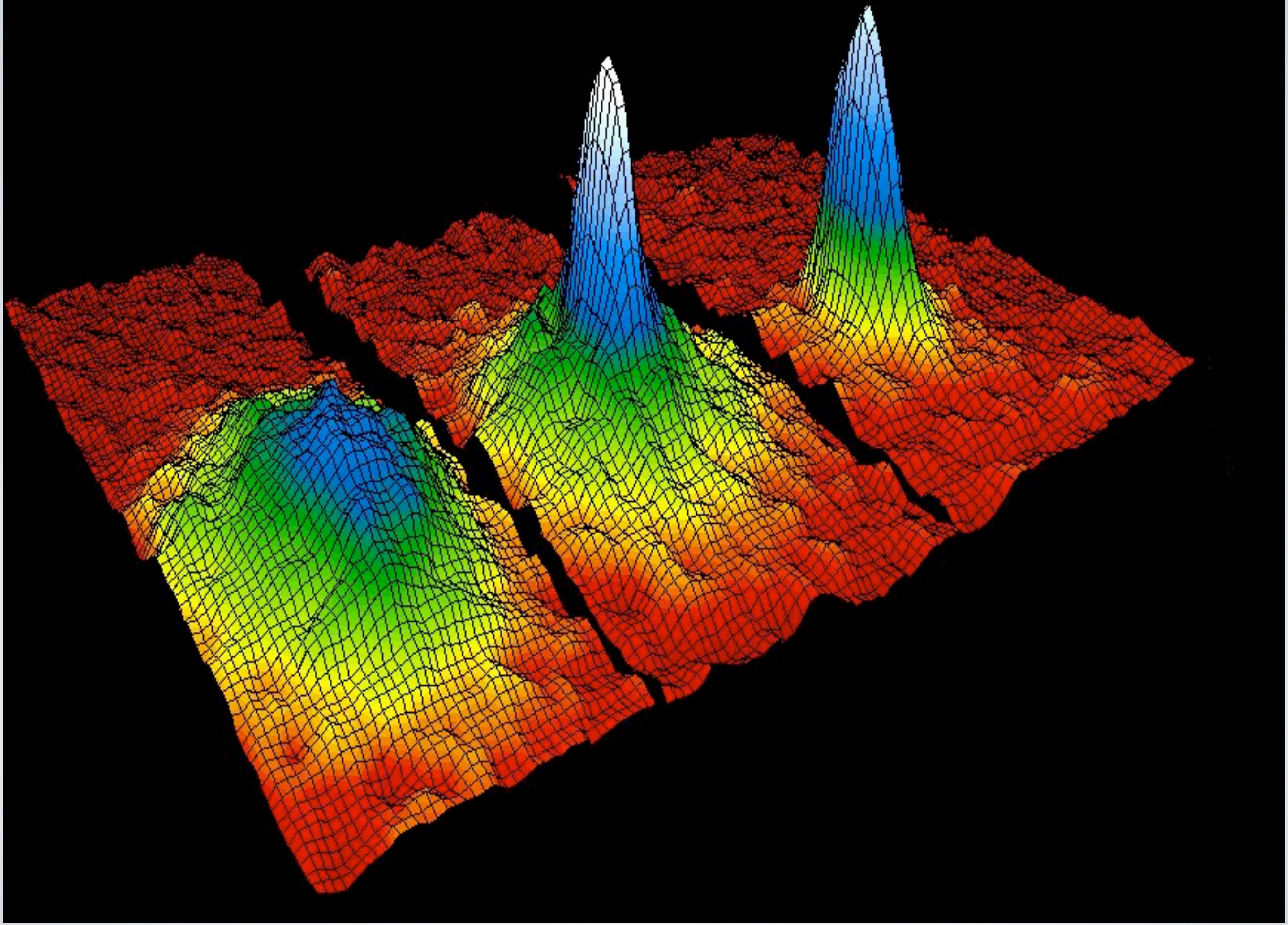
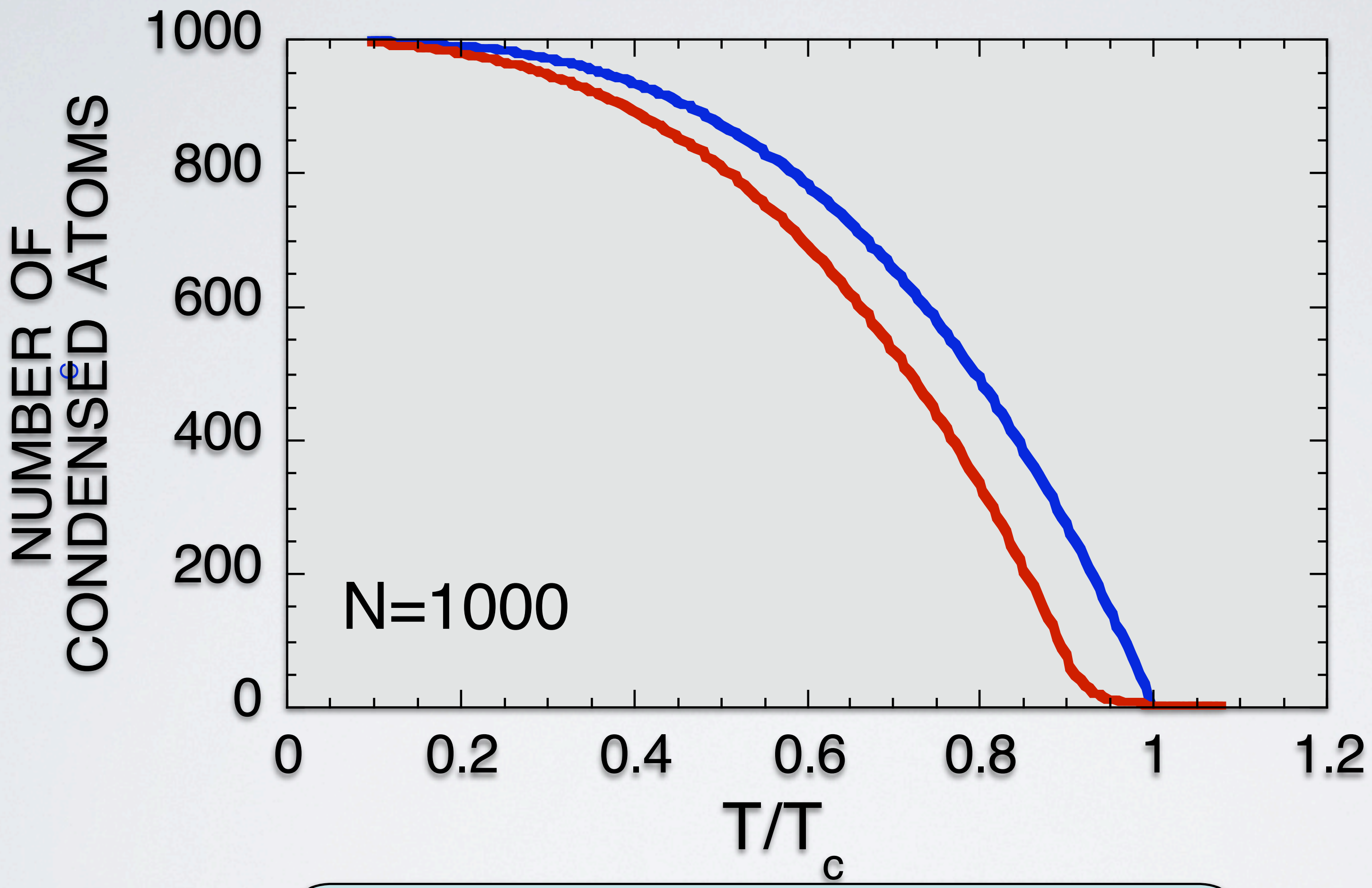


Temperature dependence of the condensed fraction



$$< N_0 > = N\left[1 - \frac{\zeta(3)}{N}\left(\frac{kT}{\hbar\omega}\right)^3\right]$$

Bose gas at zero temperature

All atoms have the same wave function. They occupy the lowest energy state of the Gross-Pitaevski equation:

$$\left[\frac{\vec{p}^{\,2}}{2m}+V_{trap}(\vec{r})+Ng\,|\,\psi(\vec{r},t)\,|^2\right]\psi(\vec{r},t)=i\hbar\frac{\partial}{\partial t}\psi(\vec{r},t)$$

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Hartree point of view

Full theory in second quantization:

$$H = \int \hat{\Psi}^{+}\left[\frac{\vec{p}^{2}}{2m}+V_{trap}\right]\hat{\Psi}d^{3}r+\frac{g}{2}\int \hat{\Psi}^{+}\hat{\Psi}^{+}\hat{\Psi}\hat{\Psi}d^{3}r$$

Heisenberg equation for atom field

$$\left[\frac{\vec{p}^{2}}{2m}+V_{trap}(\vec{r})+g\,|\,\hat{\Psi}(\vec{r},t)\,|^{2}\right]\hat{\Psi}(\vec{r},t)=i\hbar\frac{\partial}{\partial t}\hat{\Psi}(\vec{r},t)$$

Bogoliubov approximation: $\hat{\Psi}=\sqrt{N}\psi+\hat{\delta}$

condensate

thermal

$$\left[\frac{\vec{p}^{2}}{2m}+V_{trap}(\vec{r})+Ng\,|\,\psi(\vec{r},t)\,|^{2}\right]\psi(\vec{r},t)=i\hbar\frac{\partial}{\partial t}\psi(\vec{r},t)$$

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Heisenberg equation for atom field

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condensate **thermal**

$$\left[\frac{\vec{p}^{2}}{2m}+V_{trap}(\vec{r})+Ng\,|\,\psi(\vec{r},t)\,|^{2}\right]\psi(\vec{r},t)=i\hbar\frac{\partial}{\partial t}\psi(\vec{r},t)$$

What did we learn from Quantum Optics?

