4 Cwiczenia

27 października 2015
15:25

1. operacje lokalne

Mamy stan $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\leftrightarrow\updownarrow\rangle - |\updownarrow\leftrightarrow\rangle\right)$

- Na lewy foton: prawy działamy ćwierćfalówkami
- Na lewy półfalówka:

Ogólne operacje lokalne $U_1 \otimes U_2$

Pytanie: Czy da się wytworzyć splątanie operacjami lokalnymi?

1. Stany Bella

Baza produktowa $|00\rangle, |01\rangle, |10\rangle, |11\rangle$

Baza splątana:

$$|\phi_\pm\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

$$|\psi_\pm\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$$

Jak mając $|\psi_-\rangle$ wytworzyć inne operacjami lokalnymi?

Gęste kodowanie.

2. Dwa spiny: $H = \frac{\hbar\omega}{2}\left(\vec{\sigma}_1 \circ \vec{\sigma}_2\right)$

Jak startując ze stanu niesplątanego wytworzyć stan splątany

$$\frac{\hbar\omega}{2}\left(\sigma_x \otimes \sigma_x + \sigma_y \otimes \sigma_y + \sigma_z \otimes \sigma_z\right) =$$

$$= \frac{\hbar\omega}{2}\left(\begin{bmatrix} 0&0&0&1\\ 0&0&1&0\\ 0&1&0&0\\ 1&0&0&0 \end{bmatrix} + \begin{bmatrix} 0&0&0&-1\\ 0&0&1&0\\ 0&1&0&0\\ -1&0&0&0 \end{bmatrix} + \begin{bmatrix} 1&&&\\ &-1&&\\ &&-1&\\ &&&1 \end{bmatrix} \right) =$$

$$= \begin{bmatrix} 1&&&\\ &-1&2&\\ &2&-1&\\ &&&1 \end{bmatrix}$$

Wartości własne: $(-1-\lambda)^2 - 4 = (-3-\lambda)(1-\lambda)$

$\lambda_1 = 1 \quad \lambda_2 = -3$

$$|e_1\rangle = |\psi_+\rangle \quad |e_2\rangle = |\psi_-\rangle$$

$$|01\rangle = \frac{1}{\sqrt{2}}\left(|\psi_+\rangle + |\psi_-\rangle\right)$$

$$e^{-\frac{iH}{\hbar}t}|01\rangle = \frac{1}{\sqrt{2}}\left(e^{-\frac{i\omega}{2}t}|\psi_+\rangle + e^{\frac{3i\omega t}{2}}|\psi_-\rangle\right)$$

$$= \frac{e^{\frac{i\omega t}{2}}}{\sqrt{2}}\left(e^{-i\omega t}|\psi_+\rangle + e^{i\omega t}|\psi_-\rangle\right) =$$

$$= \left(|01\rangle \cos\omega t - i\sin\omega t\,|10\rangle\right)$$

Wybierajac $\omega t = \frac{\pi}{4}$ $\quad \frac{1}{\sqrt{2}}\left(|01\rangle - i|10\rangle\right)$

Operacja lokalnymi mozna juz dostac stan Bella.

2. Lamanie nierownosci Bella ze stanem: (nie max. splatanym)

$$|\psi\rangle = \sqrt{p}\,|01\rangle - \sqrt{1-p}\,|10\rangle$$

$$p(a,b) = \left|\langle a|\otimes\langle b|\left(\sqrt{p}\,|01\rangle - \sqrt{1-p}\,|10\rangle\right)\right|^2 =$$

$$= \left|\sqrt{p}\cos\frac{\theta_a}{2}\sin\frac{\theta_b}{2}e^{-i\varphi_b} - \sqrt{1-p}\cos\frac{\theta_b}{2}\sin\frac{\theta_a}{2}e^{-i\varphi_a}\right|^2 =$$

$$= p\cos^2\frac{\theta_a}{2}\sin^2\frac{\theta_b}{2} + (1-p)\cos^2\frac{\theta_b}{2}\sin^2\frac{\theta_a}{2}$$

$$-\sqrt{p}\sqrt{1-p}\,\frac{1}{4}\sin\theta_a\sin\theta_b\left(e^{i(\varphi_a-\varphi_b)} + e^{-i(\varphi_a-\varphi_b)}\right) =$$

$$= p\,\frac{1}{4}(1+\cos\theta_a)(1-\cos\theta_b) + (1-p)\frac{1}{4}(1+\cos\theta_b)(1-\cos\theta_a)$$

