

# Supersolid - yet another exotic state of quantum condensed matter

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# To remember on supersolids

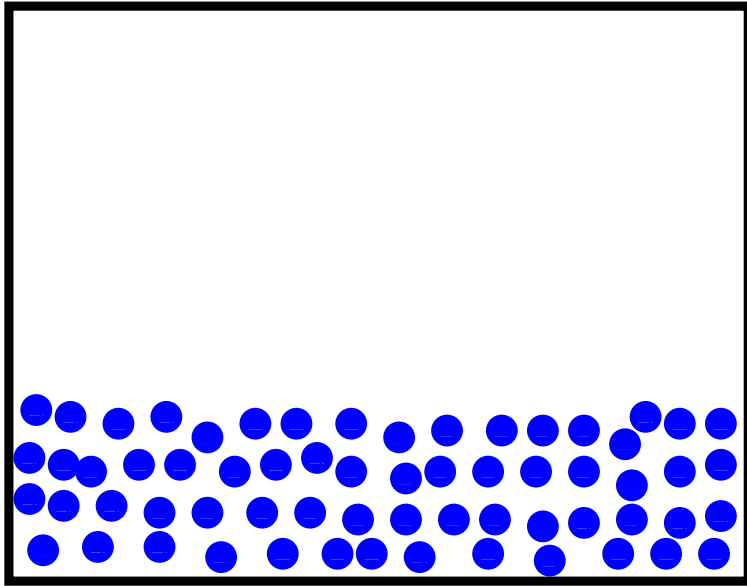
A system where its elements simultaneously