- It is useful to know the modes of the problem
- If a given mode is highly occupied, the quantum description (creation and annihilation operators) is replaced by a classical (complex amplitude) one.

$$\boxed{\hat{a} \leftrightarrow \alpha}$$

$$\boxed{\hat{a} \leftrightarrow \alpha}$$

Why annihilation operators are **less** important for atoms?

- photons are easily born and destroyed - atoms not
- photon modes well defined by boundary conditions - in general case atomic modes depend on interaction and temperature

Convenient definition of atomic modes:

$$\rho(\vec{r},\vec{r}\,') = \sum_j \frac{< n_j >}{N} \varphi_j^*(\vec{r})\varphi_j(\vec{r}\,')$$

single particle density matrix

Symmetry!

Box with periodic boundary conditions

$$\rho(\vec{r}-\vec{r}\,') = \sum_{\vec{p}} \frac{< n_{\vec{p}} >}{N} \; \frac{\exp[i\vec{p}(\vec{r}-\vec{r}\,')]}{L^3}$$

$$\varphi_{\vec{p}} = \frac{1}{L^{3/2}}\exp[i\vec{p}\cdot\vec{r}\,]$$

$$\vec{p} = \frac{2\pi}{L}(n_1, n_2, n_3)$$

Convenient definition of atomic modes:

$$\rho(\vec{r},\vec{r}\,') = \sum_j \frac{<n_j>}{N}\varphi_j^*(\vec{r})\varphi_j(\vec{r}\,')$$

single particle density matrix

BEC - dominant mode

Symmetry!

Box with periodic boundary conditions

$$\rho(\vec{r}-\vec{r}\,') = \sum_{\vec{p}} \frac{<n_{\vec{p}}>}{N}\ \frac{\exp[i\vec{p}(\vec{r}-\vec{r}\,')]}{L^3}$$

$$\varphi_{\vec{p}} = \frac{1}{L^{3/2}}\exp[i\vec{p}\cdot\vec{r}\,] \qquad \vec{p} = \frac{2\pi}{L}(n_1,n_2,n_3)$$

Let’s look at the box:

exact Heisenberg equation for atomic field

$$\left[\frac{\vec{p}^{2}}{2m}+g\,|\hat{\Psi}(\vec{r},t)|^{2}\right]\hat{\Psi}(\vec{r},t)=i\hbar\frac{\partial}{\partial t}\hat{\Psi}(\vec{r},t)$$

mode decomposition:

$$\hat{\psi}(\vec{r},t)=\frac{1}{\sqrt{V}}\sum_{\vec{p}}\hat{a}_{\vec{p}}(t)\exp\left[i\vec{p}\cdot\vec{r}\right]$$

coupled harmonic oscillators:

$$\frac{d\hat{a}_{\vec{p}}(t)}{dt}=-i\frac{p^{2}}{2m\hbar}\hat{a}_{\vec{p}}-i\frac{2g}{V\hbar}\sum_{\vec{q}_1,\vec{q}_2}\hat{a}^{*}_{\vec{q}_1}\hat{a}_{\vec{q}_2}\hat{a}_{\vec{p}+\vec{q}_1-\vec{q}_2}$$

$$\frac{d\hat{a}_{\vec{p}}(t)}{dt} = -i\frac{p^2}{2m\hbar}\hat{a}_{\vec{p}} - i\frac{2g}{V\hbar}\sum_{\vec{q}_1,\vec{q}_2}\hat{a}^*_{\vec{q}_1}\hat{a}_{\vec{q}_2}\hat{a}_{\vec{p}+\vec{q}_1-\vec{q}_2}$$

classical substitution

$$\hat{a}_{\vec{p}} = \sqrt{N}\alpha_{\vec{p}}$$

for highly occupied modes:

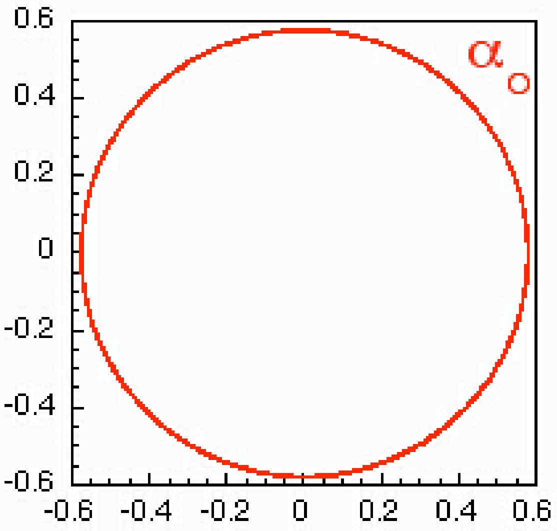
$$<\hat{a}^\dagger_{\vec{p}}\hat{a}_{\vec{p}} >= N\,|\,\alpha_{\vec{p}}\,|^2 >> 1$$

yields

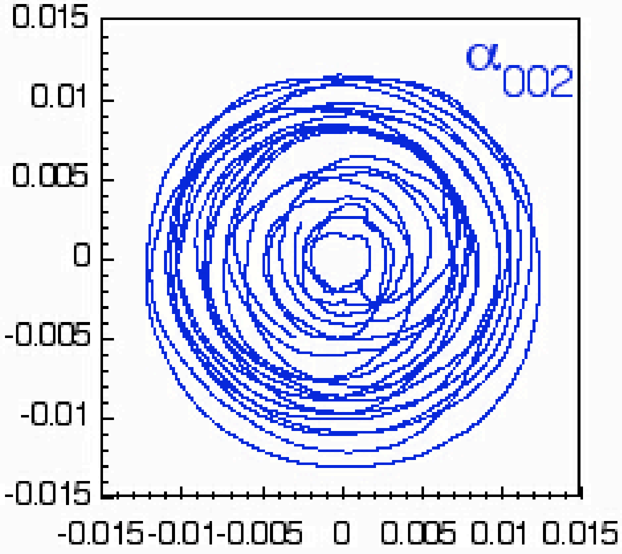
$$\frac{d\alpha_{\vec{p}}(t)}{dt} = -i\frac{p^2}{2m\hbar}\alpha_{\vec{p}} - i\frac{2gN}{V\hbar}\sum_{\vec{q}_1,\vec{q}_2}\alpha^*_{\vec{q}_1}\alpha_{\vec{q}_2}\alpha_{\vec{p}+\vec{q}_1-\vec{q}_2}$$

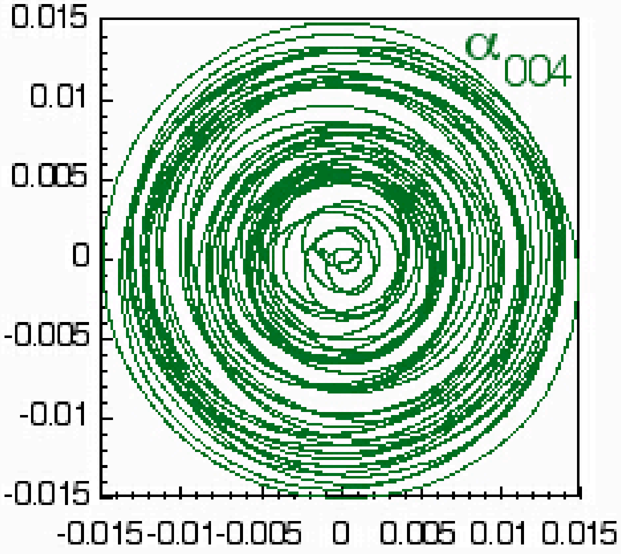
Phase space portraits of the modes

condensate



thermal





thermal

$$\Psi(\vec{r}) = \left[\Phi(\vec{r}) + u(\vec{r})\exp[-i\omega t] + v^*(\vec{r})\exp[i\omega t]\right]\exp[-i\mu t]$$

