POLS CONFERENCE
CONFORMAL AND CR GEOMETRIES, AND APPLICATIONS

Schedule

	Sunday 16 July	Monday 17 July	Tuesday 18 July
10.00-10:50	Florian Bertrand	Rod Gover	Yannick Herfray
10.50-11:20	Coffee Break	Coffee Break	Coffee Break
11.20-12:10	Dennis The	Lionel Mason	Igor Zelenko
12.10-13:00	Thomas Mettler	Boris Kruglikov	David McNutt
13.00-15:00	Lunch Break	Lunch Break	Lunch Break
15.00-15:50	Dmitri Alekseevsky	David Sykes	Andrew Waldron
15.50-16:20	Coffee Break	Coffee Break	Coffee Break
16.20-17:10	Lenka Zalabová	Omid Makhmali	Wojciech Kryński
17.10-18:00	Jerzy Lewandowski	Wojciech Kamiński	Adam Bac

Abstracts

Speaker: Dmitri Alekseevsky (A. A. Kharkevich Institute for Information Transmission Problems & University of Hradec Králové)

Title: Conformally flat homogeneous pseudo-Riemannian manifolds and conformally flat hypersurfaces

Abstract: An n-dimensional pseudo-Riemannian manifold $(M, g))$ of signature (k, ℓ) is conformally flat if locally in some coordinates the metric has the form $g = f(x)g_0$, where $g_0 = dx_1^2 + \cdots + dx_k^2 - dx_{k+1}^2 + \cdots + dx_n^2$. The theory of conformally flat manifolds had been developed by E. Cartan.

There are two approaches to conformally flat manifolds:

E. Cartan developing method and W.H. Brinkmann embedding method.

We discuss the relation between these methods and give a short review of the results about conformally flat manifolds and, in particular, homogeneous ones.

Then we present an approach for classification of homogeneous conformally flat manifolds. It is based on the theory of Lie groups and roughly speaking consists of the following two steps.

I. Describe maximal subalgebras $\mathfrak{g} \subset \mathfrak{so}(V) = \mathfrak{so}_{k+1,\ell+1}$ of the pseudo-orthogonal Lie algebra and associated connected subgroups $G \subset SO(V)$.

II. For each such subgroup $G \subset SO(V)$, describe isotropic G-orbits Gp where p is an isotropic vector in the pseudo-Euclidean vector space $V = \mathbf{R}^{k+1,\ell+1}$, which are projected onto open orbits in the conformal space $C^{k,\ell} = PV_0$.

Speaker: Adam Bac (University of Warsaw)

Title: Yang-Mills theory of the conformal Cartan connection: applications to gravity

Abstract: It is known that the equations of motion of a source-free Yang-Mills theory of the normal conformal Cartan connection are equivalent to the vanishing of the Bach tensor. We investigate the properties of the conformally invariant presymplectic potential current obtained from this theory and describe its relation to that of the standard Einstein-Hilbert Lagrangian. The pullback of our potential to the boundary of an asymptotically de Sitter spacetime turns out to coincide with the current obtained from the holographically renormalized gravitational action, which provides an alternative derivation of a symplectic structure on scri without resorting to holographic techniques.

Speaker: Florian Bertrand Beirut (American University of Beirut)

Title: Invariant curves for real analytic hypersurfaces in $\mathbf{C}^2$.

Abstract: We consider three families of complex invariant curves associated to strictly pseudoconvex hypersurfaces, namely Segre varieties, chains and extremal discs for the Kobayashi metric. These invariant curves, of different geometric nature, coincide in the case of the sphere. In this talk, I will present new characterizations of the sphere in $\mathbf{C}^2$ in term of these curves. This is joint work with G. Della Sala and B. Lamel.

Speaker: Rod Gover (University of Auckland)

Title: Poincaré–Einstein asymptotics via conformal fundamental forms, and the Poincare-Einstein Dirichlet–Neumann map

Abstract: If a manifold with boundary has on its interior a conformally compact Einstein metric, then what restrictions does that place on the geometry of the boundary embedding? It is well known that it is necessarily totally umbilic, meaning that the trace-free part of the second fundamental form must vanish. We show that the trace-free second fundamental form is the lowest order example in a series of trace-free 2-tensor conformal invariants that provide the order-by-order obstructions to the Poincaré–Einstein condition. In the case of even dimensional manifolds, probing further yields a conformal invariant that captures the image of the non-linear Poincaré–Einstein Dirichlet-Neumann map — meaning that it is the higher Neumann-type data for the Poincaré–Einstein problem with Dirichlet data a boundary conformal structure.

