

Quantum state estimation on copies

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One copy of the "unknown"

What can we learn about an unknown quantum state when we have one copy of it?

Polarization of a single photon

$$|\psi\rangle = \cos(\theta/2)|\leftrightarrow\rangle + \sin(\theta/2)e^{i\phi}|\uparrow\downarrow\rangle$$

If measuring in a certain basis (e.g. $|\leftrightarrow\rangle$, $|\uparrow\downarrow\rangle$), what can we say about the result $|\leftrightarrow\rangle$, what can we say about $|\psi\rangle$?

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Infinite number of copies

Infinite number of copies allows to determine exactly:

$$\rho = |\psi\rangle\langle\psi| = \frac{1}{2} (\mathbb{I} + \vec{n} \cdot \vec{\sigma}), \quad n_i = \text{Tr} ($$

$$\rho = \frac{1}{2} \begin{bmatrix} 1 + \langle\sigma_z\rangle & \langle\sigma_x\rangle - i\langle\sigma_y\rangle \\ \langle\sigma_x\rangle + i\langle\sigma_y\rangle & 1 - \langle\sigma_z\rangle \end{bmatrix}$$

Three types of measurements are necessary:

$$\langle\sigma_z\rangle : |\leftrightarrow\rangle, |\Downarrow\rangle$$

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In reality we always have a finite number of

$$\underbrace{|\psi\rangle \otimes \cdots \otimes |\psi\rangle}_N = |\psi\rangle^{\otimes N} - N \text{ copies}$$

The more copies we have, the easier it is to state. Different states become more orthogonal

$$\lim_{N \rightarrow \infty} \langle \psi |^{\otimes N} | \phi \rangle^{\otimes N} = \lim_{N \rightarrow \infty} \langle \psi | \phi \rangle^N =$$

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How does the quality of estimation depend on the number of copies?

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Generalized measurement

Standard measurement:

$$A = \sum_i \lambda_i P_i \quad P_i P_j = \delta_{ij} P_i \quad \sum_i P_i = I$$

observable measurement outcomes projection operator

$$p_i = \text{Tr}(\rho P_i) \quad \text{probability of the result } i$$

Generalized measurement

evolution and standard measurement on *system + ancilla*

$$p_\mu = \text{Tr}(\rho P_\mu) \quad P_\mu > 0 \quad \sum_\mu P_\mu = I$$

Quantum state
 P_μ form a Positive Operator Valued Measure (POVM)

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State estimation strategy

N copies of an unknown state: $\rho^{\otimes N}$, $\rho = |\psi\rangle$

Choose a measurement – set of P_μ

Choose a guessing strategy: $\mu \rightarrow \rho_\mu$

Quality (fidelity) of a guess: $F_\mu = \langle \psi | \rho_\mu | \psi \rangle$

Quality of estimation

$$\begin{aligned} F &= \sum_\mu p_\mu F_\mu = \sum_\mu \text{Tr}(\rho^{\otimes N} P_\mu) \langle \psi | \rho_\mu | \psi \rangle = \\ &= \langle \psi | \sum_\mu \text{Tr}(\rho^{\otimes N} P_\mu) \rho_\mu | \psi \rangle \quad \text{should not depend on } \psi \end{aligned}$$

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The optimal state estimation

The optimal estimation of a qubit state

Massar, Popescu PRL, 74, 1259 (1995)

one copy: $F = \frac{2}{3}$

N copies: $F = \frac{N+1}{N+2}$

The optimal estimation of a qudit state

Bruss, Machiavello, PLA, 253, 249 (1999)

N copies: $F = \frac{N+1}{N+d}$ obtained from the connect

In order to obtain the optimal fidelities it is necessary to perform and collective (simultaneous on many copies) measurements

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Cloning and Estimation – the Motivation