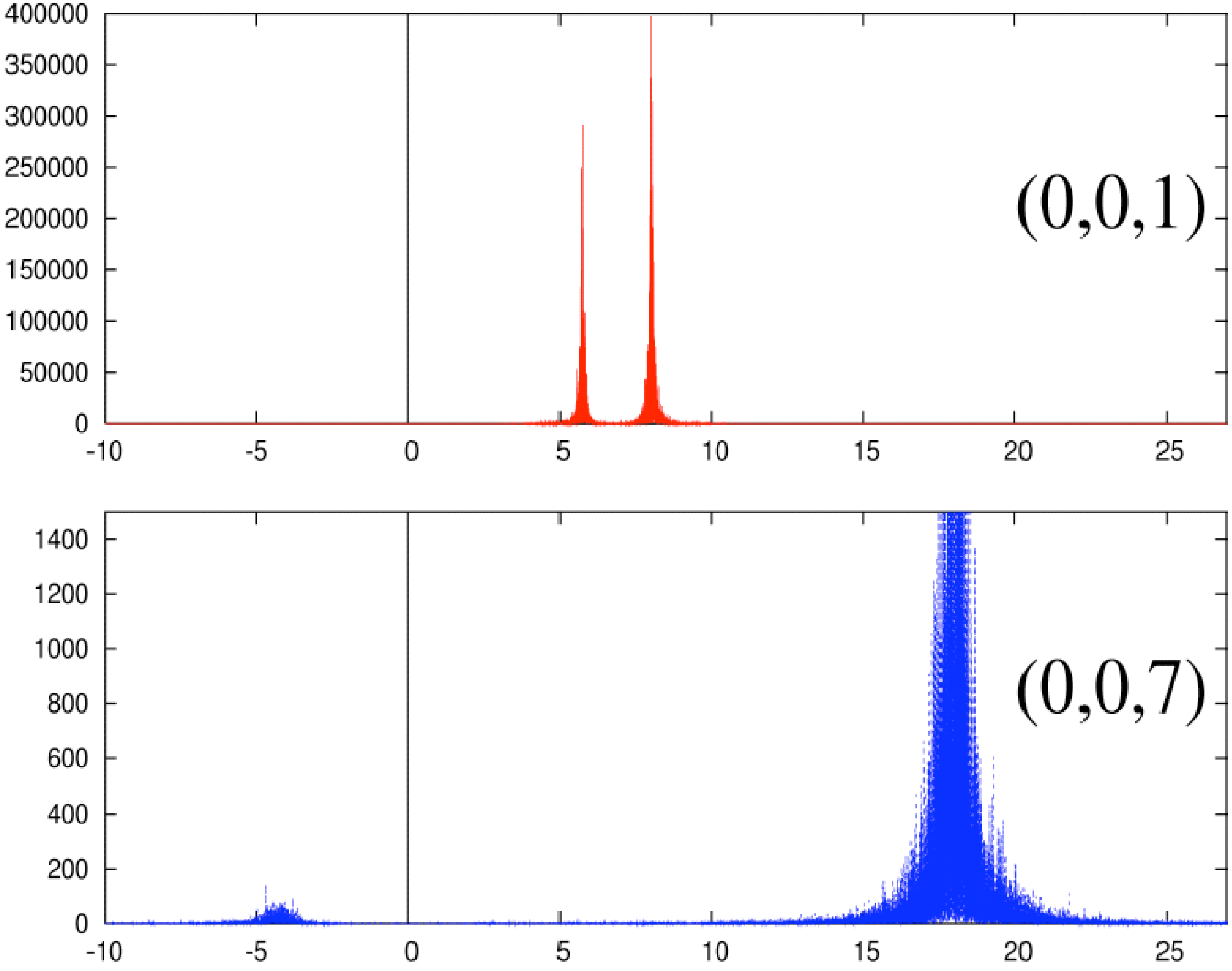
Bogoliubov-deGennes equations

$$\left[T + V + 2Ng\,|\,\Phi\,|^2 - \hbar\omega - \mu\right]u + Ng\,|\,\Phi\,|^2\,v = 0$$

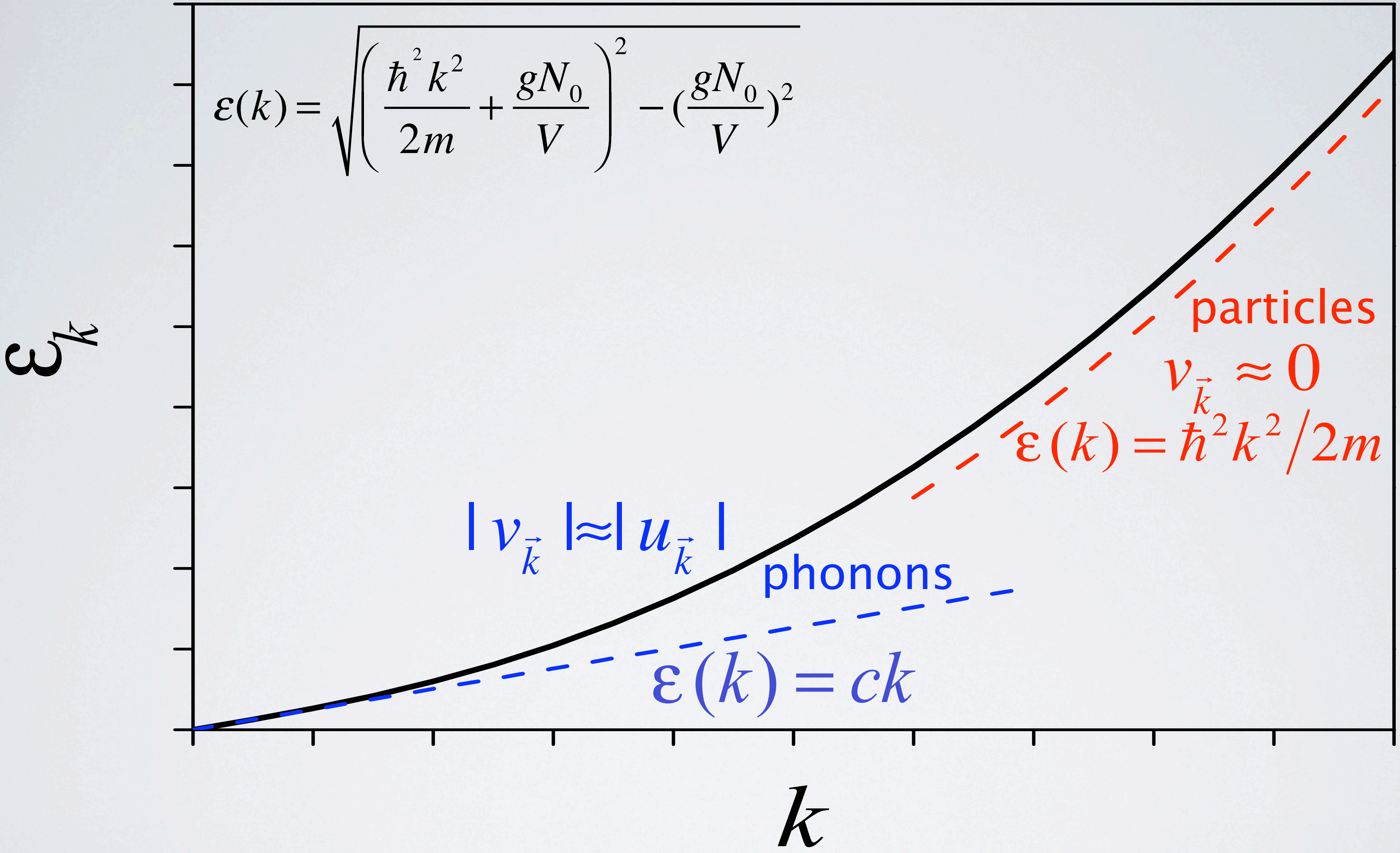
$$\left[T + V + 2Ng\,|\,\Phi\,|^2 + \hbar\omega - \mu\right]v + Ng\,|\,\Phi\,|^2\,u = 0$$