- have crystalline order, and
- flow like a liquid with no viscosity

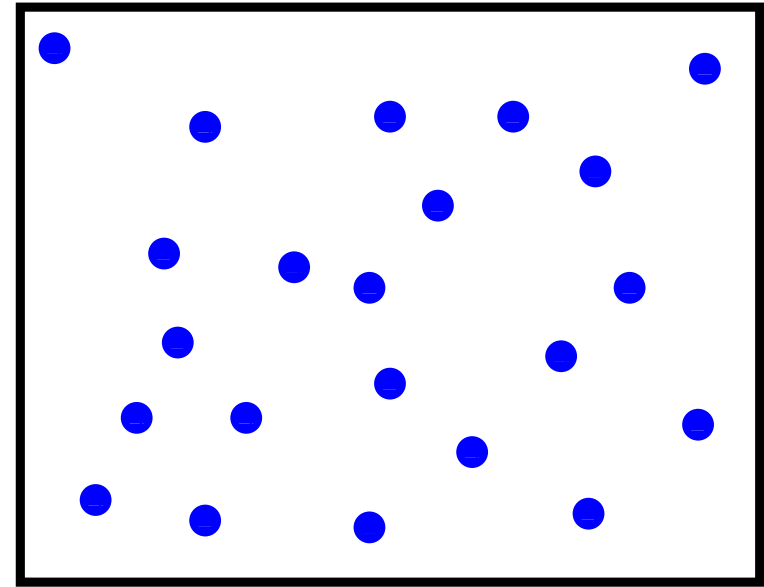
is called a **supersolid**

# Classical states of matter

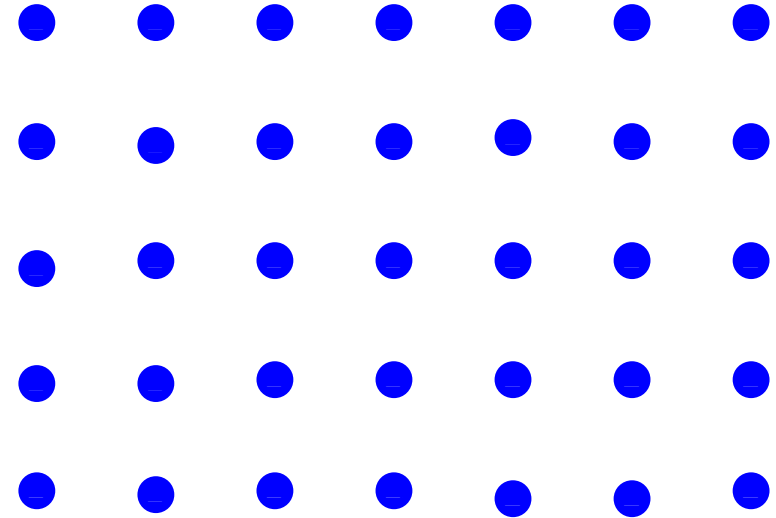
elements are basically distinguishable



liquid - ordering on short range scale

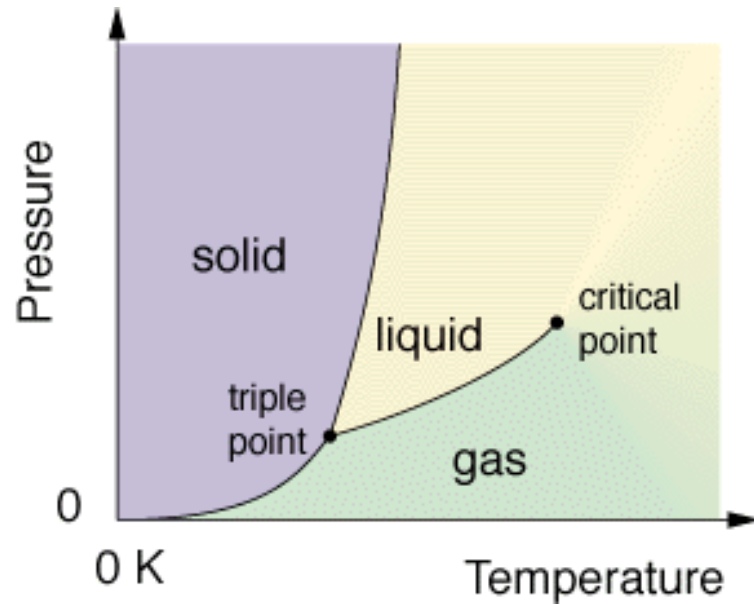


gas - no ordering at all



solid - ordering on any length scale  
Long Range Order

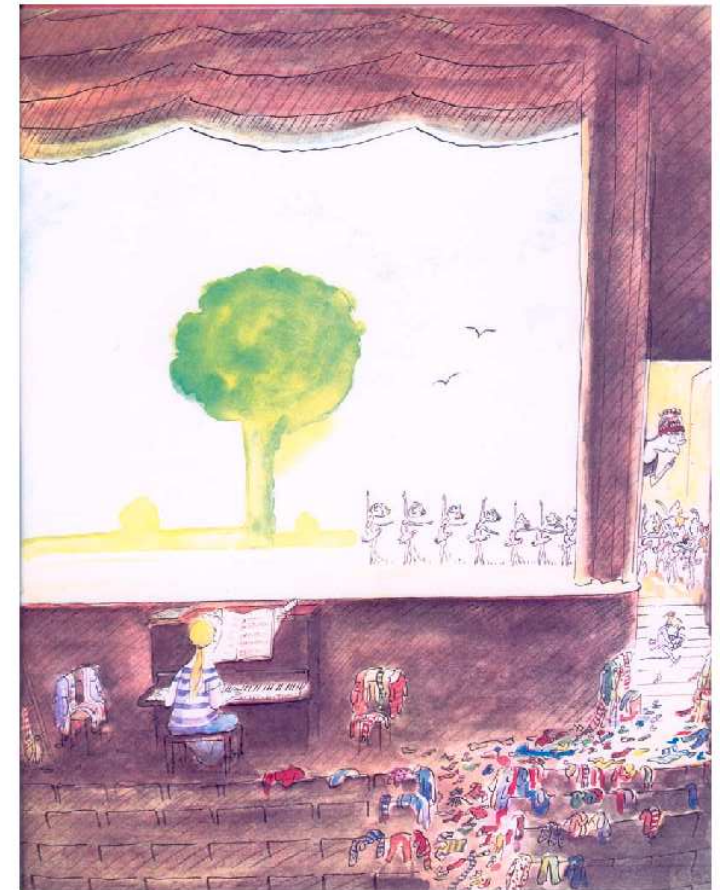
# Phase transitions - Order from chaos



## Long Range Order (LRO)

$$\langle \rho(\mathbf{r}_1)\rho(\mathbf{r}_2) \rangle \rightarrow \langle \rho(\mathbf{r}_1) \rangle \langle \rho(\mathbf{r}_2) \rangle \neq 0 \text{ for } |\mathbf{r}_1 - \mathbf{r}_2| \rightarrow \infty$$

crystal - long range correlations, rigidity



(Sempe)

# Quantum states of matter

When does quantum mechanics start to play a role?

