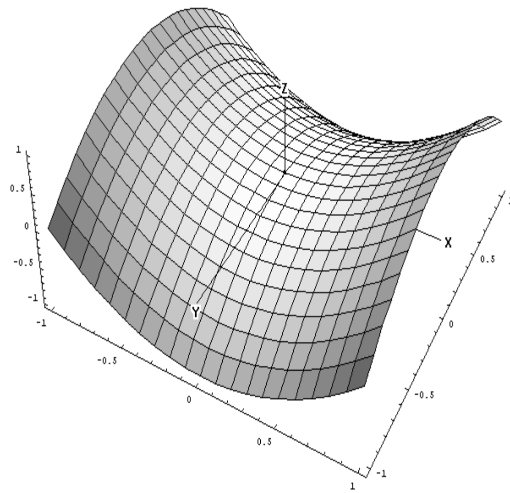
- The repulsive force between magnets acts against the gravity force. Conservation of angular momentum helps.
- Very nonuniform magnetic field. Is it a stable equilibrium?
- Potential energy

$$U(\mathbf{r}) = -\vec{\mu} \cdot \vec{\mathbf{B}}(\mathbf{r}) + mgz$$

  has to have local minimum ($\vec{\mu}$ is a magnetic moment of a top).
- For a magnetic fields in empty space we have for each component of the magnetic induction $\nabla^2 \mathbf{B}(\mathbf{r}) = 0$, which leads to

$$\nabla^2 U(\mathbf{r}) = 0,$$

  hence there is only a saddle point (not a local minimum)!
- It is in agreement with Earnshaw's theorem.



How does it just work?

M.V. Berry (also: M.D. Simon *et al.*, and S. Gov *et al.*)

Levitating top over the magnet does:

- side oscillations with frequency $\nu_{\mathrm{side}} \sim 1\mathrm{Hz}$
- precession around spinning axis with frequency $\nu_{\mathrm{precession}} \sim 5\mathrm{Hz}$
- rotation with frequency $\nu_{\mathrm{rotation}} \sim 25\mathrm{Hz}$

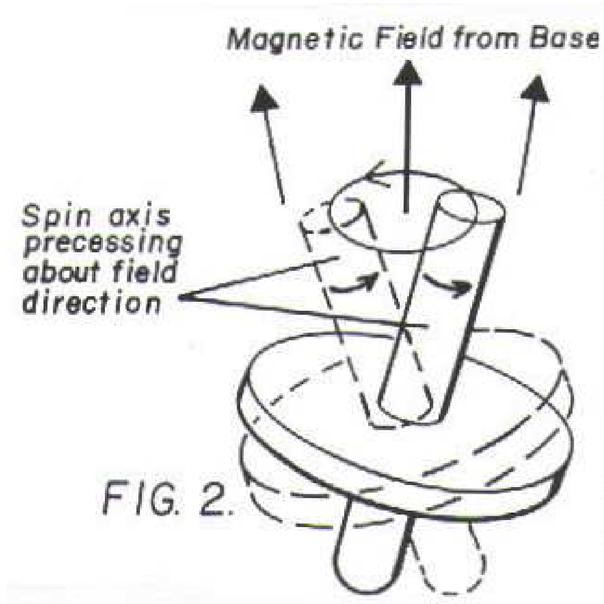


FIG. 2.

separation of time scales

$$\nu_{\mathrm{side}} \ll \nu_{\mathrm{precession}} \ll \nu_{\mathrm{rotation}}$$

Basic idea:

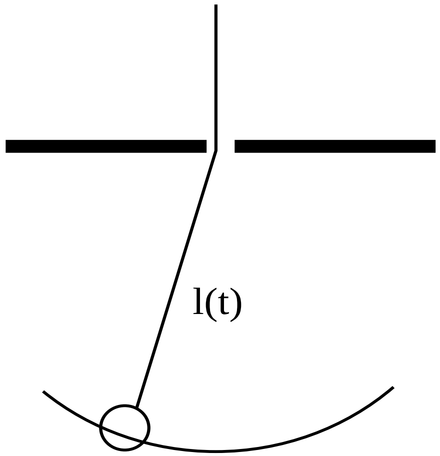
Gyroscopic effect in a continuous way adjust the precession axis in respect to the local external magnetic field $\mathbf{B}(\mathbf{r})$ (adiabatic approximation). In long time we find that in average the magnetic moment μ of the top is always antiparallel to the external field lines $\mathbf{B}(\mathbf{r})$.