$N \rightarrow M$ cloning

$$|\psi\rangle^{\otimes N} \otimes |0\rangle^{M-N} \otimes |A\rangle \quad |\Phi\rangle \in \mathcal{H}^{\otimes L}$$

input states blank states ancilla

$$\begin{aligned} \tilde{\rho} &= \text{Tr}_A |\Phi\rangle\langle\Phi| && \text{the state of } M \text{ clones} \\ \rho_1 &= \text{Tr}_{2,\dots,M} \tilde{\rho} && \text{the state of the first clone} \end{aligned}$$

$\rho_1 = \rho_2 = \dots = \rho_M =: \rho$ clones should be in optimal fidelity:

$$F = \langle\psi|\rho|\psi\rangle = \frac{M(N+1)+N(d-1)}{M(N+d)}$$

Clones are correlated!: $\tilde{\rho} \neq \rho^{\otimes M}$

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Cloning and Estimation – the Motivation

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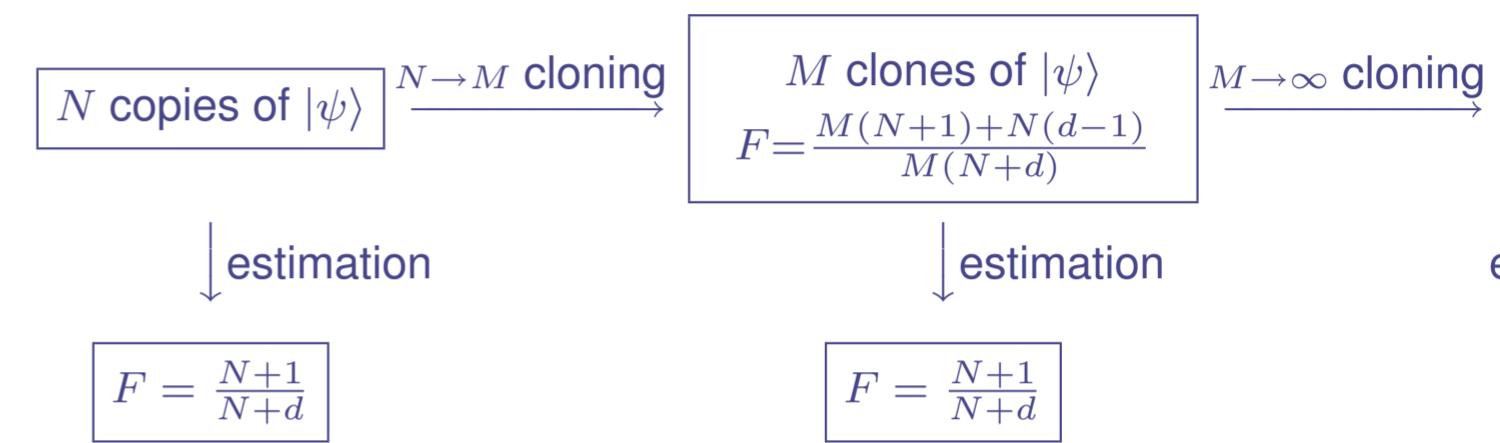
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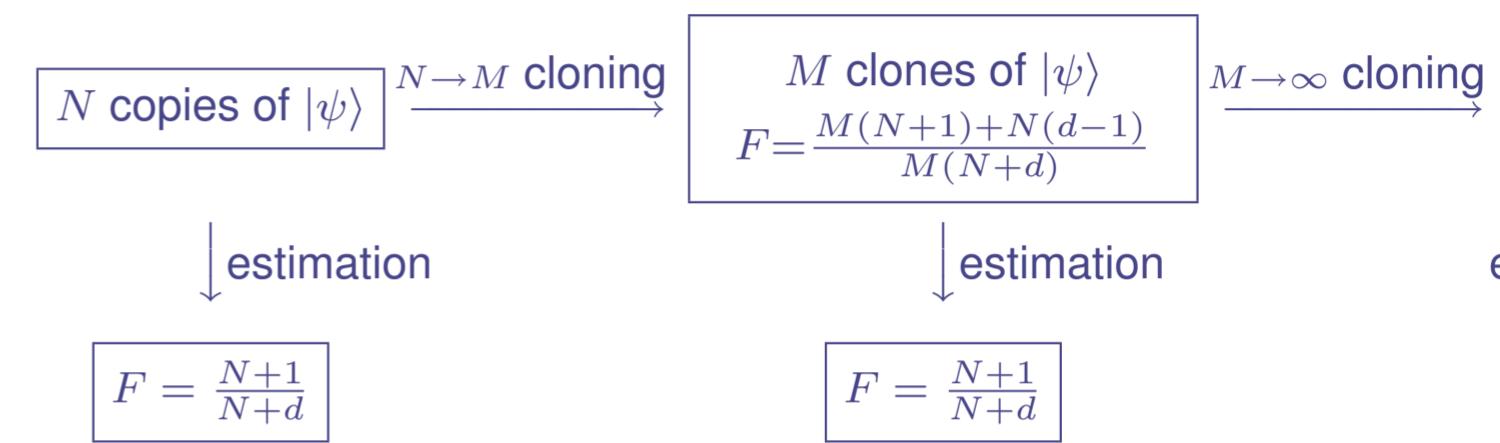


