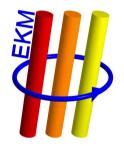
# Kinks in the dispersion of strongly correlated electrons

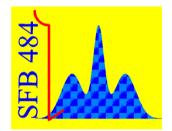
Krzysztof Byczuk

Institute of Physics, EKM, Augsburg University, Germany

May 31th, 2007



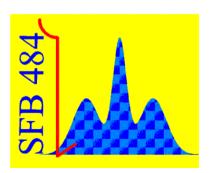




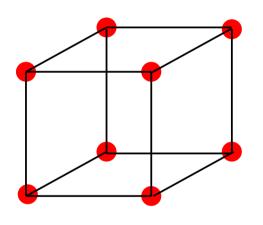
K. Byczuk, M. Kollar, K. Held, Y.-F. Yang, I.A. Nekrasov, Th. Pruschke, D. Vollhardt Nature Physics **3**, 168 (2007)

### **Collaboration**

- M. Kollar, D. Vollhardt, Augsburg, Germany
- K. Held, Y.-F. Yang, Stuttgart, Germany
- I. Nekrasov, Ekaterinburg, Russia
- T. Pruschke, Göttingen, Germany



### Standard model of quantum many-body system



#### emergent particles

quasiparticle

quasihole

holon

spinon

plasmon

magnon

phonon

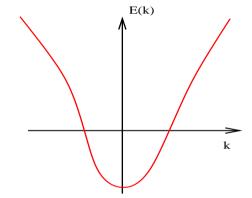
polariton

exciton

anyon

g-on

. . .



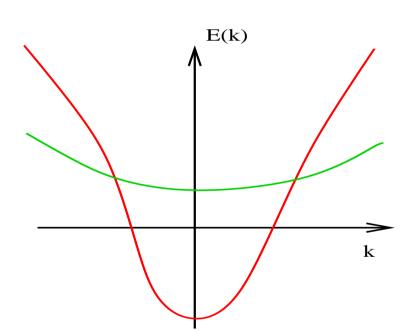
- (i) well defined dispersion relation  $E(\mathbf{k})$
- (ii) long (infinite) life-time au
- (iii) proper set of quantum numbers

(iv) statistics

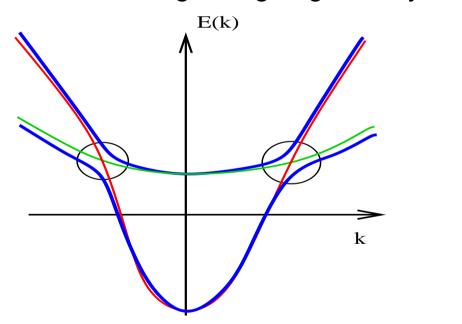
# **Dispersions and kinks**

Coupling/hybridization  $\hat{V}$  between different particles/modes

$$\langle \Psi | \hat{V} | \Phi \rangle \neq 0$$



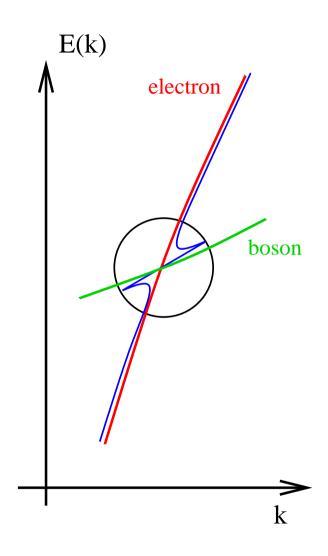
anticrossing, lifting degeneracy, ...



Df. kinks are abrupt slope changes in the dispersion relations

# Provide information on modes and couplings

# Dispersions and kinks - coupling to bosons



energy of a kink is related to energy of a bosonic fluctuation

### Dispersion of correlated electrons

One-particle spectral function - excitations at  ${\bf k}$  and  $\omega$ 

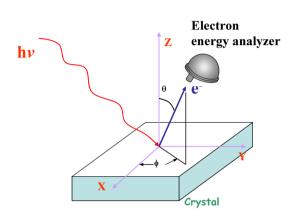
$$A(\mathbf{k}, \omega) = -\frac{1}{\pi} \operatorname{Im} \frac{1}{\omega + \mu - \epsilon_{\mathbf{k}} - \Sigma(\mathbf{k}, \omega)}$$