spectra of thermal modes

$N_0 / N = 0.76$

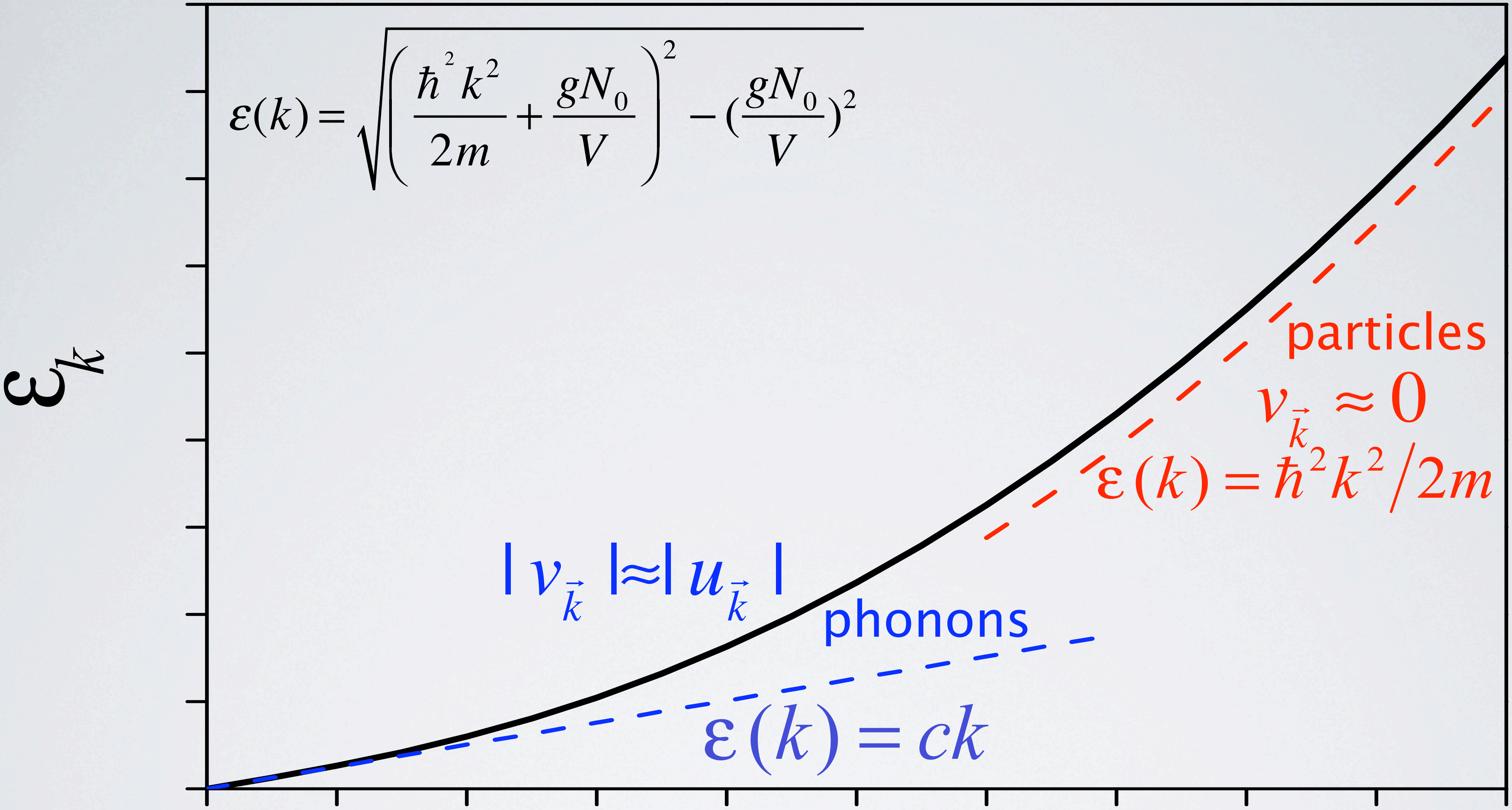


Bogoliubov–Popov spectrum of collective excitations



$$\hat{H}_B = E_0 + \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} \hat{b}^+_{\mathbf{k}} \hat{b}_{\mathbf{k}}$$

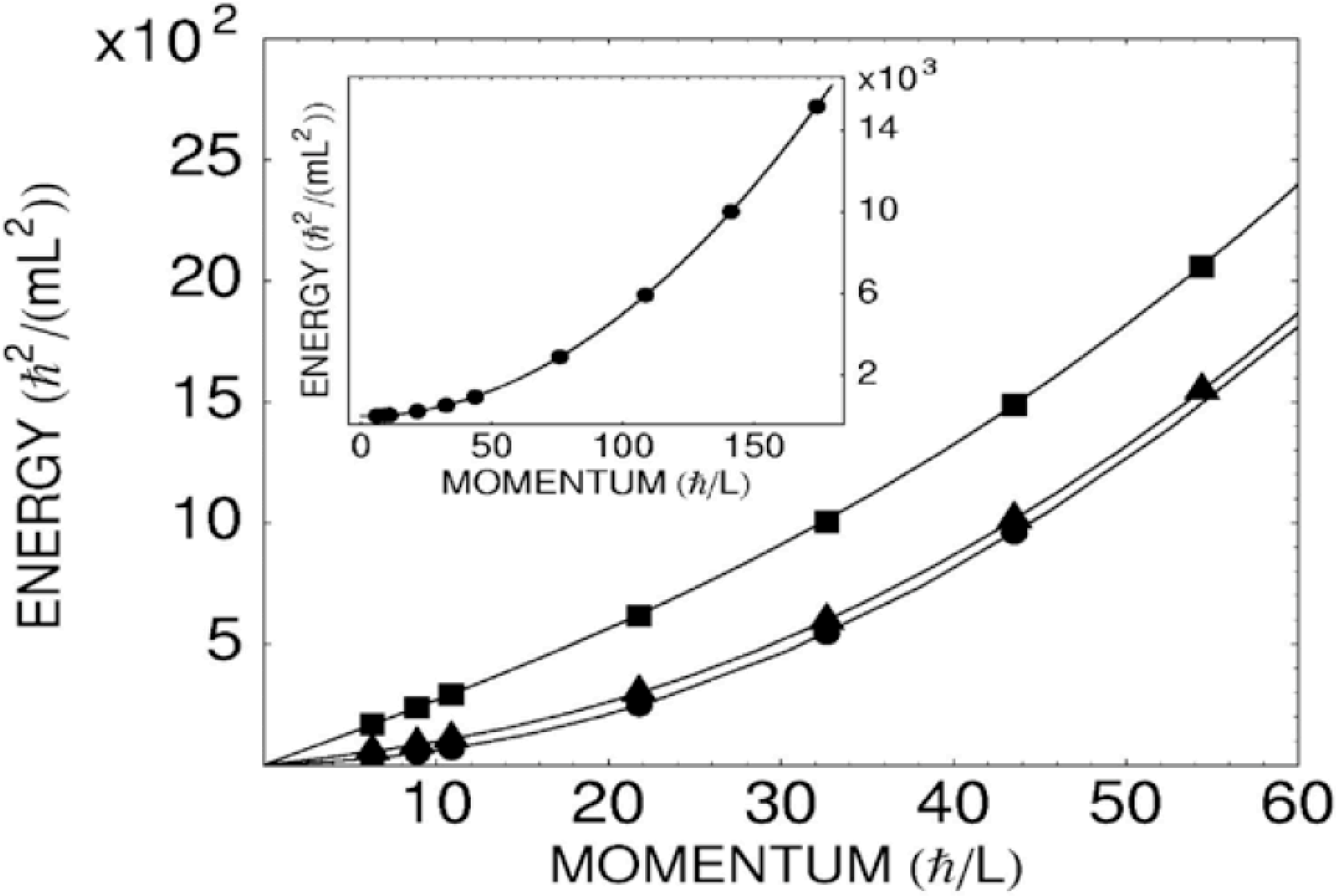
Bogoliubov–Popov spectrum of collective excitations



$$\hat{a}_{\mathbf{k}} = u_{\mathbf{k}}\hat{b}_{\mathbf{k}} + v_{\mathbf{k}}\hat{b}_{\mathbf{k}}^{+}$$

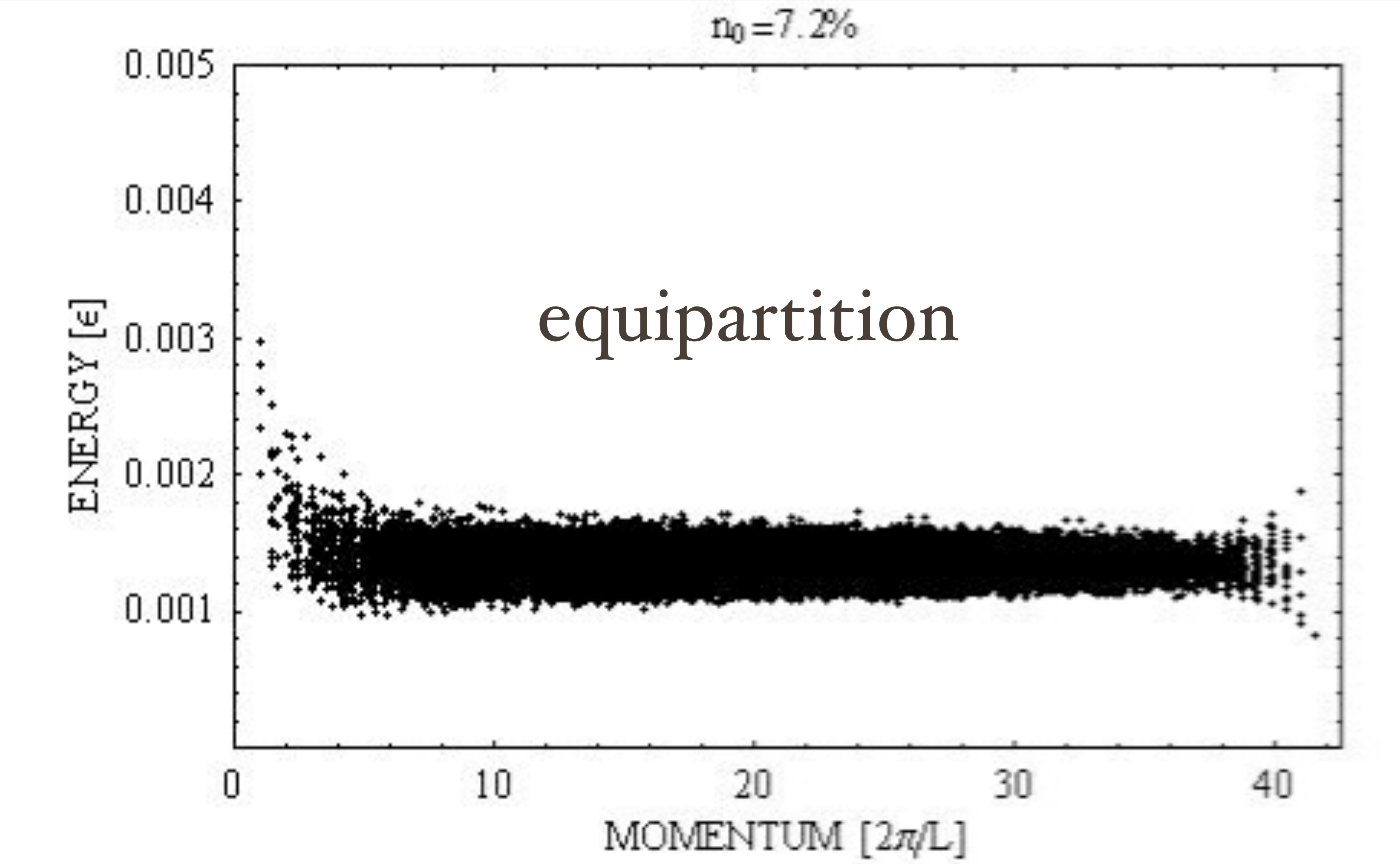
$$\hat{a}_{\mathbf{k}}^{+} = u_{\mathbf{k}}\hat{b}_{\mathbf{k}}^{+} + v_{\mathbf{k}}\hat{b}_{\mathbf{k}}$$

$$\hat{H}_B = E_0 + \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}}\hat{b}_{\mathbf{k}}^{+}\hat{b}_{\mathbf{k}}$$


x10 2
25
20
15
10
5
ENERGY ($\hbar^2/(mL^2)$)
10
20
30
40
50
60
MOMENTUM ($\hbar$/L)
x10 3
14
10
6
2
ENERGY ($\hbar^2/(mL^2)$)
0
50
100
150
MOMENTUM ($\hbar$/L)