Adiabatic invariant

Df.
Adiabatic invariant is a quantity which is (approximately) constant in time when parameters characterizing the system change slowly (adiabatically).

Example:
Mathematical pendulum with mass m. Length of the pendulum changes slowly

$$\frac{1}{l(t)}\frac{dl(t)}{dt} \ll \frac{1}{T}.$$



Then the quantity

$$E(t)\sqrt{l(t)} = \mathrm{const}$$

is an adiabatic invariant of the system.

Similarly, $T(t)/\sqrt{l(t)} = \mathrm{const}$ (because $T = 2\pi\sqrt{l/g}$)
and $E(t)T(t) = \mathrm{const}$ (the last one has a unit of an action).
□

Adiabatic invariant for levitron

In optimal equilibrium conditions the angle

$$\angle(\vec{\mu}(t), \vec{\mathbf{B}}(\mathbf{r}(t)))$$

is an adiabatic invariant of the system.

When there exists this (approximate) adiabatic invariant then the potential energy is a function of $B = \sqrt{B_x^2 + B_y^2 + B_z^2}$ (but not of a vector $\mathbf{B}$) and possesses a local minimum

$$U = U(|\mathbf{B}(\mathbf{r})|) = -|\vec{\mu}||\vec{\mathbf{B}}(\mathbf{r})| \cos\{\angle[\vec{\mu}(t), \vec{\mathbf{B}}(\mathbf{r})]\} + mgz.$$

Stable equilibrium can be effectively reached.

This fact is in agreement with Earnshaw's theorem because the system is dynamical.

Theoretical model of levitron

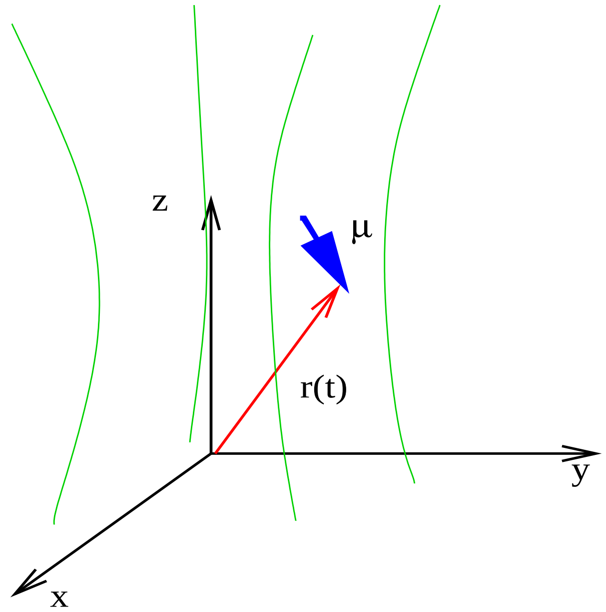
Potential energy

$$U(\mathbf{r}) = mgz(t) - \vec{\mu}\cdot\vec{\mathbf{B}}(\mathbf{r}(t))$$

Equations of motion

$$\frac{d\vec{\mathbf{p}}(t)}{dt} = \vec{\mathbf{F}}(\mathbf{r}(t)), \quad \text{where} \;\; \mathbf{F} = -\nabla \mathrm{U} = -\mathrm{mg}\hat{\mathrm{e}}_{\mathrm{z}} + \nabla\tilde{\mu}(\mathrm{t})\cdot\tilde{\mathbf{B}}(\mathbf{r}(\mathrm{t}))$$

$$\frac{d\vec{\mathbf{L}}(t)}{dt} = \vec{\mathbf{M}}(\mathbf{r}(t)), \quad \text{where} \;\; \tilde{\mathbf{M}} = \tilde{\mu}\times\tilde{\mathbf{B}}(\mathbf{r}(\mathrm{t}))$$