Dispersion relation  $E_{\mathbf{k}}$ 

$$E_{\mathbf{k}} = \{ \omega \text{ where } A(\mathbf{k}, \omega) = \max \}$$

Dispersion relation is experimentally measured

### **Angular Resolved Photoemission Spectroscopy**

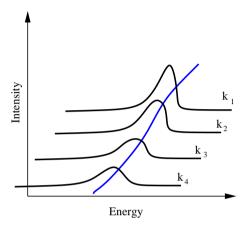


$$k_x = k\cos\phi$$

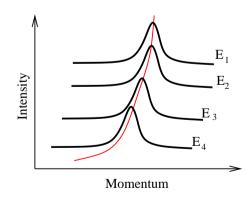
$$k_y = k \sin \phi$$

$$E = k^2/2m$$

energy resolution 1meV

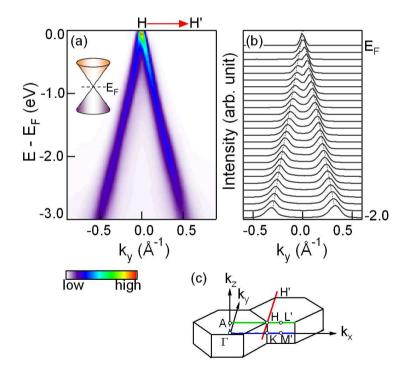


energy distribution curve (EDC)



momentum distribution curve (MDC)

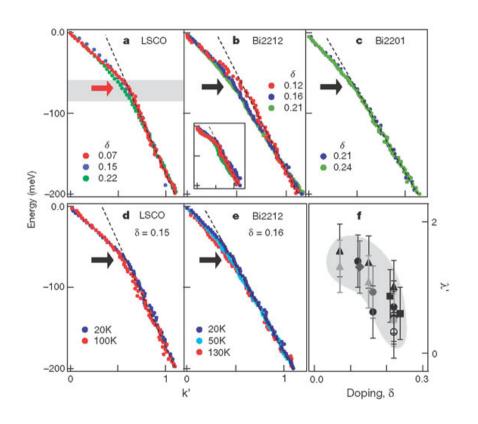
# **ARPES** and graphene



Dirac linear dispersion relation for graphene

cond-mat/0608069

### **Kinks in HTC**

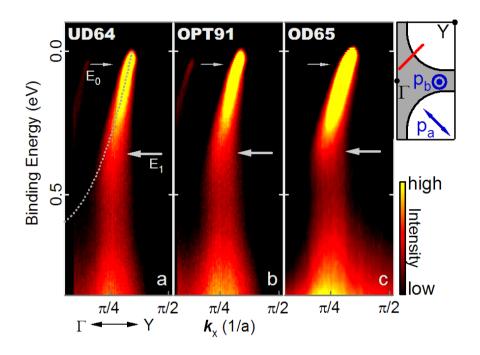


cond-mat/0604284

Kinks at 40 - 70meV

electron-phonon or electron-spin fluctuations coupling

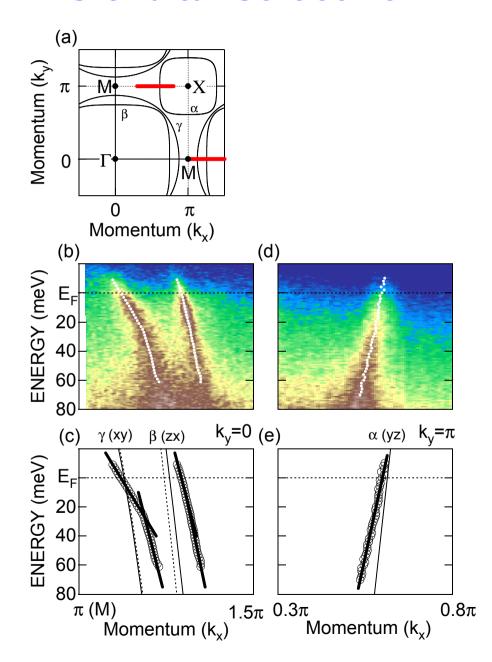
### "Waterfalls" in HTC



different HTC systems, cond-mat/0607319

Kinks seen experimentally between 300-800 meV Origin: phonos, spin fluctuations, not known yet