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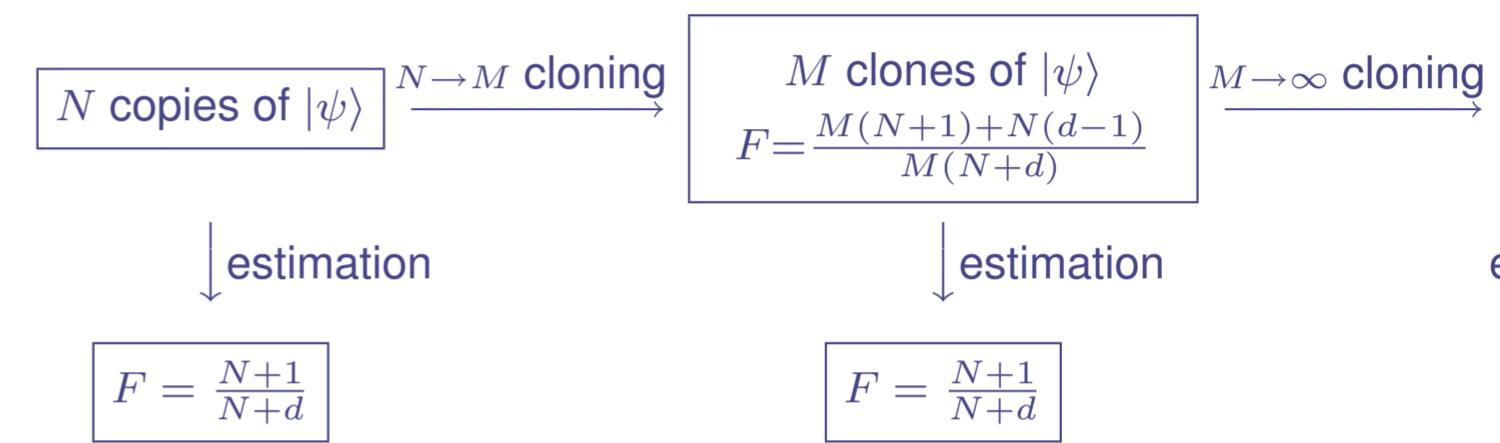