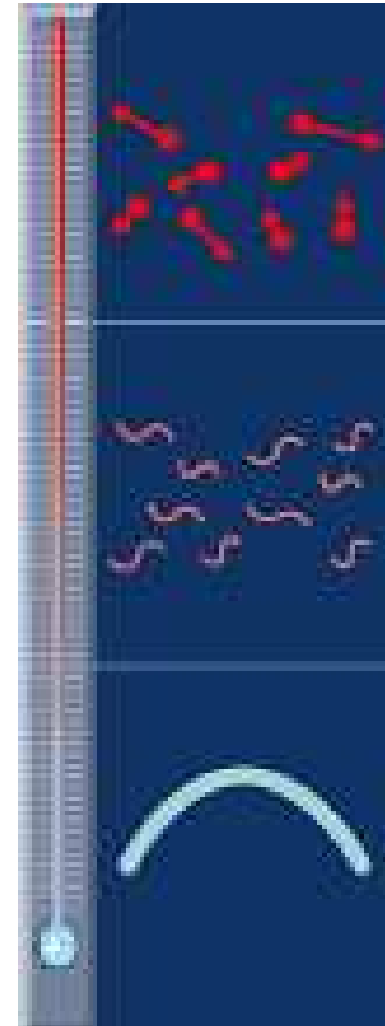
Thermal de Broglie wavelength:  $\lambda_{dB} = \sqrt{2\pi\hbar^2/mk_B T}$

Typical distance between atoms:  $d$

Quantum behavior if  $\lambda_{dB} \gtrsim d$

Neon (Ne) at  $T_c = 27\text{K}$ :  $\lambda_{dB} \approx 0.07\text{nm}$ ,  $d \approx 0.3\text{nm}$

Helium (He) at  $T_c = 4.2\text{K}$ :  $\lambda_{dB} \approx 0.4\text{nm}$ ,  $d \approx 0.27\text{nm}$



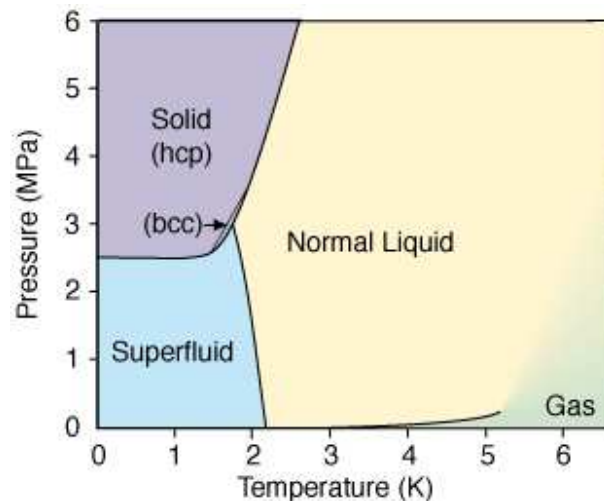
# Quantum states of matter

atoms are **indistinguishable** - bosons and fermions

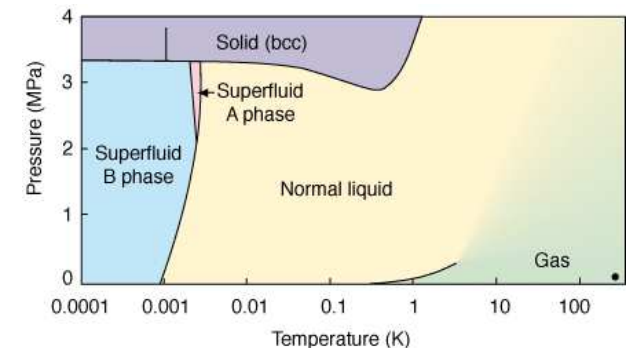
*quantum gas, quantum liquid, superfluid, quantum crystal, supersolid, ...*

$$\lambda_{dB} \gtrsim d$$

Isotopes:  $^4\text{He}$  (2p2n2e - “bosons”) and  $^3\text{He}$  (2p1n2e - “fermions”)



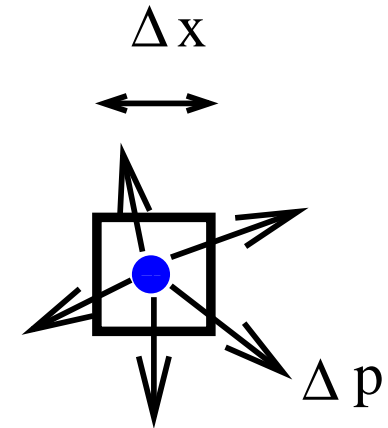
Quantum  
liquids at  $T=0\text{K}$ !



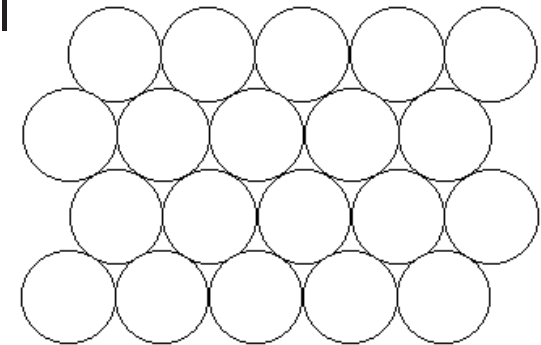
# Why quantum liquids at T=0K?