Question of temperature

$$\varepsilon(p)N_{\vec{p}} = kT \qquad \varepsilon(p)\,|\,\alpha_{\vec{p}}\,|^2 = k_B T \,/\, N$$



equipartition

We are solving lattice version of the Gross-Pitaevski equation...

$$\psi = \frac{1}{\sqrt{V}} \sum_{\vec{p}} \alpha_{\vec{p}}(t) \exp[-i\vec{p} \cdot \vec{r}]$$

We are solving lattice version of the Gross-Pitaevski equation...

$$\psi = \frac{1}{\sqrt{V}} \sum_{\vec{p}} \alpha_{\vec{p}}(t) \exp[-i\vec{p} \cdot \vec{r}]$$

Contradiction?

Resolution: exposure time and/or spatial resolution - coarse graining

pure state

- $$\rho(t) = \frac{1}{N} < \hat{\Psi}^{+}\hat{\Psi} > \approx \psi^{*}(\vec{r},t)\psi(\vec{r}\,',t)$$

- $$\overline{\rho}(T) = \frac{1}{2\Delta}\int_{T-\Delta}^{T+\Delta}\rho(t)dt = \sum_{\vec{p}} |\alpha_{\vec{p}}|^{2} \frac{\exp[i\vec{p}\cdot(\vec{r}-\vec{r}\,')]}{V}$$

modes dephasing time

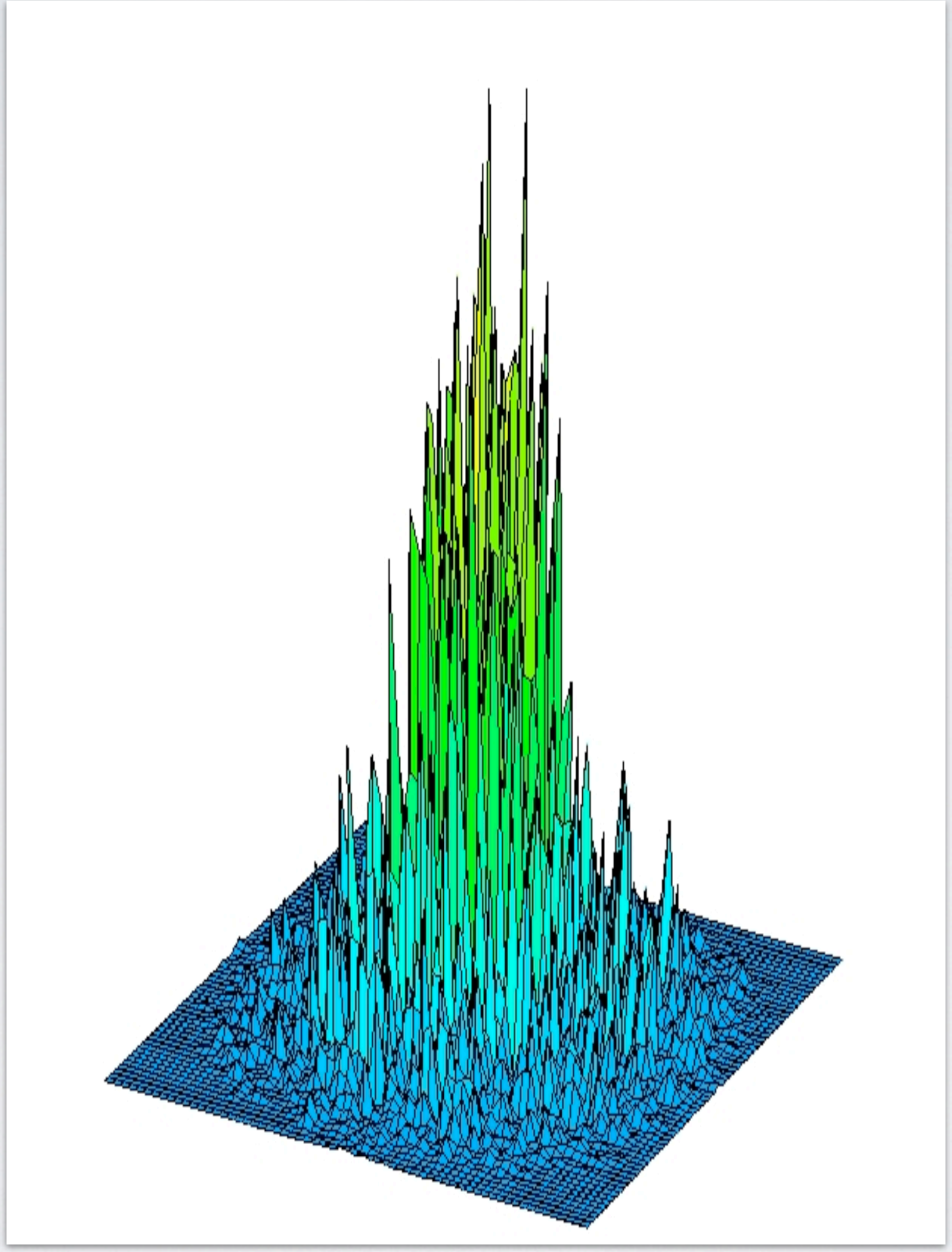
mixed state

- $$\overline{\rho}(\vec{r},\vec{r}\,') = \frac{1}{V}\int_{V} \rho(\vec{r}+\vec{R},\vec{r}\,'+\vec{R})d^{3}R$$

Our strategy:

- choose a suitable spatial lattice
- compute the ground state of GP equation
- scramble its amplitude and/or phase to inject desired amount of energy
- run the GP evolution to thermal equilibrium
- diagnose the coarse-grained density matrix

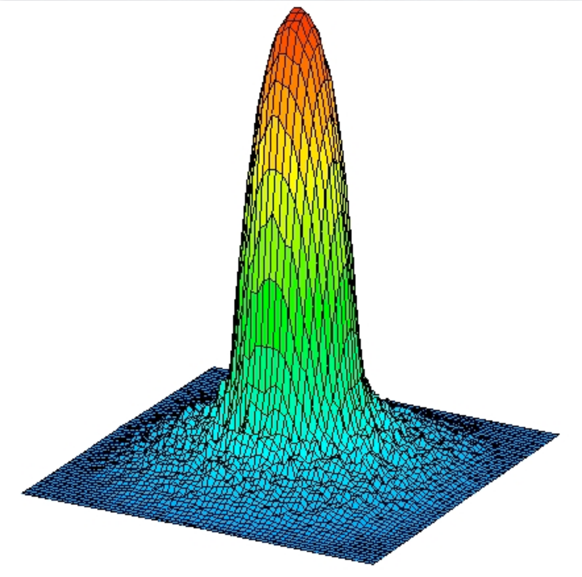
high energy solution of GP equation at the steady state evolution



