Quantum state estimation on co
copies

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One copy of the "unknown"

What can we learn about an unknown quan
when we have one copy of it?

Polarization of a single photon

$$|\psi\rangle = \cos(\theta/2)|\leftrightarrow\rangle + \sin(\theta/2)e^{i\phi}|\updownarrow\rangle$$

If measuring in a certain basis (e.g. $|\leftrightarrow\rangle, |\updownarrow\rangle$),
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$$\rho = |\psi\rangle\langle\psi| = \frac{1}{2}\left(\mathbb{I} + \vec{n}\cdot\vec{\sigma}\right), \quad n_i = \mathrm{Tr}\,($$

$$\rho = \frac{1}{2}\left[\begin{array}{cc} 1+\langle\sigma_z\rangle & \langle\sigma_x\rangle - i\langle\sigma_y\rangle \\ \langle\sigma_x\rangle + i\langle\sigma_y\rangle & 1-\langle\sigma_z\rangle \end{array}\right.$$

Three types of measurements are necess

$\langle\sigma_z\rangle : \quad |\leftrightarrow\rangle, |\updownarrow\rangle$

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In reality we always have a finite number of

$$\underbrace{|\psi\rangle \otimes \cdots \otimes |\psi\rangle}_{N} = |\psi\rangle^{\otimes N}$$ – N copie

The more copies we have, the easier it is to
state. Different states become more orthog

$$\lim_{N\to\infty} \langle\psi|^{\otimes N}|\phi\rangle^{\otimes N} = \lim_{N\to\infty} \langle\psi|\phi\rangle^{N} =$$

What is the optimal procedure to estimate t

How does the quality of estimation depend
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Standard measurement:

$$A = \sum_i \lambda_i P_i \qquad P_i P_j = \delta_{ij} P_i \qquad \sum_i P$$

observable   measurement outcomes   projection op

$p_i = \mathrm{Tr}\,(\rho P_i)$ probability of the res

Generalized measurement

evolution and standard measurement on *system + ancill*

$$p_\mu = \mathrm{Tr}\,(\rho P_\mu) \qquad P_\mu > 0 \qquad \sum_\mu P_\mu$$

Quantum st

P_μ form a Positive Operator Valued Measure (POVM)

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State estimation strategy

N copies of an unknown state: $\rho^{\otimes N}$, $\rho = |\psi\rangle$

Choose a measurement – set of P_μ

Choose a guessing strategy: $\mu \to \rho_\mu$

Quality (fidelity) of a guess: $F_\mu = \langle\psi|\rho_\mu|\psi\rangle$

Quality of estimation

$$F = \sum_\mu p_\mu F_\mu = \sum_\mu \mathrm{Tr}\left(\rho^{\otimes N} P_\mu\right) \langle\psi|\rho_\mu|\psi\rangle =$$

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The optimal estimation of a qubit state

Massar, Popescu PRL, 74, 1259 (1995)

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The optimal estimation of a qudit state

Bruss, Machiavello, PLA, 253, 249 (1999)

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Cloning and Estimation – the Moti

$N \to M$ cloning

$$|\psi\rangle^{\otimes N} \otimes |0\rangle^{M-N} \otimes |A\rangle \qquad |\Phi\rangle \quad \in \mathcal{H}^{\otimes}$$

input states blank states ancilla

$$\tilde{\rho} = \text{Tr}_A |\Phi\rangle\langle\Phi| \qquad \text{the state of M clon}$$

$$\rho_1 = \text{Tr}_{2,\ldots,M}\tilde{\rho} \qquad \text{the state of the firs}$$

$\rho_1 = \rho_2 = \cdots = \rho_M =: \rho$ clones should b

optimal fidelity:

$$F = \langle\psi|\rho|\psi\rangle = \frac{M(N+1)+N(d-1)}{M(N+d)}$$

Clones are correlated!: $\tilde{\rho} \neq \rho^{\otimes M}$

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$$|\psi\rangle^{\otimes N} \otimes |0\rangle^{M-N} \otimes |A\rangle \qquad |\Phi\rangle \quad \in \mathcal{H}^{\otimes}$$

input states blank states ancilla

$$\tilde{\rho} = \text{Tr}_A |\Phi\rangle\langle\Phi| \qquad \text{the state of M clon}$$

$$\rho_1 = \text{Tr}_{2,\ldots,M} \tilde{\rho} \qquad \text{the state of the firs}$$

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How correlations between copies influence
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Permutation invariant states

Permutation invariance

$$\Pi\tilde{\rho}\Pi^\dagger = \tilde{\rho}, \quad \forall_{\Pi\in \text{repr}(S_N)}$$

N qubits (N spin $1/2$)

a convenient basis, under action of $SU(2)^{\otimes N}$:

$$|j, m, \alpha\rangle, \quad j = 0, \ldots, N/2;\, m = -j, \ldots, j;\, \alpha$$

total angular momentum projection on the "z" axis equivalent re

- for a given j, α; vectors $|j, m, \alpha\rangle$ support an irreducib of $SU(2)$
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States from the optimal cloning machine ar
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$$= 0 \oplus 0 \oplus 1 \oplus 1 \oplus 1 \oplus 2$$ fully symmetric su

$$\tilde{\rho} =$$

States from the optimal cloning machine ar
on the symmetric subspace - very bad esti

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Fidelity of estimation

Arbitrary permutationally invariant state:

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$p_j \geq 0$, $\sum_j p_j = 1$, $\tilde{\rho}_j$ - are density matrices on $2j+1$ di

Optimal estimation fidelity

$$F = \frac{1}{2}\left(1 + \sum_{j=0(\frac{1}{2})}^{N/2} \frac{p_j \mathrm{Tr}(J_z \tilde{\rho}_j)}{j+1}\right)$$

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Fidelity dependence on η:

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Product state

Product state: $\tilde{\rho} = \rho^{\otimes 4}$

Fidelity dependence on η:

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- Problem of estimation on correlated copies qubit permutation invariant states
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