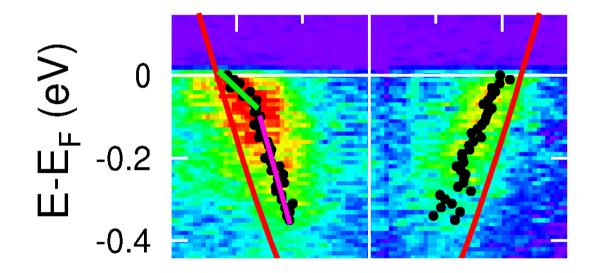
### Kinks orbital selective



Kink at 30meV in  $\gamma$ -band only

 $Sr_2RuO_4$ , cond-mat/0508312

### More examples of kinks in ARPES



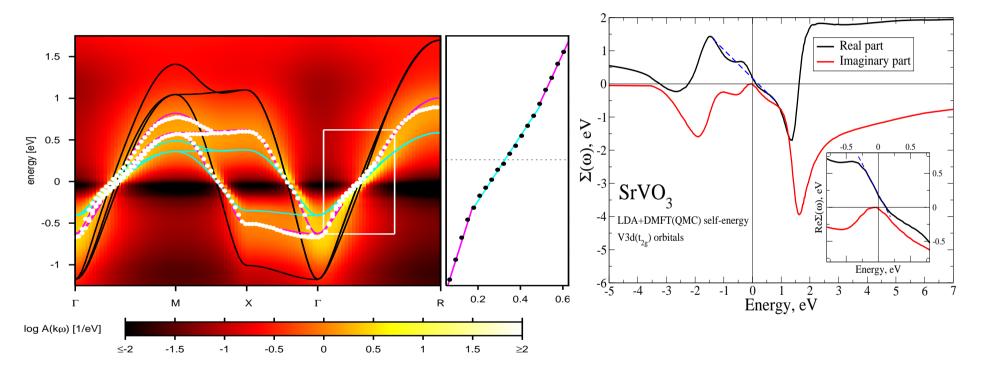
SrVO<sub>3</sub>, cond-mat/0504075

Kinks seen experimentally at 150 meV Pure electronic origin?

# Kinks in LDA+DMFT study of SrVO<sub>3</sub>

plain band model with local correlations, no other bosons, ... but kinks!

I.A. Nekrasov *et al.*, cond-mat/0508313, PRB (2006)



$$G_{\mathbf{k}}(\omega) = \frac{1}{\omega + \mu - \epsilon_{\mathbf{k}} - \Sigma(\omega)} \longrightarrow E_{\mathbf{k}} + \mu - \epsilon_{\mathbf{k}} - \operatorname{Re}\Sigma(E_{\mathbf{k}}) = 0$$

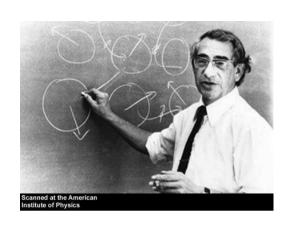
Not found in SIAM with simple hybridization function! → DMFT self-consistency effect

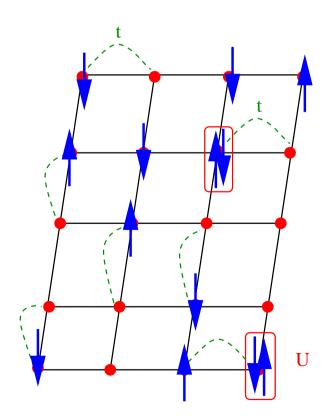
# New purely electronic mechanism

- in strongly correlated systems
- characteristic energy scale
- range of validity for Fermi liquid theory