- Heisenberg uncertainty principle

$$\Delta x \Delta p \gtrsim \frac{\hbar}{2}$$



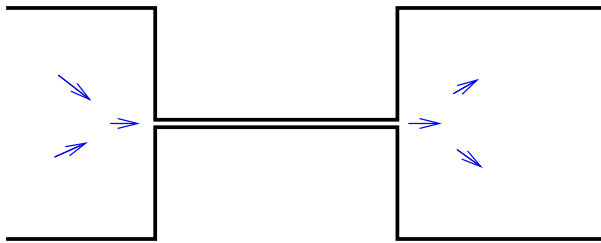
- the more localized particle in a space the faster it moves - zero point motions
- He at  $p_{atm}$  does not crystallize due to zero point motions
- kinetic energy is large as compared to  $v(r) < 0$  potential
- He crystallizes at  $p \gg p_{atm}$  due to hard core repulsion
- (as hard bowls form dense packed structure (hcp))



# Superfluid quantum liquid $T \leq T_\lambda$

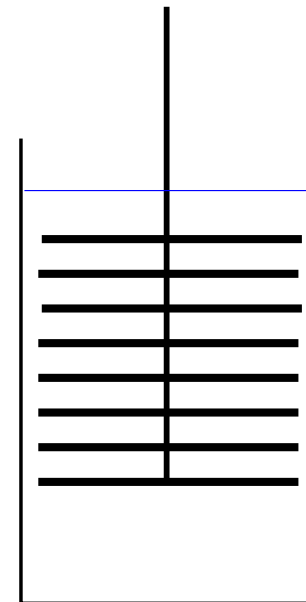
- frictionless flow through capillaries (**superflow**), creeping, fountain effect, ...

*P. Kapitsa, Nature* **141**, 74 (1938); *J.F. Allen and D. Meisener, ibid.*, 75 (1938); *E.L. Andronikashvili* 1946



$$\frac{\Delta p}{L} \sim \eta \frac{v}{R^2}$$

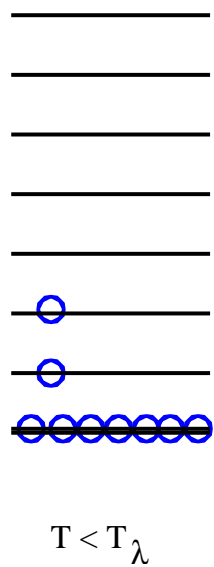
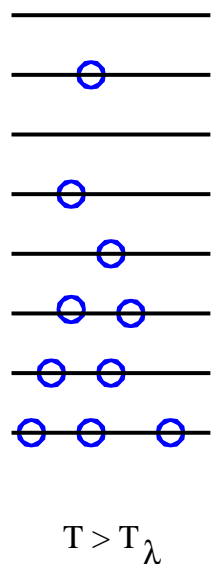
$\Delta p = 0$  and  $v > 0$  means zero viscosity  $\eta = 0$





# Microscopic theory of superfluid $^4\text{He}$

Type of Bose - Einstein condensation of interacting bosons



- macroscopic number  $N_0$  of bosons are in the lowest energy state
- $N_0$  bosons have the same wave function
- one-particle density matrix

$$\rho(\mathbf{r} - \mathbf{r}') = \langle \psi^\dagger(\mathbf{r})\psi(\mathbf{r}') \rangle$$

describes long-range order

$$\langle \psi^\dagger(\mathbf{r})\psi(\mathbf{r}') \rangle \rightarrow \langle \psi^\dagger(\mathbf{r}) \rangle \langle \psi(\mathbf{r}') \rangle = N_0/V = n_0$$

gdy  $\mathbf{r} - \mathbf{r}' \rightarrow \infty$

Off-diagonal long-range order (ODLRO)

# Macroscopic wave function of the condensate

- there exists a wave function of the condensate

$$\Psi(\mathbf{r}; T) = \sqrt{\rho_s(\mathbf{r})} e^{i\theta(\mathbf{r})}$$

order parameter

- flow velocity of the condensate ( $\rho_s = \text{const.}$ )