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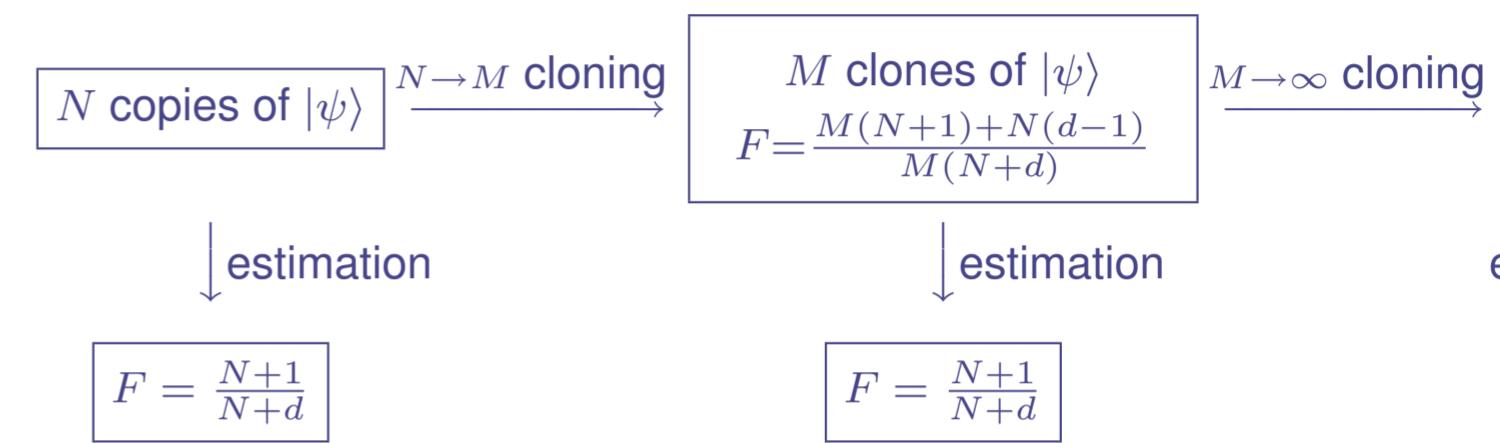


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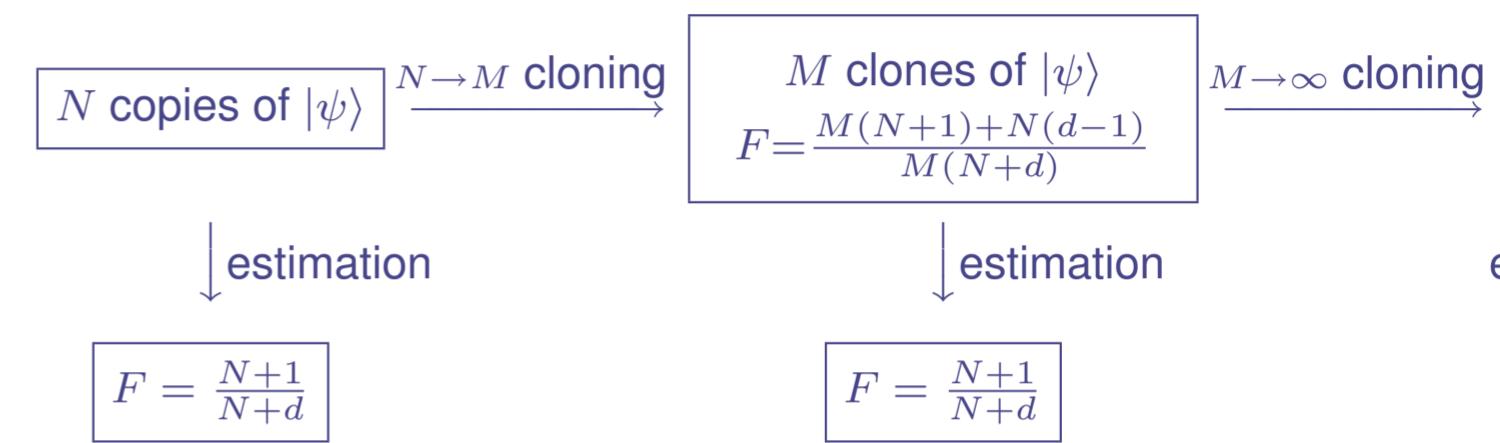