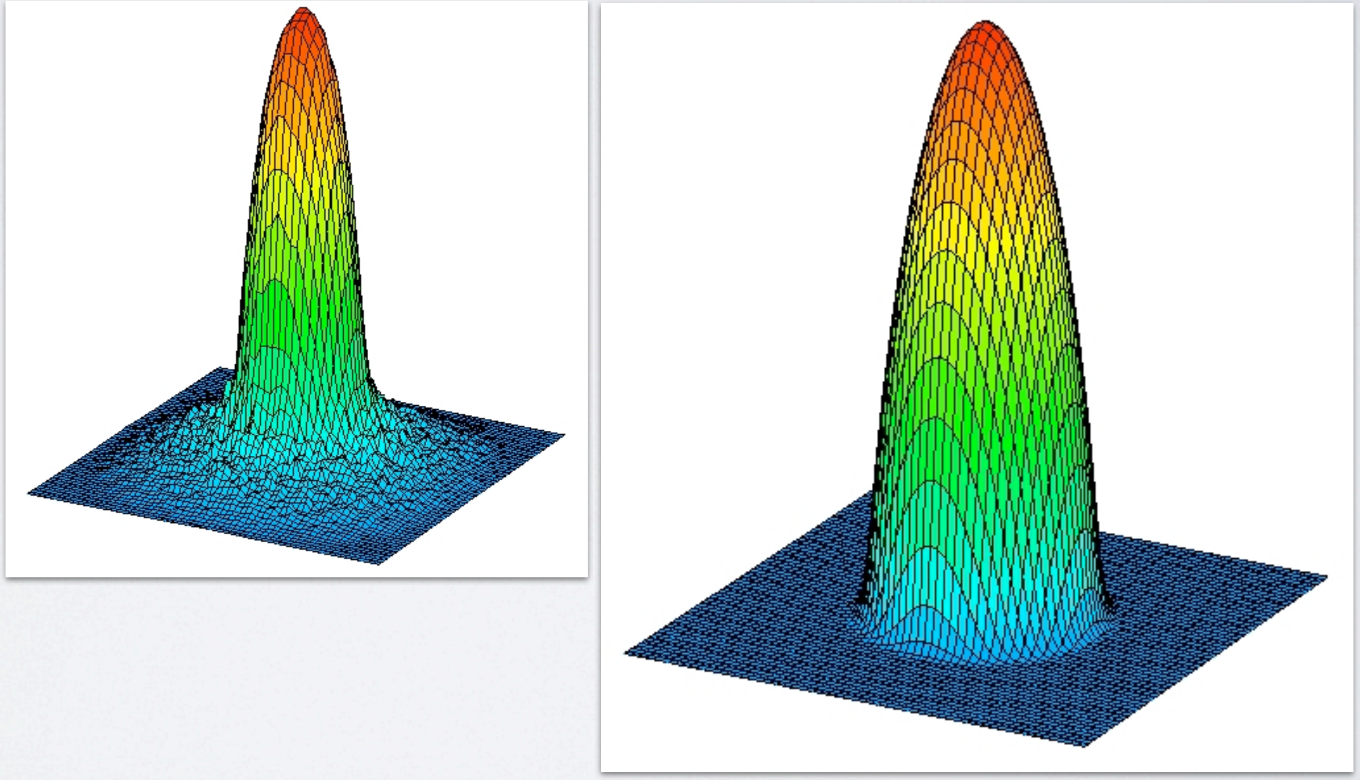
high energy solution of GP equation at the steady state evolution

Bose gas at nonzero temperature according to CFA.

Bose gas at nonzero temperature according to CFA.



Bose gas at nonzero temperature according to CFA.



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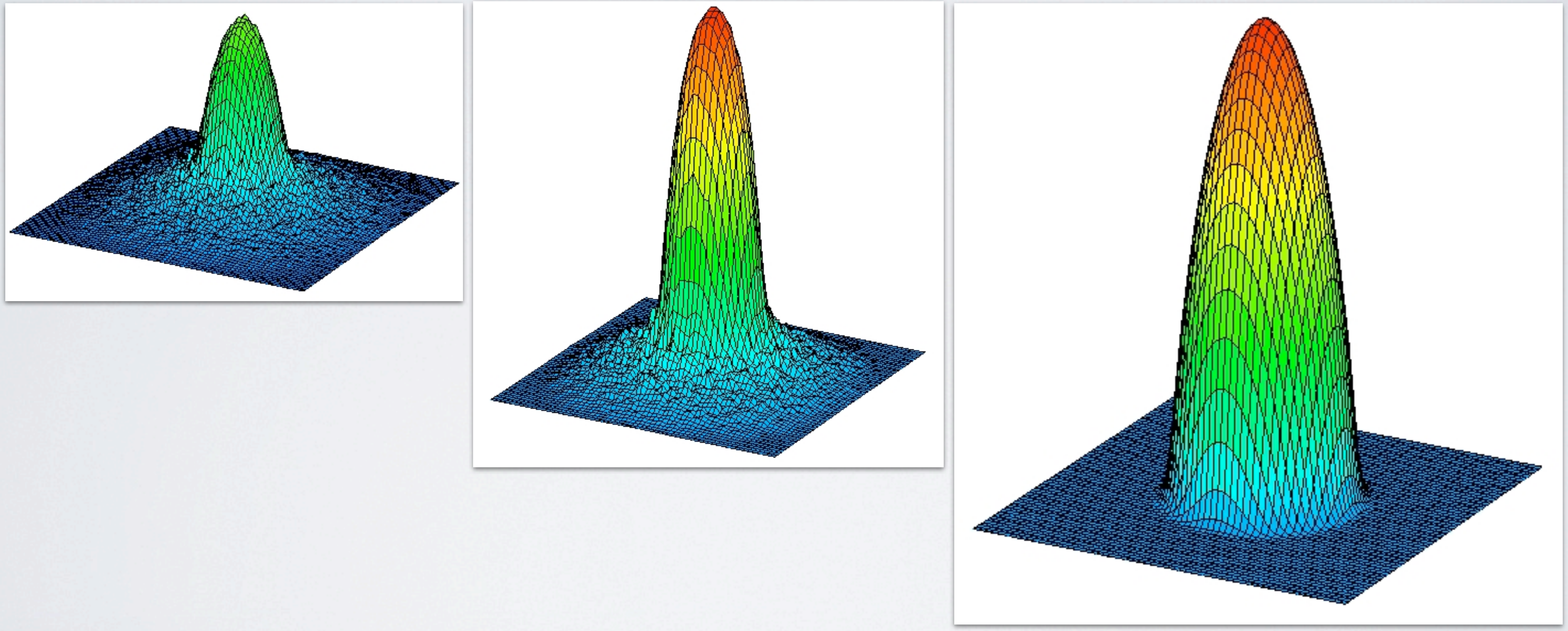

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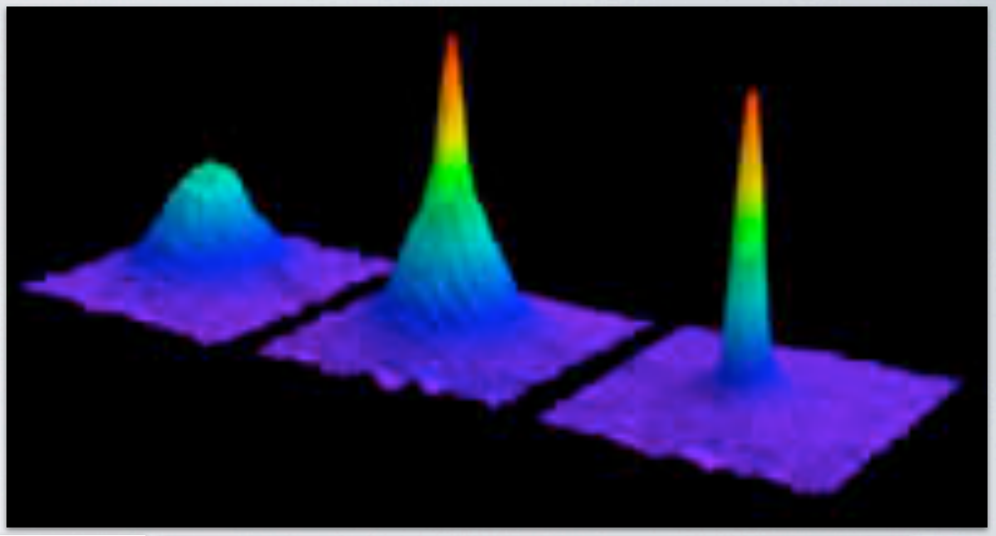
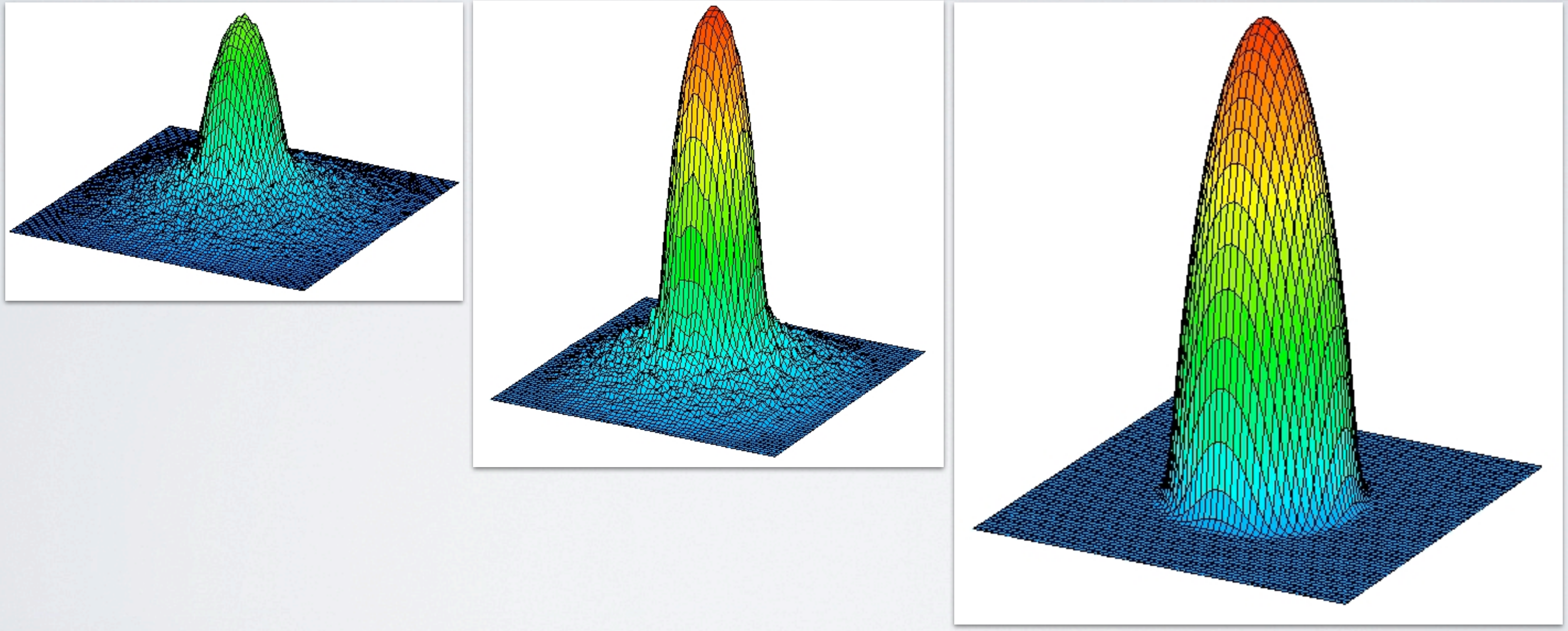