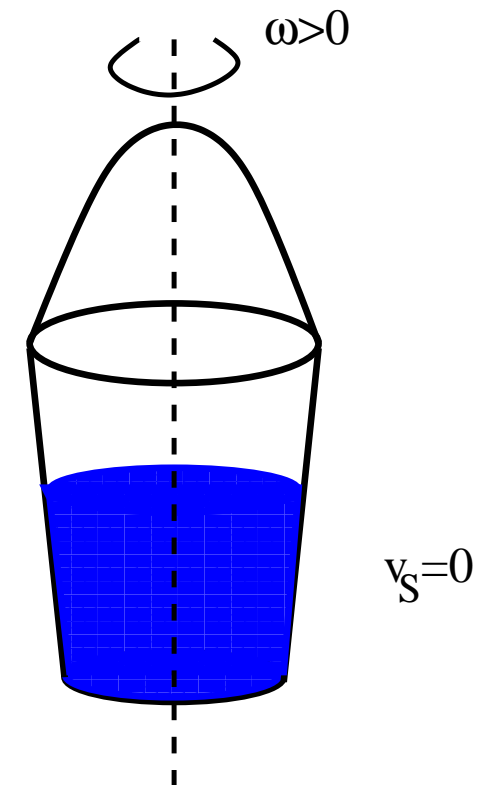
$$\mathbf{v}_s = \frac{\hbar}{m} \nabla \theta(\mathbf{r})$$

- in 'no holes space' flow is rotationless

$$\nabla \times \mathbf{v}_s = 0$$

- in rotating vessel with  $^4\text{He}$ , superfluid does not move

- non-classical moment of inertia (NCMI)



*London 1954, Hess i Fairbank 1967*

# Non-classical moment of inertia (NCRI)

Free energy

$$F(\Omega) = F_0 + \frac{1}{2}I_{\text{class}}\Omega^2$$

with  $F_0$  free energy at  $\Omega = 0$  and moment of inertia  $I_{\text{class}} = NmR^2$

NCRI means that at small  $\Omega$  new term appears

$$\Delta F(\Omega) = -\frac{1}{2}\frac{\rho_s}{\rho}I_{\text{class}}\Omega^2$$

Two fluid system (normal and superfluid) has smaller moment of inertia

$$I_{\text{NCRI}} = \left(1 - \frac{\rho_s}{\rho}\right) I_{\text{class}}$$

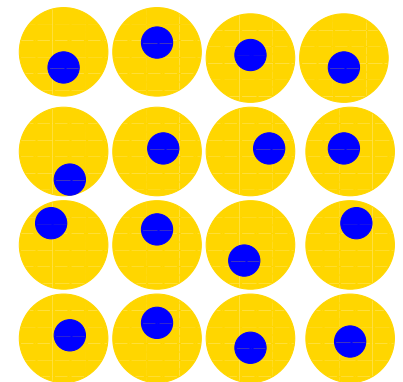
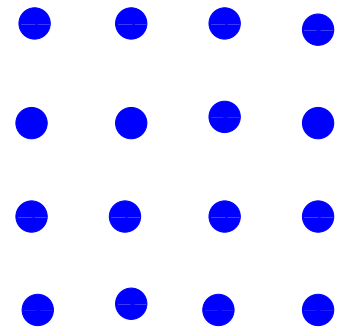
# Quantum crystals

- solid He at high pressure (25atm for  $^4\text{He}$  and 30atm for  $^3\text{He}$ )
- amplitude of zero motions is 30% of a lattice constant

classical crystal - small zero motions  $\Delta x \ll a$

quantum crystal - large zero motions  $\Delta x \gg a$

- in quantum crystals atoms are indistinguishable



# Quantum crystals - Supersolids

Can a quantum crystal be superfluid?

*M. Wolfke, Ann. Acad. Sci. Techn. Varsovie* **6**, 14 (1939)

diagonal long-range order

$$\langle \rho(\mathbf{r}_1)\rho(\mathbf{r}_2) \rangle \rightarrow \langle \rho(\mathbf{r}_1) \rangle \langle \rho(\mathbf{r}_2) \rangle \neq 0$$

off-diagonal long-range order

$$\langle \psi^\dagger(\mathbf{r})\psi(\mathbf{r}') \rangle \rightarrow \langle \psi^\dagger(\mathbf{r}) \rangle \langle \psi(\mathbf{r}') \rangle \neq 0$$

for  $|\mathbf{r}_1 - \mathbf{r}_2| \rightarrow \infty$

Solid-state rigidity and fluidity with zero viscosity

## Supersolid

# Quantum crystals - Commensurate insulators

**Insulating** variational wave function (*O. Penrose, L. Onsager 1951, 1956; C.N. Yang 1962*)

$$\Psi_G^{PO}(\mathbf{r}_1, \dots, \mathbf{r}_N) = \sqrt{\frac{1}{N!}} \sum_P \prod_{j=1}^N W(\mathbf{R}_j - \mathbf{r}_{Pj})$$

with  $W(\mathbf{R}_j - \mathbf{r}_{Pj})$  localized Wannier wave functions,  $N_L = N$

$$\langle \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}') \rangle = \frac{1}{N} \sum_{j=1}^N W^*(\mathbf{R}_j - \mathbf{r}) W(\mathbf{R}_j - \mathbf{r}') \rightarrow 0$$

no ODLRO (some delocalization, exchange is needed)