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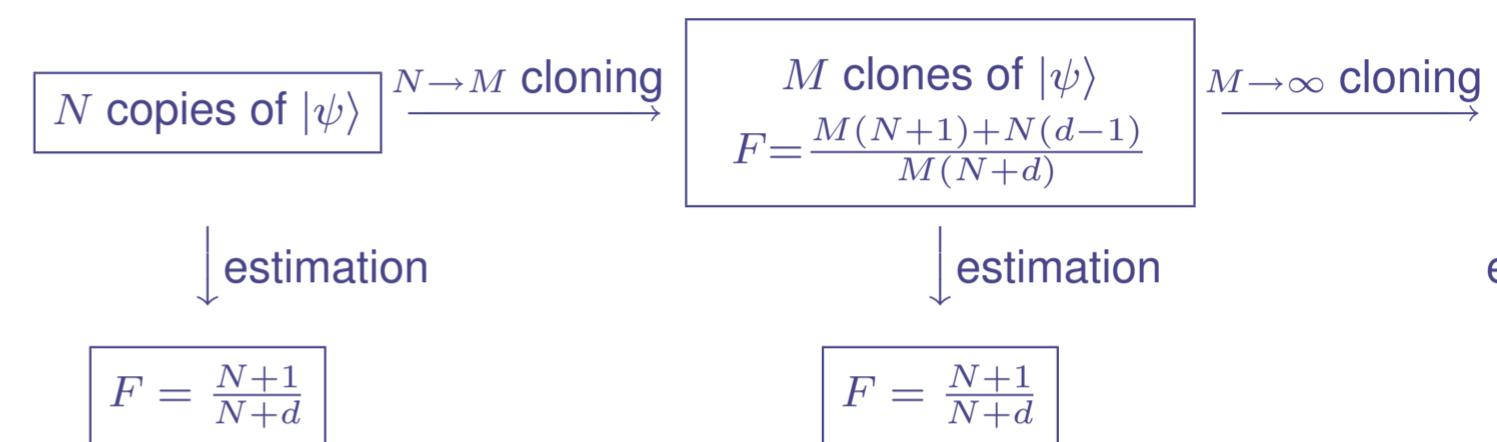


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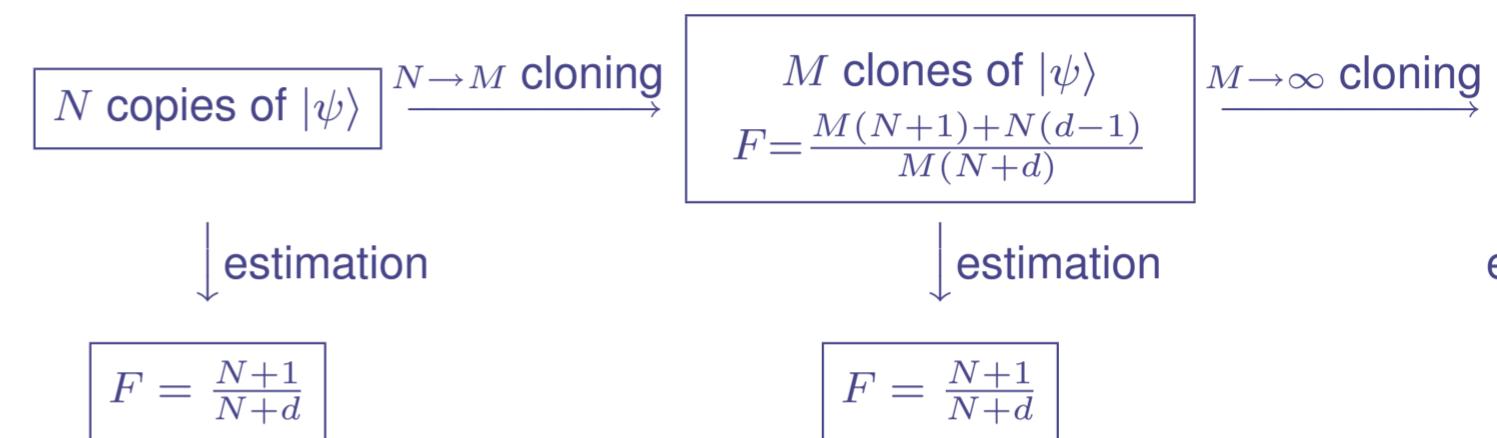