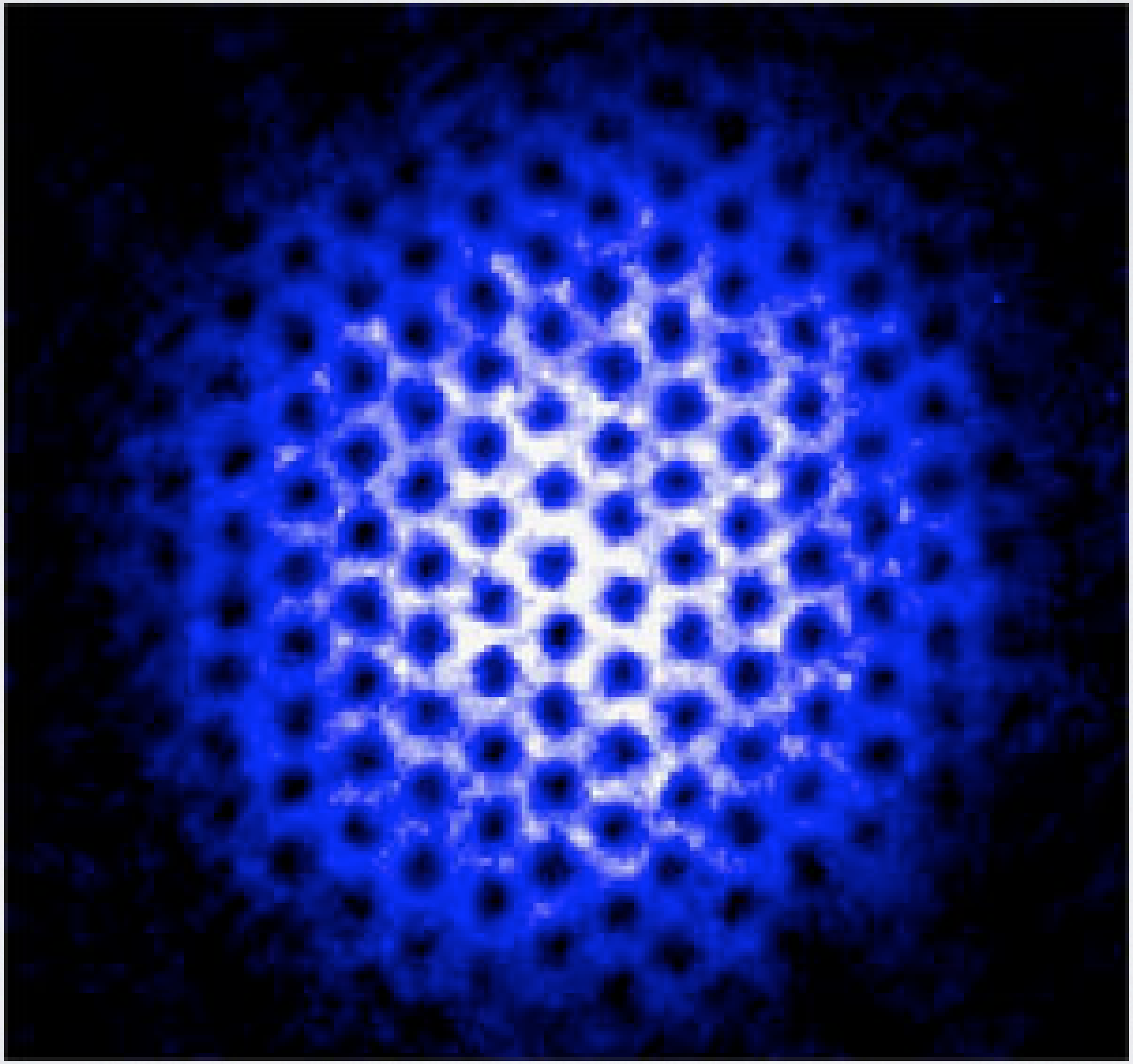


vortices in the condensate:

$$\Psi(r,\vartheta,\varphi)=\Phi_m(r,\vartheta)\exp[im\varphi]$$

multiplicity

lattice of singly charged vortices
E. Cornell



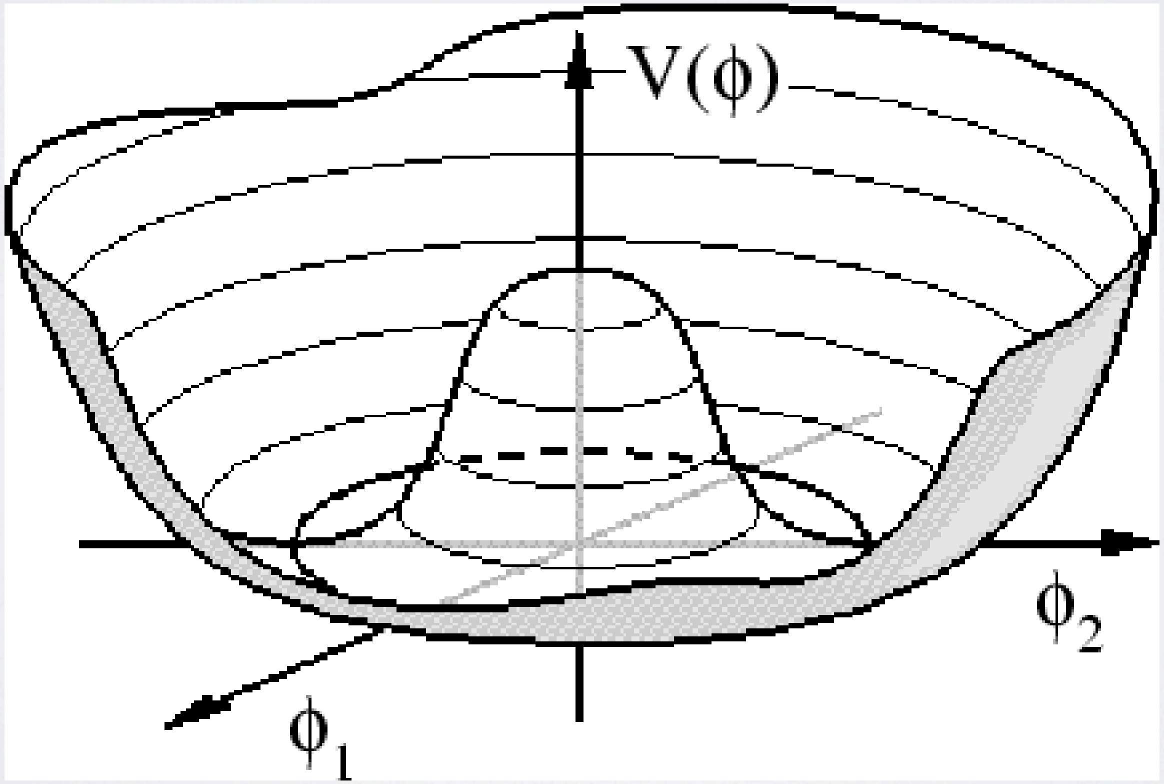
new possibilities - optical dipole traps

atoms are dragged into high/low intensity region
depending on the sign of detuning

K. Helmerson, M.F. Andersen, C. Ryu, P. Cladé, V. Natarajan, A.
and W.D. Phillips

Generating persistent currents states of atoms using orbital angular momentum of photons