Speaker: Yannick Herfray (University of Tours)

Title: Asymptotics in General Relativity from the perspective of Cartan geometry

Abstract: There are several classical notions of asymptotic flatness in General Relativity: The exact procedures and geometries are however very different depending on whether one considers an asymptote in null, time-like, or space-like directions. I will recall these classical notions and show that, despite their apparent important differences, these can nonetheless all be derived from one unique concept, curved obit decomposition of Cartan geometry and that there is therefore a profound unity in these notions of asymptotics.

Speaker: Wojciech Kamiński (University of Warsaw)

Title: On classification of conformal anomalies

Abstract: Anomalies in quantum field theories are described by certain cohomology groups. It is important to understand the structure of these groups. Among various anomalies, the one related to conformal gauge transformations was long resisting classification. Deser and Schwimmer determined the non-trivial structure of the group in '93,

but the classification was proven much later by Boulanger. Not surprisingly, many important objects in conformal geometry (for example Branson curvature or ambient metric ambiguities) appear naturally in this subject. I will present an alternative approach to classification problem by the ambient metric construction and conformal jet isomorphism.

Speaker: Boris Kruglikov (UiT The Arctic University of Norway)
Title: CR blow-ups and symmetry
Abstract: I will discuss the blow-up construction in CR geometry and apply it to the symmetry dimension problem: what are the bounds on the Lie algebras of infinitesimal analytic transformations of CR structures? I will also discuss the recent progress on the Beloshapka conjecture for real hypersurfaces in complex spaces.

Speaker: Wojciech Kryński (IMPAN)
Title: On two constructions in the path geometry: dancing and chains
Abstract: Given a path geometry on a surface M, there are two constructions via which the 3- dimensional projectivized tangent bundle PTM can be endowed with a path geometry. One of these constructions is given by the well-known class of chains. The other construction is referred to as the dancing construction. In this talk I will provide necessary and sufficient conditions that determine whether a path geometry in dimension three arises via one of these two constructions. I'll also discuss a generalization of the dancing construction, referred to as freestyling. Based on a joint work with Omid Makhmali.

Speaker: Omid Makhmali (University of Granada)
Title: Weyl metrizability of 3-dimensional projective structures and CR submanifolds
Abstract: A projective structure is Weyl metrizable if it has a torsion-free representative that preserves a conformal structure. It was shown by Mettler that projective structures on a surface are Weyl metrizable. The choice of a Weyl structure can be interpreted as a section of a disk bundle which gives rise to a holomorphic curve in a complex surface. In this talk we interpret Weyl metrizability of 3-dimensional projective structures as certain 5-dimensional nondegenerate CR submanifolds in a class of 7-dimensional 2-nondegenerate CR structures.

Speaker: Lionel Mason (University of Oxford)
Title: Holomorphic discs for SD Einstein metrics from conformal boundary data
Abstract: In split signature, global twistor constructions for conformally self-dual (SD) gravity and Yang-Mills construct solutions from twistor data that can be expressed in terms of free functions without gauge freedom. This is developed for asymptotically flat SD gravity to give a fully nonlinear encoding of the asymptotic gravitational data in terms of a real homogeneous generating function h on the real twistor space. This can be used to study the problem of filling in data from the conformal boundary $S^2 \times S^1$ into the interior split signature self-dual space. On the boundary the conformal structures must be 'Zollfrei' in the sense of Guillemin and this work extends his study of these conformal structures.

Speaker: David McNutt (UiT The Arctic University of Norway)
Title: Conformally covariant curvature invariants and black hole detection

Abstract: Locating the event horizon of a dynamical black hole solution is difficult to accomplish, due to its teleological nature. However, if the black hole solution is conformally related to a stationary black hole solution, then the event horizon is a quasi-local surface, known as a conformal Killing horizon. Without knowledge of the corresponding conformal Killing vector field, can a defining function be found to characterize this null hypersurface?