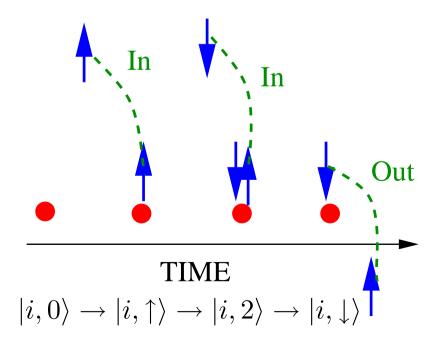
### Hubbard model for strongly correlated electrons

$$H = -\sum_{ij\sigma} t_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$





#### Local Hubbard physics



### All what we know about Hubbard model

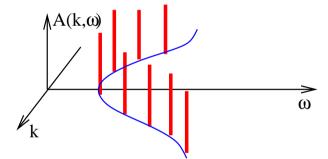
Solved in U = 0 limit (non-interacting limit)

$$G_{\sigma}(\mathbf{k}, \omega) = \frac{1}{\omega + \mu - \epsilon_{\mathbf{k}}}$$

Dispersion relation

$$\epsilon_{\mathbf{k}} = \sum_{j(i)} t_{ij} e^{i\mathbf{k}(\mathbf{R}_i - \mathbf{R}_j)}$$

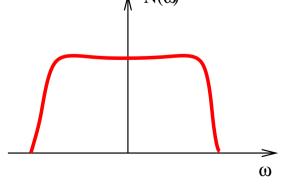
Spectral function - one-particle excitations



$$A_{\sigma}(\mathbf{k},\omega) \equiv -\frac{1}{\pi} \text{Im} G(\mathbf{k},\omega) = \delta(\omega + \mu - \epsilon_{\mathbf{k}})$$

Density of states (DOS) - thermodynamics

$$N_{\sigma}(\omega) \equiv \sum_{\mathbf{k}} A(\mathbf{k}, \omega) = \sum_{\mathbf{k}} \delta(\omega + \mu - \epsilon_{\mathbf{k}})$$



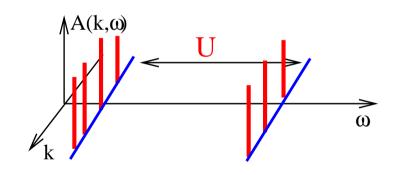
### All what we know about Hubbard model

Solved in t = 0 limit (atomic limit)

$$G_{\sigma}(\mathbf{k},\omega) = \frac{1 - n_{-\sigma}}{\omega + \mu} + \frac{n_{-\sigma}}{\omega + \mu - U} = \frac{1}{\omega + \mu - \Sigma_{\sigma}(\omega)}$$

#### Real self-energy

$$\Sigma_{\sigma}(\omega) = n_{\sigma}U + \frac{n_{-\sigma}(1 - n_{-\sigma})U^{2}}{\omega + \mu - (1 - n_{-\sigma})U}$$

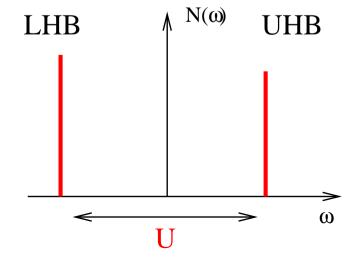


#### Spectral function

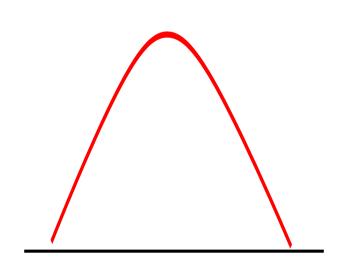
$$A_{\sigma}(\mathbf{k},\omega) = (1 - n_{-\sigma})\delta(\omega + \mu) + n_{-\sigma}\delta(\omega + \mu - U)$$

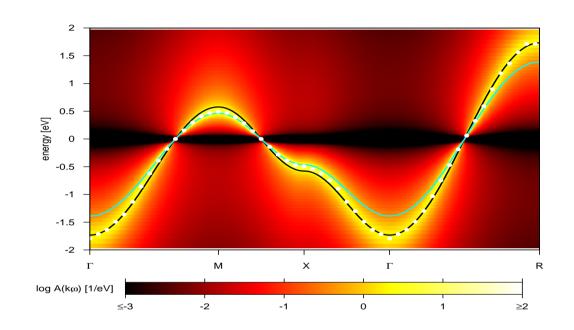
Green function and self-energy are local,