Two approximations:

- fast rotation $\nu_{\text{rotation}} \gg \nu_{\text{precession}}$
- fast precession $\nu_{\text{precession}} \gg \nu_{\text{side}}$

Fast rotation $\nu_{\text{rotation}} \gg \nu_{\text{precession}}$

$$\vec{\mathbf{L}} \;||\; \vec{\omega} \;||\; \vec{\mu}$$

hence

$$\vec{\mathbf{L}} = I\vec{\omega} = I\omega\frac{\vec{\mu}}{\mu}.$$

Equation of motion for $\mathbf{L}$ can be expressed by $B(t) = |\mathbf{B}(t)|$ and a temporal direction $\mathbf{b}(t)$ of the field 'as seen by' the top

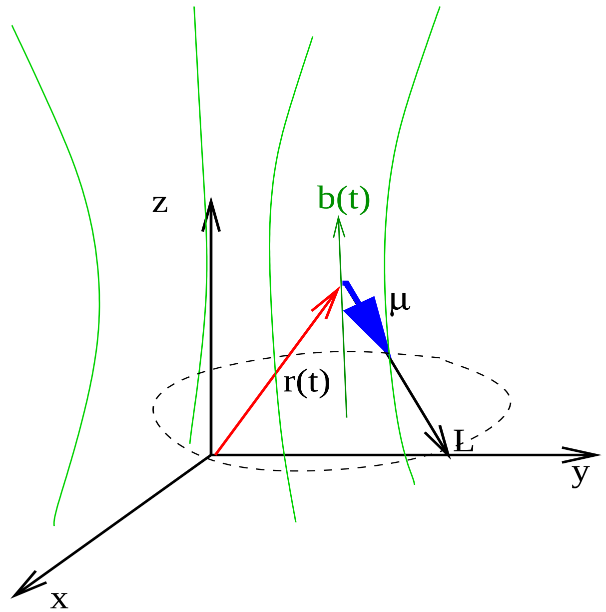
$$\mathbf{B}(t) = B(t)\mathbf{b}(t).$$

Hence

$$\frac{d\mathbf{L}(t)}{dt} = \Omega(t)\mathbf{b}(t) \times \mathbf{L}(t),$$

where

$$\Omega(t) = -\frac{\mu B(t)}{I\omega} = -\frac{\mu B(t)}{L}.$$



Precession of the top around temporal axis $b(t)$
with a conserved length of $\mathbf{L}$.

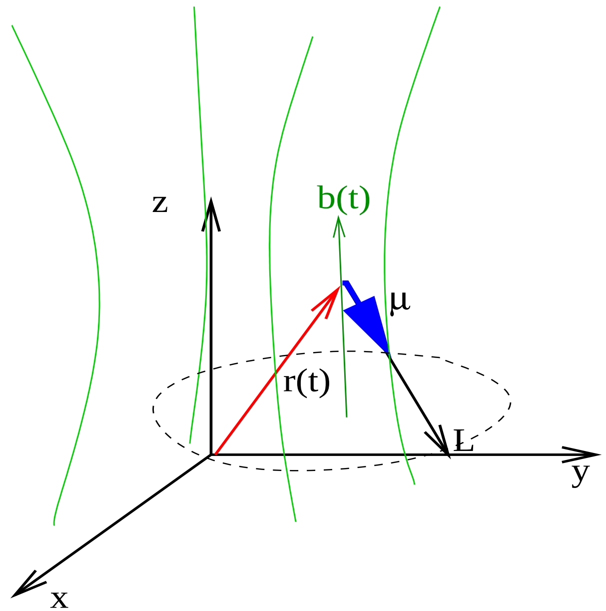
Fast precession $\nu_{\text{precession}} \gg \nu_{\text{side}}$