Nuclear Physics A, 790, (2007) 705-712



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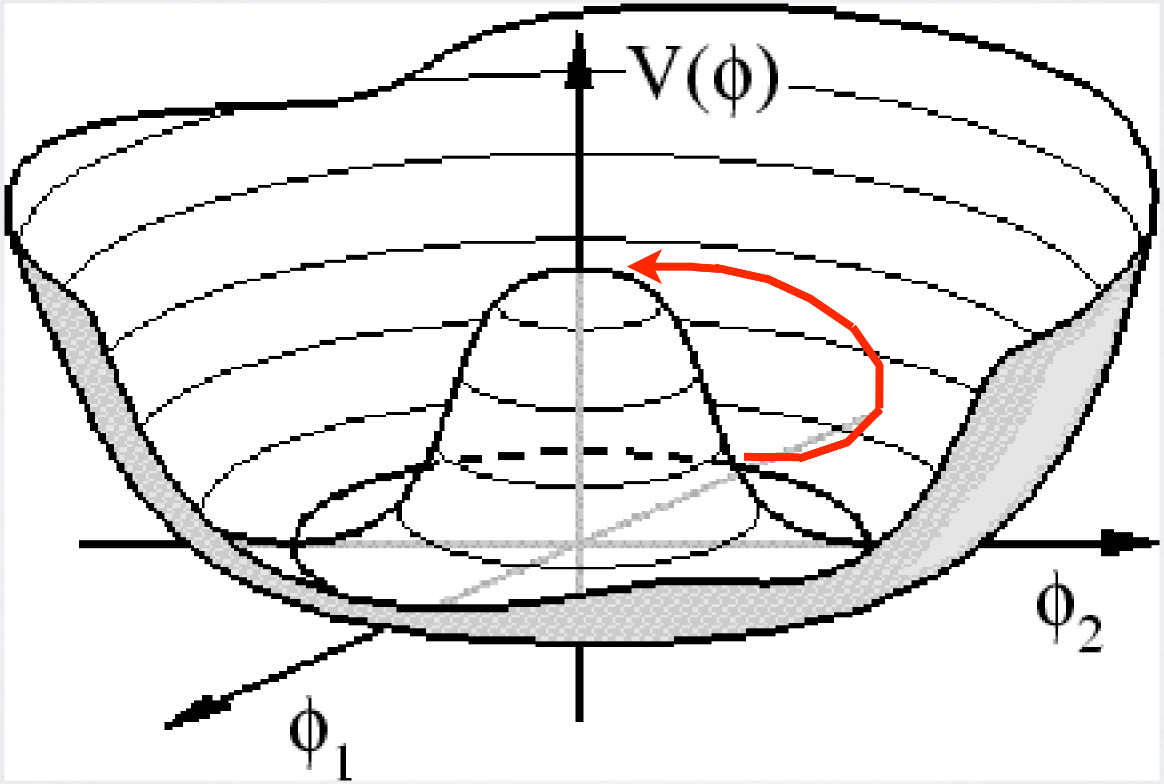
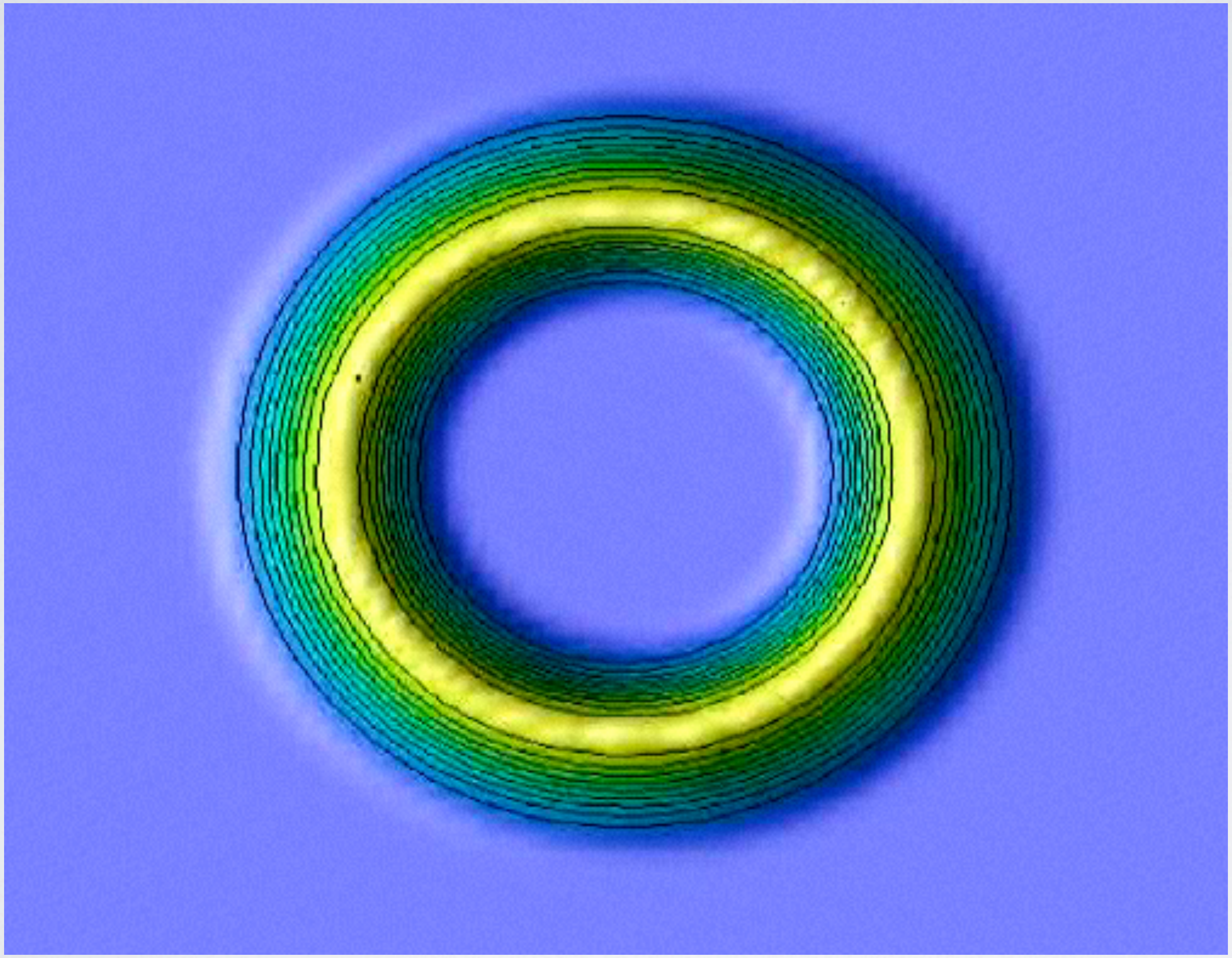
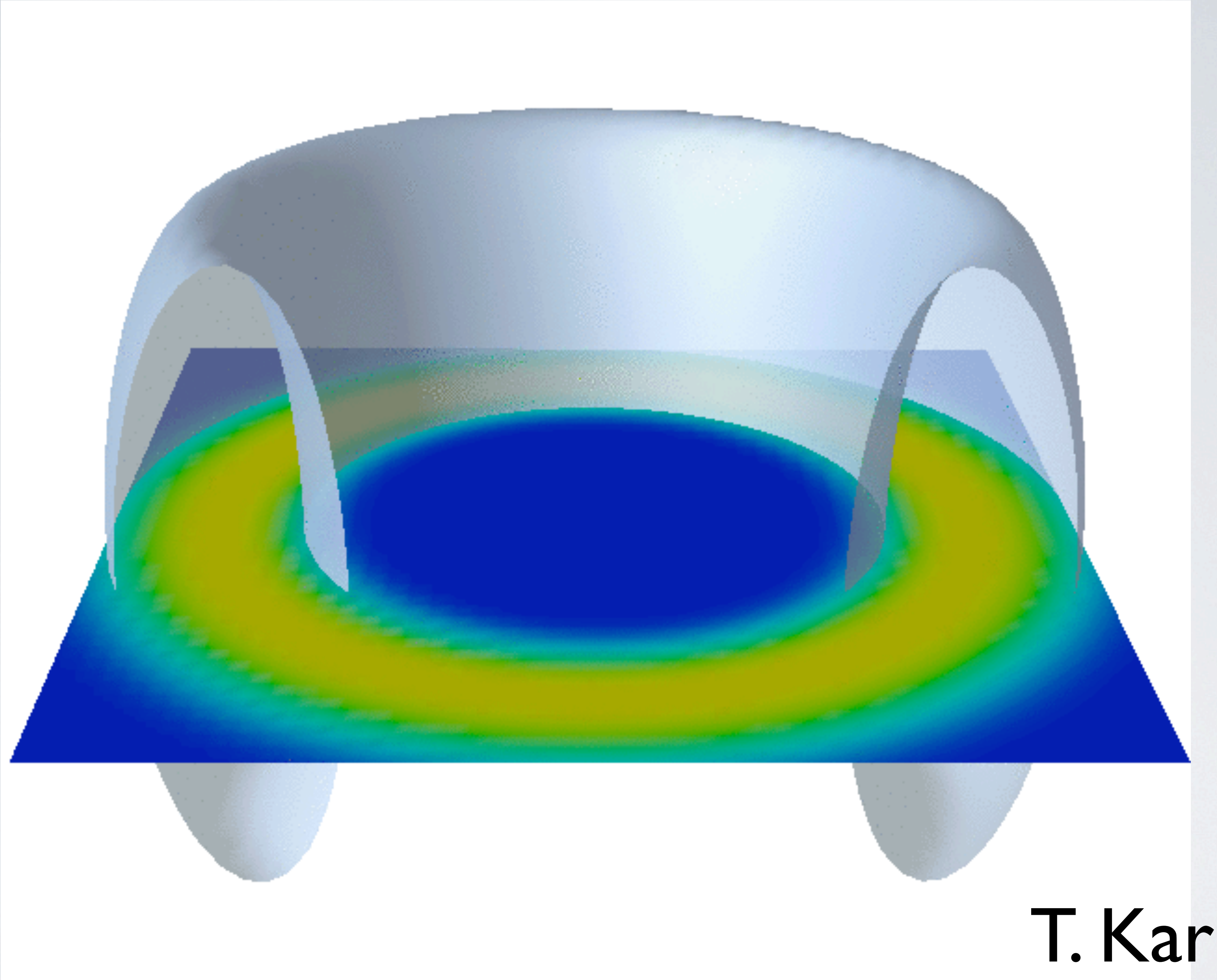


scenario of Phillips' experiment for m=5



T. Karpiuk



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