i.e. k independent



# Weakly correlated system

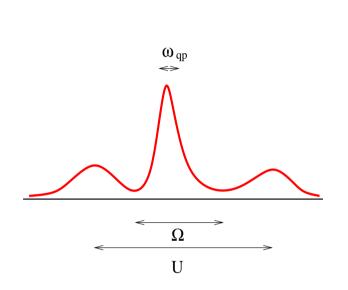


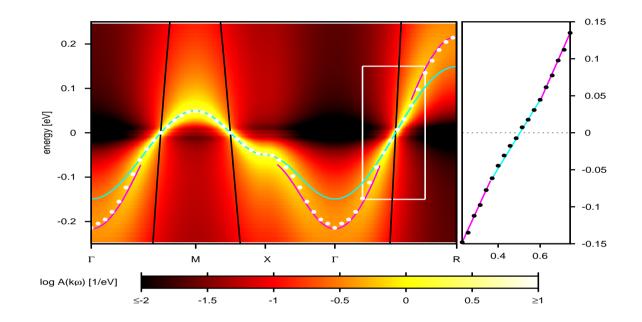


Fermi liquid 
$$Z_{FL} \lesssim 1$$
:  $E_{\mathbf{k}} = Z_{FL} \epsilon_{\mathbf{k}}$  for  $|E_{\mathbf{k}}| < \omega_*$ 

$$E_{\mathbf{k}} = \epsilon_{\mathbf{k}} \text{ for } |E_{\mathbf{k}}| > \omega_*$$

### Kinks due to strong correlations





Fermi liquid  $Z_{FL} \ll 1$ :  $E_{\mathbf{k}} = Z_{FL} \epsilon_{\mathbf{k}}$  for  $|E_{\mathbf{k}}| < \omega_*$ 

Different renormalization  $Z_{CP} \ll 1$ :  $E_{\mathbf{k}} = Z_{CP} \epsilon_{\mathbf{k}} \pm c$  for  $|E_{\mathbf{k}}| > \omega_*$ 

### Mathematical explanation of kinks within DMFT

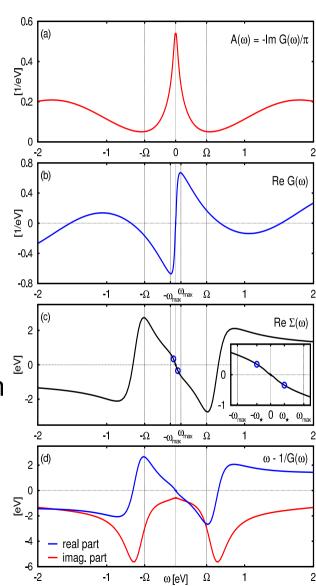
#### DMFT self-consistency condition

$$\Sigma(\omega) = \omega - 1/G(\omega) - \Delta(G(\omega))$$

$$\Delta(G(\omega)) \approx (m_2 - m_1^2)G(\omega) + \dots$$

Three-peak structure sufficient condition

Fermi-liquid for  $|\omega| < \omega_* \sim Z_{FL}$ 



# **Microscopic predictions**

#### Starting from:

ullet  $\epsilon_{\mathbf{k}}$  - bare dispersion relation

$$G_0(\omega) = \sum_{\mathbf{k}} \frac{1}{\omega - \epsilon_{\mathbf{k}}}$$

 $\bullet$   $Z_{FL}$ 

we predict that:

### **Microscopic predictions**

Kink position

$$\omega_* = 0.41 Z_{FL} \frac{\text{Im}1/G_0}{\text{Re}G_0'/G_0^2}$$

Intermediate energy regime

$$Z_{CP} = Z_{FL} \frac{1}{\text{Re}G_0'/G_0^2}$$

- Change in the slope  $Z_{FL}/Z_{CP}$  interaction independent
- ullet Curvature of the kink  $\sim Z_{FL}^2$
- ullet Sharpness of the kink  $\sim 1/Z_{FL}^2$
- ullet Sharper for stronger U

# Outlook: possible origin of the "waterfalls"

"Waterfalls": kinks at  $\omega_{\star} \approx 300\text{-}400 \text{ meV}$  in cuprates

crossover to Hubbard bands?