$$|\Omega(t)| \gg |\frac{db(t)}{dt}|$$

- Then $\mathbf{L}$ and $\mathbf{b}$ are correlated with each other such that $\mathbf{L} \cdot \mathbf{b}$ is the adiabatic invariant (the angle between $\mathbf{L}$ and $\mathbf{b}$ is constant).
- The quantity $\mu_B \equiv \vec{\mu}(t) \cdot \vec{\mathbf{b}}(t)$ is also the adiabatic invariant ($\vec{\mathbf{L}} \;\; || \;\; \vec{\omega} \;\; || \;\; \vec{\mu}$).
- Potential energy

$$U(\mathbf{r}) = mgz - \vec{\mu}(t) \cdot \vec{\mathbf{B}}(t) = mgz - \vec{\mu}(t) \cdot \vec{\mathbf{b}}(t)B(t) = mgz - \mu_B B(t)$$

depends on the length $|\mathbf{B}|$ and hence can possess a local minimum!



Conditions consistency

Note that

- fast rotations $\nu_{\rm rotation} \gg \nu_{\rm precession}$ means that ω is large
- fast precession $\nu_{\rm precession} \gg \nu_{\rm side}$ means that $\Omega = \mu_B / I\omega$ is large

The angular velocity for the spinning top is bounded from below and above

$$\omega_{min} \lesssim \omega \lesssim \omega_{max}$$

to occur a stable dynamical levitation.

Range of stability at OZ axis

Existence of extrema:

$$-mg + \mu \cdot \frac{\partial \mathbf{B}(0, z_0)}{\partial z} = 0,$$

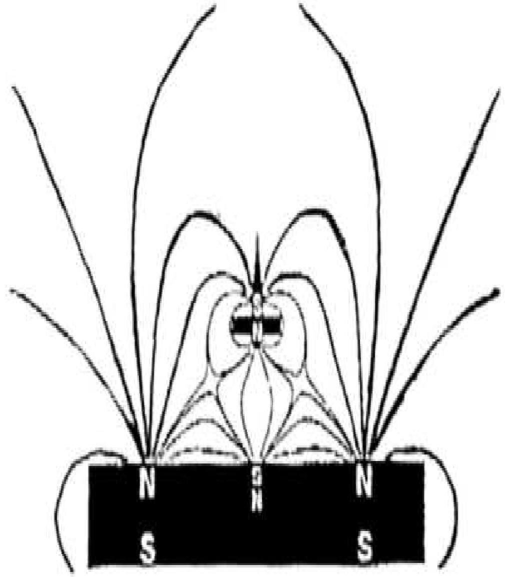
$$\frac{\partial \mathbf{B}(0, z_0)}{\partial \rho} = 0.$$

Existence of local minimum:

$$\frac{\partial^2 B(0, z_0)}{\partial z^2} > 0,$$

$$\frac{\partial^2 B(0, z_0)}{\partial \rho^2} > 0.$$

$B = \sqrt{B_\rho^2 + B_z^2}$ and axial symmetry assumed. $\mathbf{B}$.



Single loop with current

As an example one considers the magnetic field coming from a single loop of radius a with a current, i.e.