$$-\sqrt{p(1-p)}\,\frac{1}{2}\sin\theta_a\sin\theta_b\cos(\varphi_a-\varphi_b) =$$

$$= \frac{1}{4}\Big(1 - \cos\theta_a\cos\theta_b + (2p-1)(\cos\theta_a - \cos\theta_b)$$

$$- 2\sqrt{p(1-p)}\sin\theta_a\sin\theta_b\cos(\varphi_a-\varphi_b)\Big)$$

$$\begin{cases} |\vec{a}\rangle = \cos\frac{\theta_a}{2}|0\rangle + \sin\frac{\theta_a}{2}e^{i\varphi_a}|1\rangle \\ |-\vec{a}\rangle = \sin\frac{\theta_a}{2}|0\rangle - \cos\frac{\theta_a}{2}e^{i\varphi_a}|1\rangle \end{cases} \qquad \begin{matrix}\theta_a \to \pi - \theta_a \\ \varphi \to \varphi + \pi\end{matrix}$$

$$\langle\sigma_a \otimes \sigma_b\rangle =$$

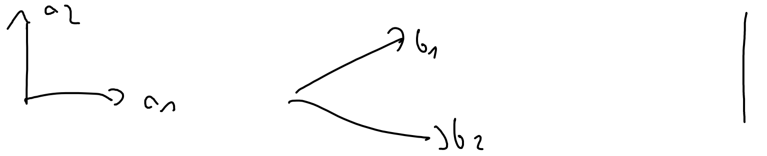
$$= \frac{1}{4}\Big(1 - \cos\theta_a\cos\theta_b + (2p-1)(\cos\theta_a - \cos\theta_b) - 2\sqrt{p(1-p)}\sin\theta_a\sin\theta_b\cos(\varphi_a-\varphi_b)$$

$$+1 - \cos\theta_a\cos\theta_b - (2p-1)(\cos\theta_a - \cos\theta_b) - 2\sqrt{p(1-p)}\sin\theta_a\sin\theta_b\cos(\varphi_a-\varphi_b)$$

$$-1 - \cos\theta_a\cos\theta_b + (2p-1)(-\cos\theta_a - \cos\theta_b) - 2\sqrt{p(1-p)}\sin\theta_a\sin\theta_b\cos(\varphi_a-\varphi_b)$$

$$-1 - \cos\theta_a\cos\theta_b + (2p-1)(\cos\theta_a + \cos\theta_b \quad - \quad -\ '' - \quad)$$

$$= -\cos\theta_a\cos\theta_b - 2\sqrt{p(1-p)}\cos(\varphi_a-\varphi_b)\sin\theta_a\sin\theta_b$$



$$\langle\sigma_{a1}\otimes\sigma_{b1}\rangle = -2\sqrt{p(1-p)}\,\frac{\sqrt{2}}{2} = -\sqrt{2p(1-p)}$$

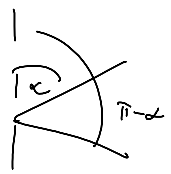
$$\langle\sigma_{a1}\otimes\sigma_{b2}\rangle = -\sqrt{2p(1-p)}$$

$$\langle\sigma_{a2}\otimes\sigma_{b1}\rangle = -\frac{\sqrt{2}}{2}$$

$$\langle\sigma_{a2}\,\sigma_{b2}\rangle = \frac{\sqrt{2}}{2}$$

$$\langle C\rangle = \sqrt{2} + 2\sqrt{2p(1-p)} = 2\sqrt{2}\left(\sqrt{p(1-p)} + \frac{1}{2}\right)$$

$$\operatorname{tg}\alpha = \frac{1}{2\sqrt{p(1-p)}}$$



$$C = -4\sqrt{p(1-p)}\sin\alpha - 2\cos\alpha$$

$$\max_{\alpha}\langle C\rangle =$$

$$\operatorname{tg}\alpha = 2\sqrt{p(1-p)}$$

$$\sin\alpha = \frac{\operatorname{tg}\alpha}{\sqrt{1+\operatorname{tg}^2\alpha}} = \frac{2\sqrt{p(1-p)}}{\sqrt{1+4p(1-p)}}$$

$$= -2\left(\cos\alpha + 2\sqrt{p(1-p)}\sin\alpha\right)$$

$$= -2\left(\frac{1 + 4p(1-p)}{\sqrt{1+4p(1-p)}}\right)$$

OK. Zawsze łamany dla dowolnego p