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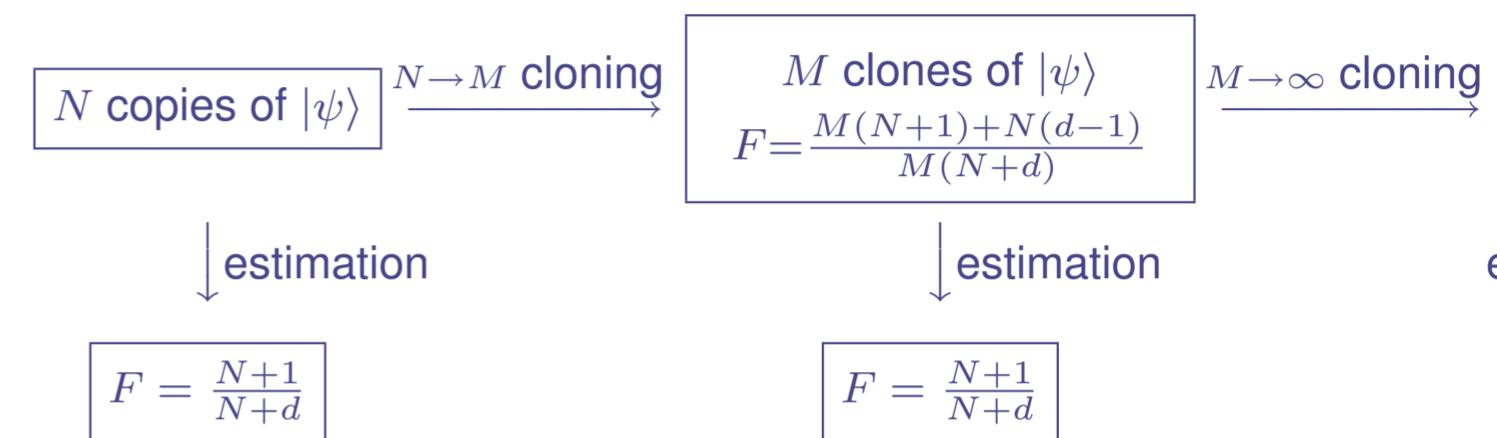


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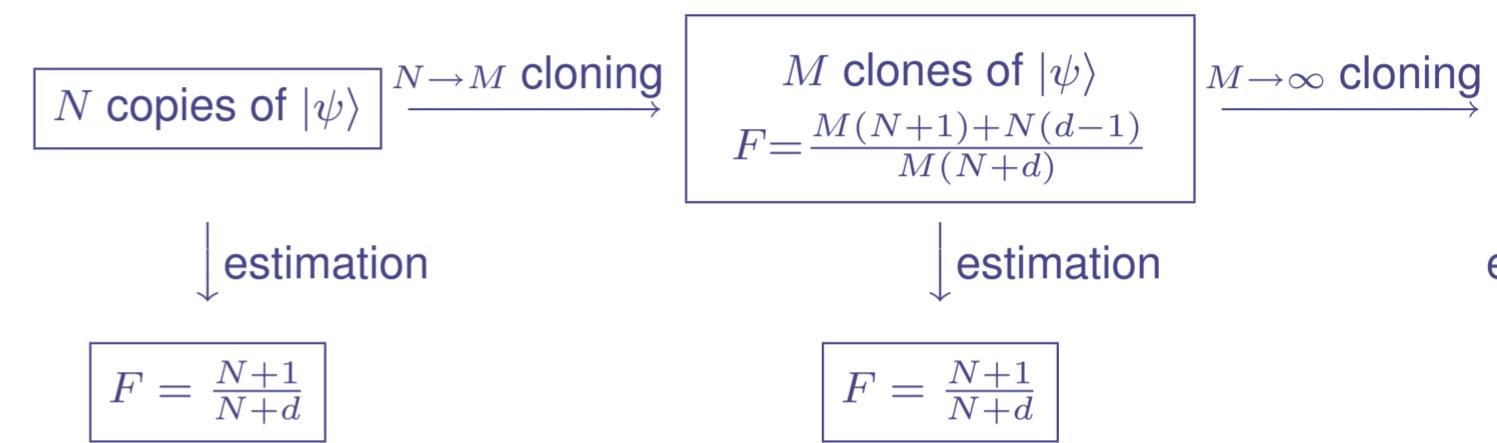