$$B_z(0, z) = \frac{B_o a^3}{(a^2 + z^2)^{\frac{3}{2}}}$$

Then we find the conditions for the stability point

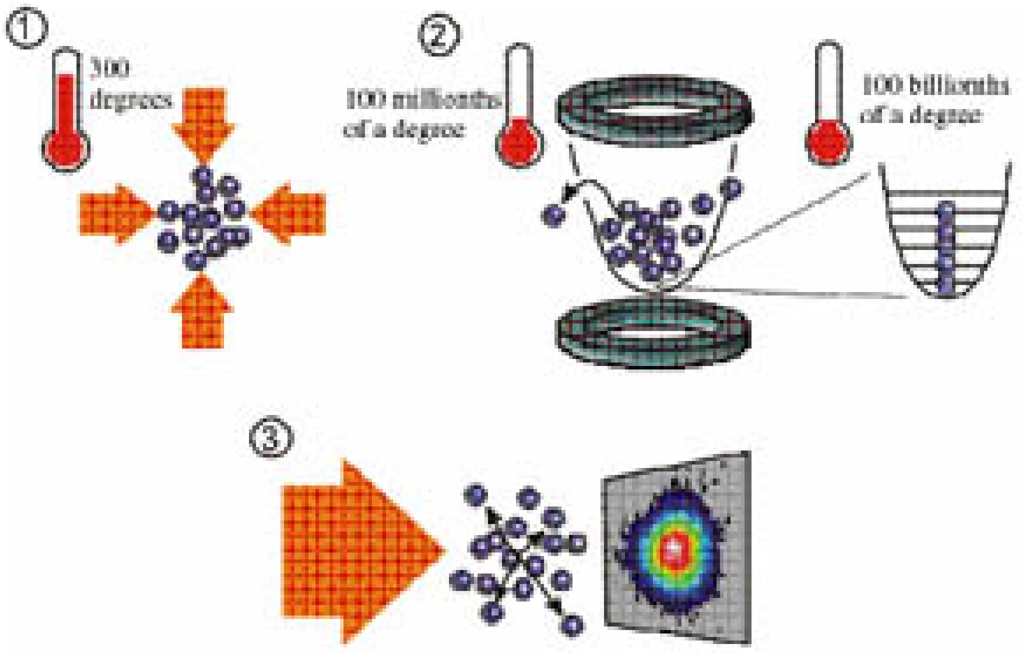
$$\frac{a}{2} < z_0 < \frac{a}{\sqrt{2}}$$

where z_0 depends explicitly on m, μ, ω and B_0.

In practice the range where the stable minimum exists is of the order 0,5cm.

Trapping ultra cold atoms

Physics of levitron is very similar to current research with neutral atoms and BEC



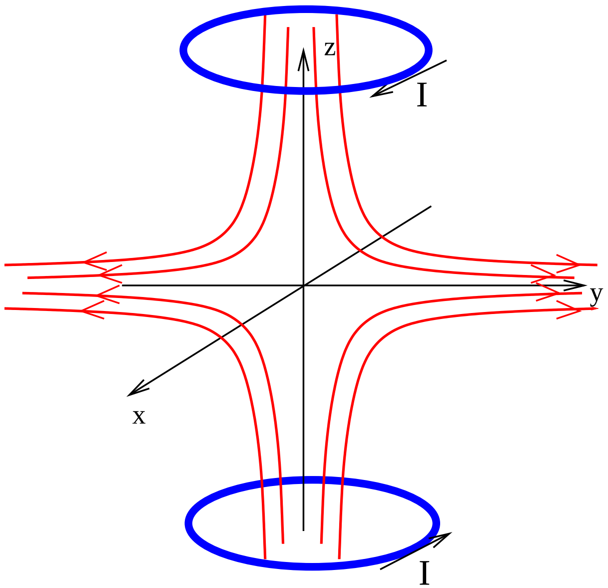
How to create a magnetic trap for atoms? Why parabolic potential so popular?

What Earnshaw says?

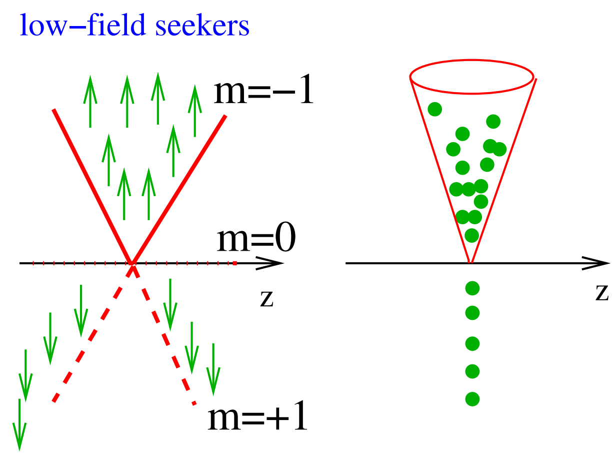
Magnetic trap for alkali atoms

- Alkali atoms have non-zero magnetic moment
- Stern-Gerlach experiment 1924 - nonuniform magnetic field separates different momenta
- Paul's trap 1985 - use anti-Helmholtz coils (applied Migdal *et al.* 1985)