Wang et al. (2006)

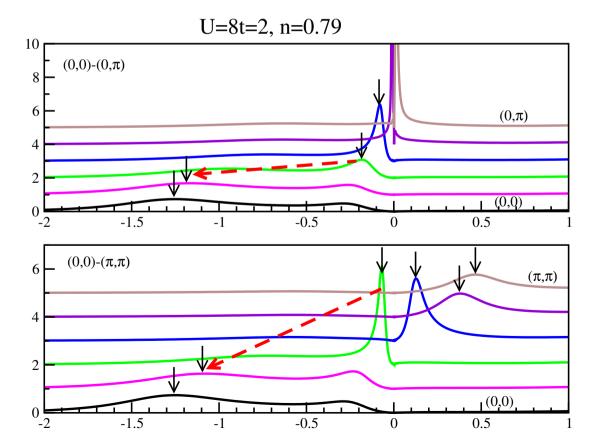
•  $U \gg t \Rightarrow$  dispersion goes from central peak to Hubbard band

K. Byczuk, M. Kollar (unpublished)

$$\Sigma(\omega) = \Sigma_0 + \frac{\Sigma_1}{\omega} + O\left(\frac{1}{\omega^2}\right)$$

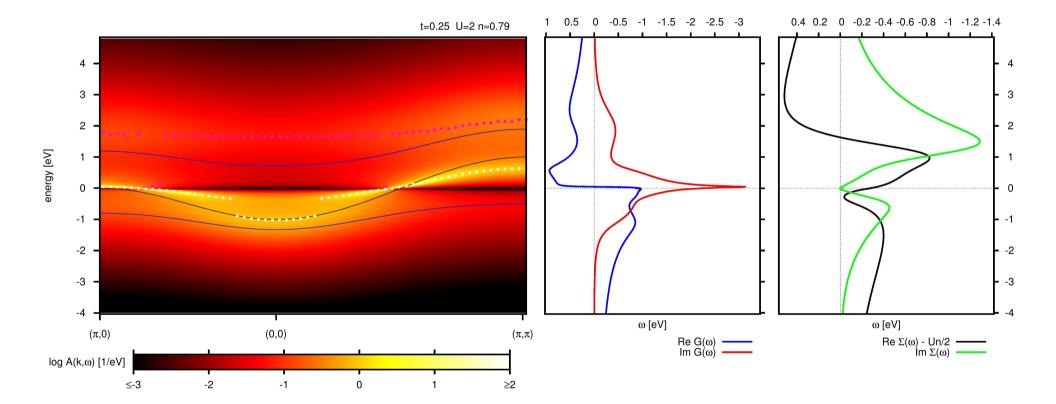
$$\downarrow \downarrow$$

$$E_{\mathbf{k}}^{\mathsf{UHB,LHB}} pprox rac{1}{2} \left[ \boldsymbol{\epsilon}_{\mathbf{k}} \pm \sqrt{\boldsymbol{\epsilon}_{\mathbf{k}}^2 + cU^2} 
ight]$$



### Crossover to Hubbard bands

Hubbard model, square lattice, DMFT(NRG), U = 8t, n = 0.79



- Im $\Sigma$  decays faster than Re $\Sigma$
- for large energies:  $E_{k}$  approaches  $E_{k}^{\text{UHB,LHB}}$
- waterfalls from central peak to LHB

### **Conclusions**

- Strong correlations (three peak spectral function) a sufficient condition for electronic kinks
- Energy scale for electronic kinks  $\omega_* = Z_{FL}D$  determined by Fermi-liquid renormalization and bare (LDA) density of states
- $\omega_*$  sets the energy scale for Fermi-liquid regime where  $E_{\bf k}=Z_{FL}\epsilon_{\bf k}$  for  $|E_{\bf k}|<\omega_*$
- Beyond Fermi-liquid regime the dispersion is still renormalized and useful  $E_{\bf k}=Z_{CP}\epsilon_{\bf k}\pm c$  for  $|E_{\bf k}|>\omega_*$  where the offset c and  $Z_{CP}$  determined by  $Z_{FL}$  and D
- Electronic kinks are within cluster extension of DMFT (DCA)  $\Sigma_{\mathbf{K}}(\omega) = \omega \frac{1}{G_{\mathbf{K}}(\omega)} \Delta(G_{\mathbf{K}}(\omega))$
- Electronic kinks and waterfalls are generic feature of strongly correlated systems