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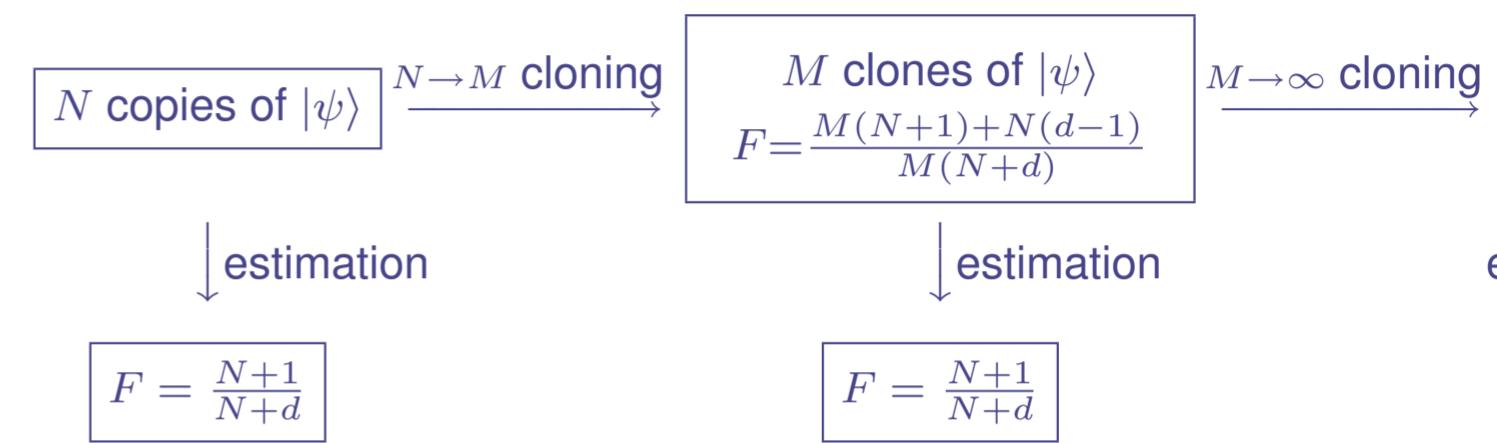


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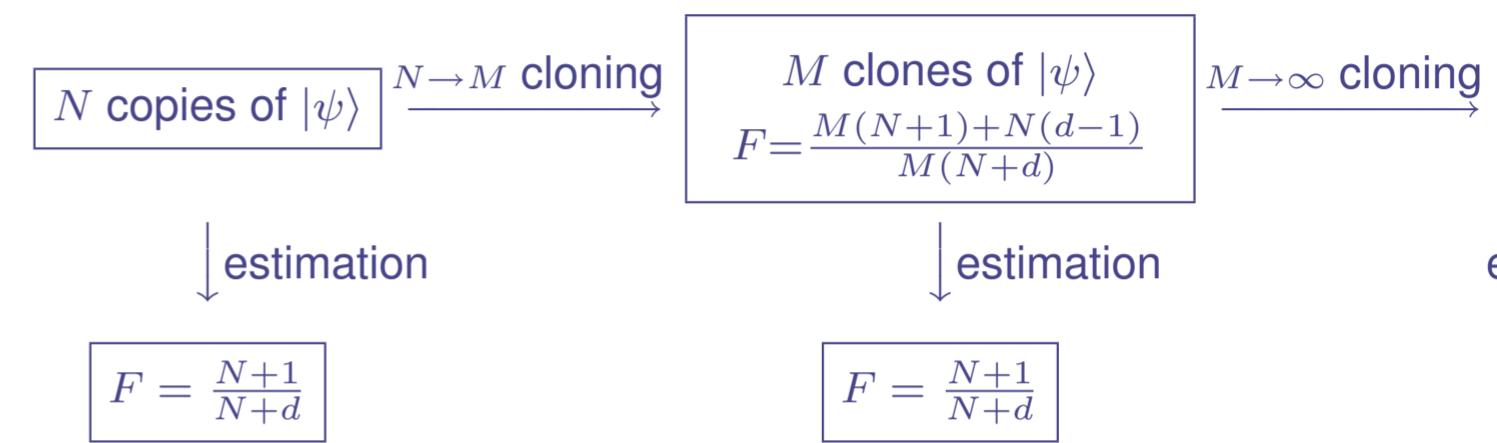