$$U = -\vec{\mu} \cdot \vec{B}(\mathbf{r})$$



$$\vec{B} = B(x, y, -2z)$$



LEVITRON-LIKE PHYSICS

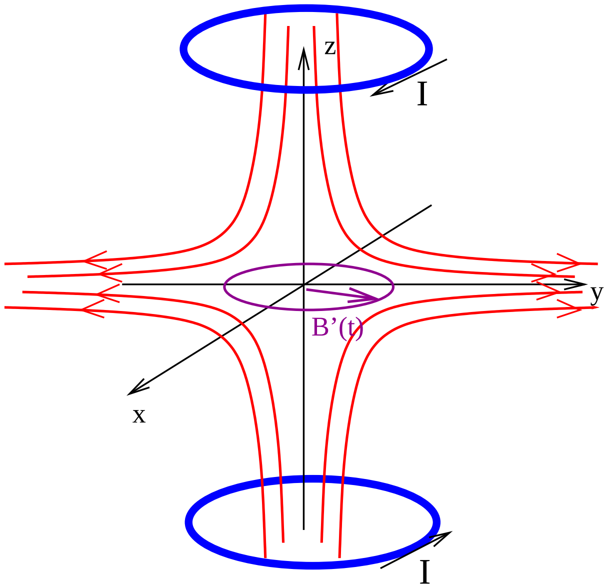
magnetic moments move adiabatically: $\angle[\vec{\mu}, \vec{B}(\mathbf{r})] = const.$

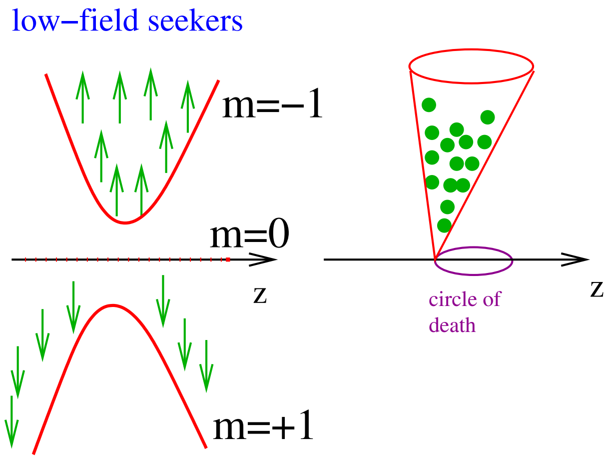
Larmor frequency $\frac{g\mu_B B}{\hbar} \equiv \omega_{L(precession)} \gg \omega_{T(side)} \sim 20Hz$

At origin $B = 0$ and adiabaticity lost, spin-flips, leak of particles!!!

Time-averaged orbiting potential (TOP)

To cure the leakage problem one applies time-dependent component of the external magnetic field





$$\vec{B} = B(x, y, -2z) + B'(\cos\omega_0 t, \sin\omega_0 t, 0) \qquad \omega_T \ll \omega_0 \sim 20kHz$$

$$\bar{B}(r) = \frac{\omega_0}{2\pi}\int_0^{2\pi/\omega_0} dt B(t) \approx B' + \frac{B^2}{4B'}(x^2 + y^2 + 8z^2) + \dots$$

$U(r) = g\mu_B \bar{B}(r)$ parabolic trap with local minimum to satisfy Earnshaw's theorem

Conclusions

- Levitron works owing to the existence of an adiabatic invariant
- Everything is in agreement with Earnshaw's theorem
- Exactly solvable model of levitron reduces to the effective theory with an adiabatic invariant
- Excellent educational toy for everybody
- Similarity to magnetic traps used to create Bose-Einstein condensate
- Ultra hot plasma, ultra cold neutrons, ...

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Happy New Year!