# Quantum crystals - Commensurate supersolids

**Bose-Einstein condensate** variational wave function with exchange  
(*L. Reatto 1969, G.V. Chester 1970, A. Leggett 1970*)

$$\Psi_G^{BEC}(\mathbf{r}_1, \dots, \mathbf{r}_N) = \prod_{i=1}^N \left( \frac{1}{\sqrt{N}} \sum_{j=1}^N W(\mathbf{R}_j - \mathbf{r}_i) \right)$$

density wave modulation with macroscopic condensation of all  $N$  bosons in one-particle state  $\phi(\mathbf{r}) = \frac{1}{\sqrt{N}} \sum_{j=1}^N W(\mathbf{R}_j - \mathbf{r})$

$$\langle \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}') \rangle \rightarrow \rho_s \phi^*(\mathbf{r}) \phi(\mathbf{r}')$$

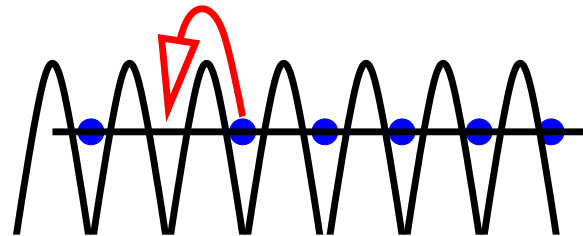
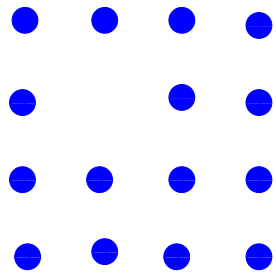
**Zero probability to occur in Nature a commensurate supersolid in continuous space**  
(*N. Prokof'ev, B. Svistunov 2005*)

$$N_L = N \times k$$

condition can happen by accident or be designed in laboratory (cold atoms)

# Incommensurate quantum crystals - vacancies

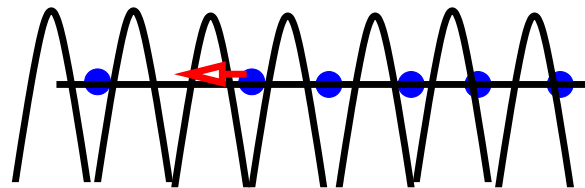
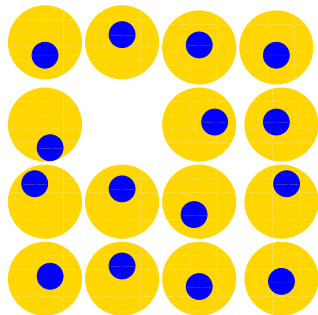
a vacancy moves backward as compared with movements of atoms



## classical crystal

to overcome energy barrier  
activation energy  $\Delta$  needed

$$p \sim \exp(-\Delta/k_B T)$$



## quantum crystal

atoms tunnel

vacancy is not static

vacancy becomes a quasiparticle

**vacanson, defecton**

$$|\mathbf{k}\rangle = \sum_i e^{i\mathbf{k}\mathbf{R}_i} |\mathbf{R}_i\rangle$$

coherent quasiparticles with dispersion  $\epsilon(\mathbf{k})$



# Incommensurate quantum crystals - Supersolid

*Andreev, Lifshitz (1969), Chester (1970), Leggett (1970), ...*

- defectons in solid  $^4\text{He}$  are bosons
- BEC condensation of defectons below  $T_c$
- BEC defectons are superfluid
- BEC defectons do not move with the system
- moment of inertia should be reduced