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Estimation on correlated copies

How correlations between copies influence estimation quality (RDD *Phys. Rev. A* 71, 062321

$\rho = \eta|\psi\rangle\langle\psi| + (1 - \eta)\mathbb{I}/d$ – density matrix of

$\tilde{\rho} \neq \rho^{\otimes N}$ – correlated state of N copies

different $\tilde{\rho}$, can have the same reduced matrix

Given $\tilde{\rho}$ we want to estimate $|\psi\rangle$. Fidelity

$$F = \langle\psi| \sum_{\mu} \text{Tr}(\tilde{\rho} P_{\mu}) \rho_{\mu} |\psi\rangle$$

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Permutation invariant states

Permutation invariance

$$\Pi \tilde{\rho} \Pi^\dagger = \tilde{\rho}, \quad \forall_{\Pi \in \text{repr}(S_N)}$$

N qubits (N spin 1/2)

a convenient basis, under action of $SU(2)^{\otimes N}$:

$$|j, m, \alpha\rangle, \quad j = 0, \dots, N/2; m = -j, \dots, j; \alpha$$

total angular momentum projection on the "z" axis equivalent re

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States from the optimal cloning machine are on the symmetric subspace - very bad estimator

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$$\begin{aligned}\frac{1}{2} \otimes \frac{1}{2} \otimes \frac{1}{2} \otimes \frac{1}{2} &= (0 \oplus 1) \otimes \frac{1}{2} \otimes \frac{1}{2} = (\frac{1}{2} \oplus \frac{1}{2} \oplus \frac{1}{2} \oplus \frac{1}{2}) \\ &= 0 \oplus 0 \oplus 1 \oplus 1 \oplus 1 \oplus 2 \quad \text{fully symmetric subspace}\end{aligned}$$

$$\tilde{\rho} =$$

States from the optimal cloning machine are on the symmetric subspace - very bad estimator

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Fidelity of estimation

Arbitrary permutationally invariant state:

$$\tilde{\rho} = \sum_{j=0(\frac{1}{2})}^{N/2} \frac{p_j}{d_j} \tilde{\rho}_j \otimes \mathbf{1}_{d_j} \quad d_j - \text{multiplicity of representation}$$

$p_j \geq 0, \sum_j p_j = 1, \tilde{\rho}_j$ - are density matrices on $2j + 1$ dimensions

Optimal estimation fidelity

$$F = \frac{1}{2} \left(1 + \sum_{j=0(\frac{1}{2})}^{N/2} \frac{p_j \text{Tr}(J_z \tilde{\rho}_j)}{j + 1} \right)$$

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The optimal state for state estimation

The best state $\tilde{\rho}$ for estimation

Subspaces with high j are bad for the state

Yet, necessary to give required reduced density matrix

Fixing the reduced density matrix (fixing η):

$$\rho = \eta|\psi\rangle\langle\psi| + (1 - \eta)\mathbb{I}/d$$

one can ask, what is the state $\tilde{\rho}$ optimal for estimation of $|\psi\rangle$?

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State from the optimal cloning machine

State supported on the symmetric subspace

Fidelity dependence on η :

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Product state

Product state: $\tilde{\rho} = \rho^{\otimes 4}$

Fidelity dependence on η :

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qubit permutation invariant states

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