Solitons in BEC

Solitons

What is Soliton

Optical Soliton

What does Soliton look like

Soliton in Collision
Solitons in BEC

- Dark solitons ($g > 0$)
- Quantum pressure
- Interactions
- Filled solitons
- Gap solitons
- "Negative mass"

Bright solitons ($g < 0$)


$N_{\text{Soliton}} < 10^4$

Quasi-1D regime

Collapse for $E_{\text{int}} > E_{\text{radial}}$

Theoretical model

Gross-Pitaevskii Equation (NRS)

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \right) + \hbar V(x, y, z, t)\psi + NU_0 |\psi|^2 \psi$$

Atom-atom interactions

$$U_0 = \frac{4\pi a_0 \hbar^2 N}{m}$$

Harmonic Potential

$$V(x, y, z, t)$$
Atom Lasers

Atom laser: a coherent beam or pulse of atoms.

Max Planck Quantum Optics: Outcoupling via spin flip (2001)

Yale: Gravity-induced tunneling through a lattice (1998)

Phase imprinting

\[ \Psi(x) = \sqrt{a} \left( e^{-\frac{x}{\lambda}} + \sqrt{1 - \frac{\lambda}{\Delta}} \cdot \tanh \left( \frac{x - \frac{\lambda}{2}}{\lambda} \right) \right) \]

Figure 1: (a) Density and phase distribution of a dark soliton state with \( \Delta \Phi_x = \pi \). The density maximum has a width of \( \sim \lambda \). (b) Phase imprinting potential, \( U_{\Phi} \), and associated phase distribution.
DARK SOLITON CREATION BY PHASE IMPRINTING

**NIST**: Science 287, 97 (1999)
Aspect Ratio ~ 2

**Hannover**: PRL, 83, 5198 (1999)
Aspect Ratio ~ 30

**LASER**

\[ \Phi \rightarrow \Phi e^{i\theta} \]

**EXPERIMENT**

**THEORY**

**Soliton trains**
Observations of Bright Solitons

- **Bright soliton in a waveguide**
  - Rolls down potential hill

- **Bright soliton train**
  - 4 or 5 solitons oscillate and interact in harmonic trap
    - Rice University (2002)

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**Fig. 1.** Experimental setup for soliton production. \(^7\)Li atoms are evaporatively cooled in aloff-family magnetic trap and transferred into a crossed optical dipole trap in state \(|F = 1, m_F = 1\rangle\) where they Bose condense. Magnetic tuning of the scattering length to positive, zero, and negative values is performed with the two pinch coils (PC). Switching-off the vertical trapping beam (VB) allows propagation of a soliton in the horizontal 1D waveguide (HB). Absorption images of solitons and BECs are recorded on a charge-coupled device camera in the \(x, z\) plane.
Manipulating interactions: soliton formation

One prepares a lithium condensate with repulsive interaction and place it in a quasi-1D geometry (EELS):

\[ h\omega_L \ll k_B T, \mu \]

One switches interactions towards \( a=0 \) (ideal gas) or \( a<0 \) (attractive)
Dispersion and fundamental solitons

Gap Solitons

Narrow momentum distribution and small nonlinearity

\[ \Psi(x,t) = A(x,t) \Phi_{n, k_0}(x) e^{-i \frac{\hbar}{\tau} E_0(k_0)t} \]

Nonlinear Schrödinger equation for the envelope

\[ i \hbar \left( \frac{\partial}{\partial t} + \nabla^2 \right) A(x,t) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} A(x,t) + \alpha |A(x,t)|^2 A(x,t) \]
Gap Solitons
Variational approach

Equation of motion:
\[ i \frac{\partial u}{\partial x} = - \frac{1}{2} \frac{\partial^2 u}{\partial x^2} + |u|^2 u \]

Lagrangian density:
\[ L = i(\overline{u}_x u - uu_x) - |u|^2 - |u|^4 \]

Ansatz:
\[ u = A(z) \exp \left( - \frac{x^2}{2W(z)^2} + ib(z)x^2 + i\phi(z) \right) \]

\[ L = \frac{\sqrt{\pi}}{4W} A^2 \left( 4\phi'W^2 + b'W^4 + 1 + W^{-2} - A^2 \sqrt{2W^2} + b^2W^4 \right) \]

Reduced dynamics:
\[ \ddot{W} = \frac{1}{W^3} - \frac{\sqrt{2E}}{2} \frac{1}{W^2} \]

Stability in 1D

\[ \ddot{W} = - \frac{\partial U}{\partial W} \]

\[ U = \frac{1}{2W^2} - \frac{\sqrt{2E}}{2W} \]
Stability in > 1D

Equation of motion:
\[ i \frac{\partial u}{\partial z} = -\frac{1}{2} \Delta u + |u|^2 u \]

Lagrangian Density:
\[ L = i(u \dot{u}^* - u^* \dot{u}) - |u_x|^2 - |u|^4 \]

2D
\[ U = \frac{A}{W^2} \]

3D
\[ U = \frac{A}{2W^2} - \frac{A}{W^3} \]

Stability in 3D

\[ \ddot{W} = -\frac{\partial U}{\partial W} \]

\[ U = \frac{1}{2W^2} - \frac{\sqrt{2}}{2W^3} \]
Gap Solitons

\[ \hbar \frac{\partial \Psi}{\partial t} = \left[ -\frac{\hbar^2}{2m} \Delta + U(r, t) + \frac{4\pi a h^2}{m} |\Psi|^2 \right] \Psi \]

\[ U(r, t) = f_1(t) \sin^2 \left( \frac{2\pi z}{\lambda} \right) + \frac{m}{2} \left[ \omega_1^2 \sigma^2 + f_2(t) \omega_2^2 \right] \]

Autoformacja

\[ \Psi(\text{a.u.}) \]

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BEC in the lattice

\[ \partial_t \psi = -\frac{1}{2} \Delta \psi + U(r,t) + g(t)|\psi|^2 \psi \]

\[ U(r,t) = \epsilon(t)(1 - \cos(2z)) + f(t) \left[ \frac{\omega_z^2}{2} \rho^2 + U_0(z) \right] \]
Dynamics

Short time

Long time

Stability of the solution

fully 3D
2D versus 3D

Extra dimension can stabilize