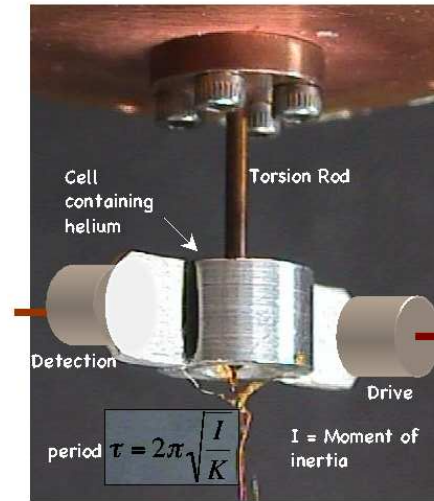
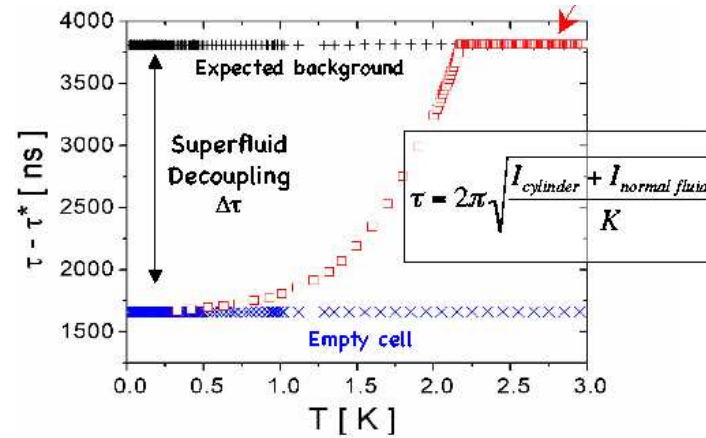
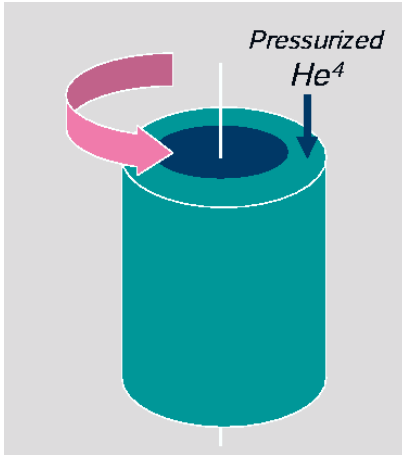


$$I(T) = I_{\text{class}} \cdot \left(1 - \frac{\rho_s(T)}{\rho}\right)$$

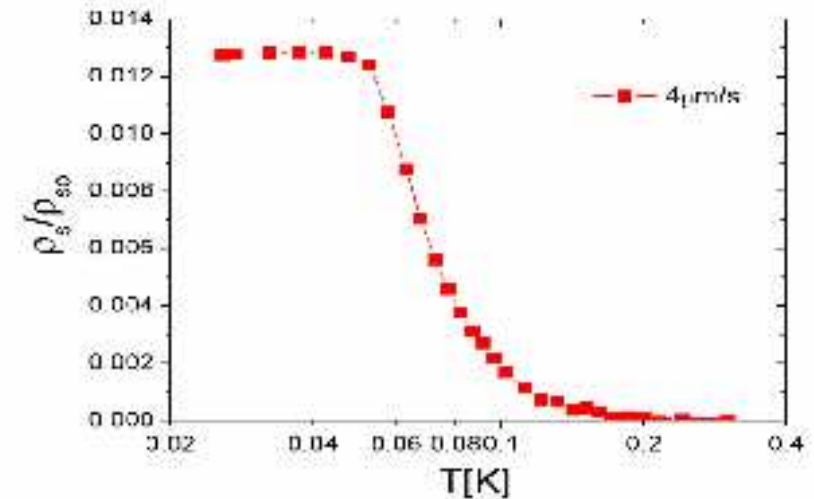
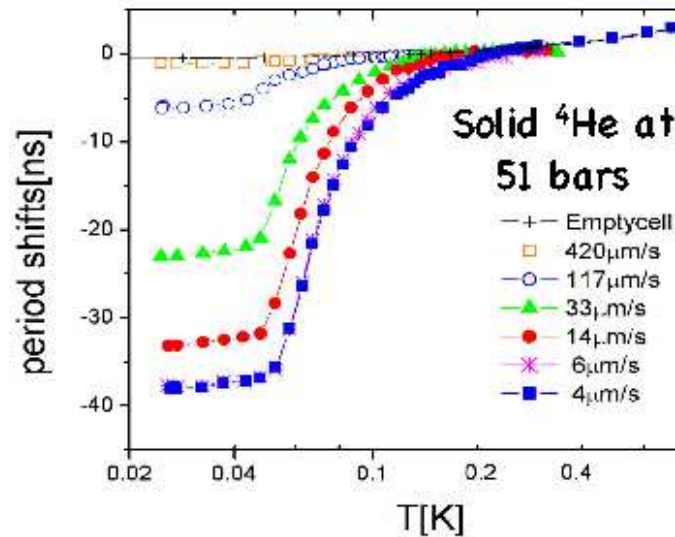
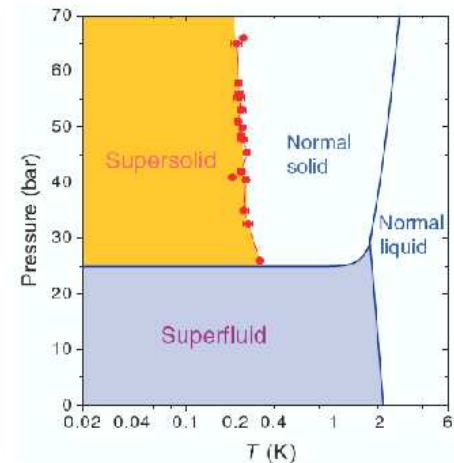
**supersolid = LRO + ODLRO**