In this talk, I will outline an approach to generate conformally covariant scalar curvature invariants and use these to locate the event horizon for such conformally transformed stationary black holes, in the sense that the hypersurface will be defined as the zero-set of a curvature invariant.

Speaker: Thomas Mettler (UniDistance Suisse)

Title: Para-Kähler Einstein metrics in parabolic geometries

Abstract: I will motivate and explain a canonical construction of almost para-Kähler Einstein metrics from a specific class of geometric structures. Additionally, I will explore the connection between these metrics and Patterson—Walker metrics, while also introducing a corresponding parabolic evolution equation.

Speaker: Jerzy Lewandowski (University of Warsaw)

Title: Uniqueness of extreme horizons.

Abstract: The study of the geometry of extreme Killing horizons (and their generalizations, extreme isolated horizons) led to the discovery of the constraints that must be satisfied by the intrinsic and extrinsic geometry of the horizon. The resulting equation of extreme horizons constrains the topology and shape of the horizon. In four-dimensional space-time, assuming axial symmetry and spherical topology, the only solutions are those defined by Kerr metrics. An argument that would imply that symmetry has been sought for more than 20 years. Recently, Dunajski and Lucietti showed that every solution of the extremality equation regular on a compact surface has the additional symmetry sought. This result has the strongest consequences in 4-dimensional space-time, but is also true in any dimension.

Speaker: David Sykes (Masaryk University)

Title: Local geometry of new 2-nondegenerate real hypersurface examples

Abstract: This talk is on recent joint work with Martin Kolar and Ilya Kossovskiy wherein we introduce a class of uniformly 2-nondegenerate CR hypersurfaces in arbitrary CR dimension greater than 2. The structures' Levi forms have submaximal rank, and the value of their CR symbols (a basic local invariant) is constant. Having constant basic local invariants leads to a subtle lo- cal equivalence problem. The talk will cover a solution to this local equivalence problem obtained using some of the techniques from Chern–Moser normal forms theory. Additional results include a classification of the considered structures' infinitesimal automorphism algebras and a classification of the homogeneous structures among them.

Speaker: Dennis The (UiT The Arctic University of Norway)

Title: Simply-transitive CR real hypersurfaces in $\mathbf{C}^3$

Abstract: Holomorphically (locally) homogeneous CR real hypersurfaces M^3 in $\mathbf{C}^2$ were classified by Elie Cartan in 1932. A folklore legend tells that an unpublished manuscript

of Cartan also treated the next dimension M^5 in $\mathbf{C}^3$ (in conjunction with his study of bounded homogeneous domains), but no paper or electronic document currently circulates.

Starting from around the year 2000, significant progress was made on this 5-dimensional classification problem, and the classification was completed in 2020. I will discuss the final aspects of this effort, in particular giving a survey of my joint work with Doubrov & Merker and how we settled the simply-transitive, Levi non-degenerate classification.

Speaker: Andrew Waldron (University of California, Davis)

Title: Conformal Compact Yang—Mills

Abstract: The Yang—Mills equations for connections on bundles over Riemannian base manifolds have myriad applications in mathematics and physics. We study the asymptotics, renormalized energy, obstruction to smooth solutions and Dirichlet-to-Neumann map for the boundary Yang—Mills problem for base manifolds equipped with a conformally compact metric.

Speaker: Lenka Zalabová (University of South Bohemia in České Budějovice & Masaryk University)

Title: First BGG operators on homogeneous parabolic geometries

Abstract: We study first BGG operators and their solutions on homogeneous conformal geometries. We develop an invariant calculus that allows us to find solutions explicitly using only algebraic computations. We demonstrate our result on examples of homogeneous conformal geometries mainly from general relativity. Finally, we comment on the first BGG operators and their solutions on CR and general parabolic geometries.

Speaker: Igor Zelenko (University of Texas, A&M)

Title: Methods of Tanaka theory in the local geometry of 2-nondegenerate CR structures of hypersurface type

Abstract: In the talk I will describe two possible modifications of Tanaka theory for construction of the canonical absolute parallelism for 2-nondegenerate CR structures of hypersurface type. The talk is based on the joint works with David Sykes and Curtis Porter.