# “Mary goes round” experiments

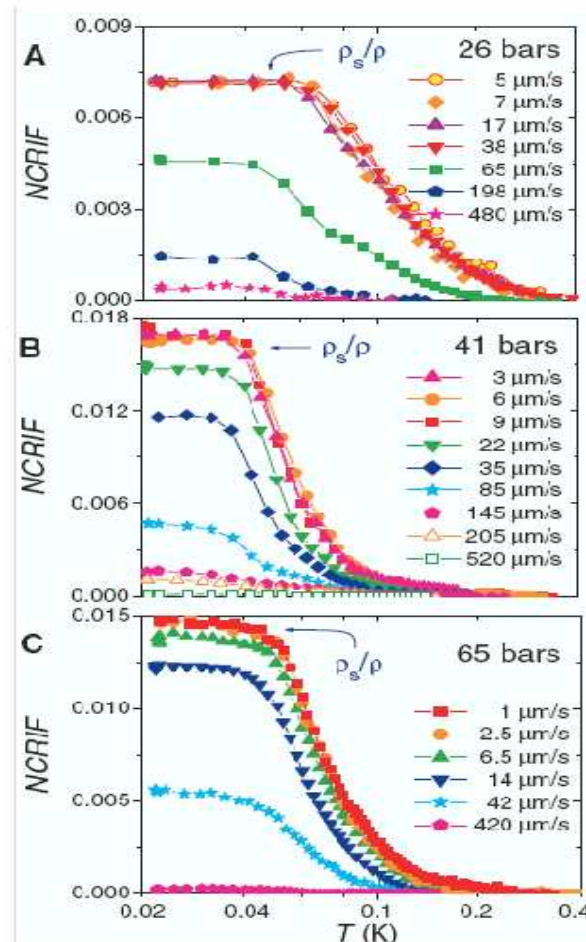
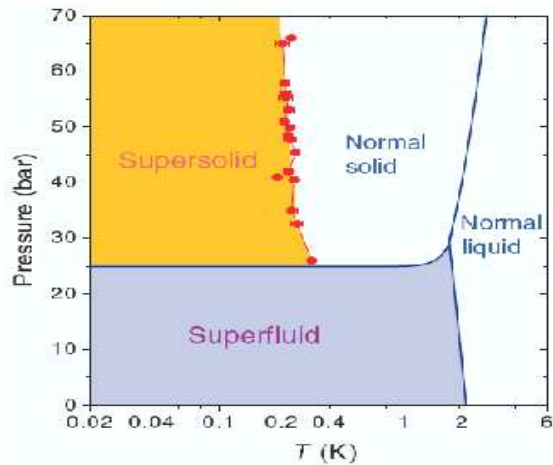
*E. Kim, M.H. Chan, Nature (2004), Science (2004), J. Low Temp. Phys. (2005)*



in solid <sup>4</sup>He below 0.2K decrease of oscillation period - moment of inertia decreases



# Supersolid?



in Kim and Chan's experiments - ca. 1,3% of a crystal is a supersolid  
problems: in solid helium number of defects is orders of magnitudes smaller!

commensurate supersolid, phase separation, defects, microcrystals, vortices, glass ...???

Experiment positively reproduced in 3 other laboratories but with history depend results!

# Conclusions and outlooks

- supersolid combines crystal rigidity and superfluidity
- idea applies to H, He, cold atoms in MOT and optical lattices, or atom impurities in crystal hosts, and superconducting electrons
- Kim and Chan's experiments still call for a correct interpretation ...
- ... but they have certainly renewed an interest in **supersolids**

## Reading:

1. E. Kim, M.H. Chan, *Nature* **425**, 227 (2004); *Science* **305**, 1941 (2004).
2. *Physics Today*, November 2004, p. 23.
3. *Adv. in Physics* **56**, 381 (2007); *Contemporary Phys.* **48**, 31 (2007); *Science* **319**, 120 (2008).
4. A. Leggett, *Quantum liquids* (Oxford Univ. Press 2006).
5. K. Byczuk, *Postepy Fizyki* **58**, 194 (2007).