Ocena: $\frac{1}{3}$ zadanie + $\frac{2}{3}$ egzamin

Nielsen, Chuang „Quantum computation & q. info."

Le Bellac „Wstep do informatyki kwantowej"

Quantum Information

Open quantum systems
- int. with enviroment
- decoherence
- generalized measurements

Fundamental concepts
- states discrimination
- entanglement

Quantum communication
- classical & quantum channels capacities
- q. cryptography

Quantum computing
- q. algorithms
- q. computing implementation
- q. error correction

I. Basics of QM

- states $|\psi\rangle \in \mathcal{H}$ (pure) — complete knowledge about a system

$$|\psi\rangle = \sum_{i=1}^{d} c_i |i\rangle, \quad \sum |c_i|^2 = 1 \quad c_i \in \mathbb{C}$$

effectively $2d-2$ real parameters $\quad |\psi\rangle \equiv e^{i\varphi}|\psi\rangle$

- measurements (van Neuman projective)

$$\sum_i P_i = \mathbb{1}, \quad P_i^2 = P_i \qquad p_i = \langle\psi|P_i|\psi\rangle$$

i — measurements outcomes

$$|\psi\rangle \xrightarrow{i} |\psi_i\rangle = \frac{P_i|\psi\rangle}{\sqrt{p_i}}$$

$|\psi_i\rangle$ — conditional state

rank-1 projective measurement: $P_i = |i\rangle\langle i|$

- oberrvables

if we associate value a_i of a physical quantity $\mathcal{A}$ with measurement outcome we can define

$A = \sum_i a_i P_i$ - observable of a physical quantity $\mathcal{A}$

average value of $\mathcal{A}$: $\langle A\rangle = \sum_i p_i a_i = \langle\psi| \sum_i a_i P_i |\psi\rangle = \langle\psi|A|\psi\rangle$

concept of physical quantity $\longrightarrow$ preffered basis in $\mathcal{H}$

- mixed states (imperfect knowledge)

$\{q_k, |\psi_k\rangle\}$ - statistical ensamble ($|\psi_k\rangle$ prepared with probability q_k)

measurement outcome probability:

$$p_i = \sum_k q_k \langle\psi| P_k |\psi\rangle = \Big\{ \langle\psi| A |\psi\rangle = \mathrm{Tr}\left(A\, |\psi\rangle\langle\psi|\right)$$

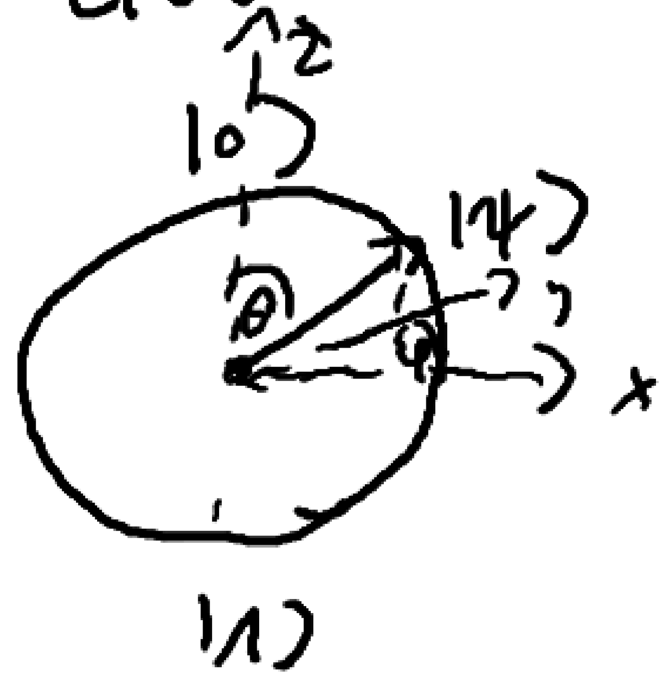
$$= \mathrm{Tr}\Big[\underbrace{\Big(\sum_k q_k P_k\Big)}_{\rho \text{ - density matrix}} |\psi\rangle\langle\psi|\Big] \qquad \rho \geq 0 \quad \mathrm{Tr}\,\rho = 1 \quad (d^2-1 \text{ real parameters})$$

(contains all the information required to predict measurement probabilities)

$\rho = \sum_k q_k |\psi_k\rangle\langle\psi_k|$ - in general non-unique decomposition (unique only when ρ has non-degenerate spectrum)

- qubit (simplest possible q. system)

$$|\psi\rangle = a|0\rangle + b|1\rangle = \cos\tfrac{\theta}{2}|0\rangle + e^{i\varphi}\sin\tfrac{\theta}{2}|1\rangle, \qquad \theta \in [0,\pi], \quad \varphi \in [0, 2\pi[$$



$$|\pm\rangle_x = \frac{1}{\sqrt{2}}\left(|0\rangle \pm |1\rangle\right)$$

$$|\pm\rangle_y = \frac{1}{\sqrt{2}}\left(|0\rangle \pm i|1\rangle\right)$$

$$|+\rangle_z = |0\rangle \qquad |-\rangle_z = |1\rangle$$

$$|\psi\rangle\langle\psi| = \begin{bmatrix} \cos^2\frac{\theta}{2} & \cos\frac{\theta}{2}\sin\frac{\theta}{2}e^{-i\varphi} \\ \cos\frac{\theta}{2}\sin\frac{\theta}{2}e^{i\varphi} & \sin^2\frac{\theta}{2} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1+\cos\theta & \sin\theta\, e^{-i\varphi} \\ \sin\theta\, e^{i\varphi} & 1-\cos\theta \end{bmatrix} =$$

$$= \frac{1}{2}\left(\mathbb{1} + \cos\theta\,\sigma_z + \sin\theta\cos\varphi\,\sigma_x + \sin\theta\sin\varphi\,\sigma_y\right) =$$

$$= \frac{1}{2}\left(\mathbb{1} + \vec{m}\cdot\vec{\sigma}\right) \qquad \vec{m} = \begin{bmatrix} \sin\theta\cos\varphi \\ \sin\theta\sin\varphi \\ \cos\theta \end{bmatrix} \qquad |\vec{m}| = 1$$

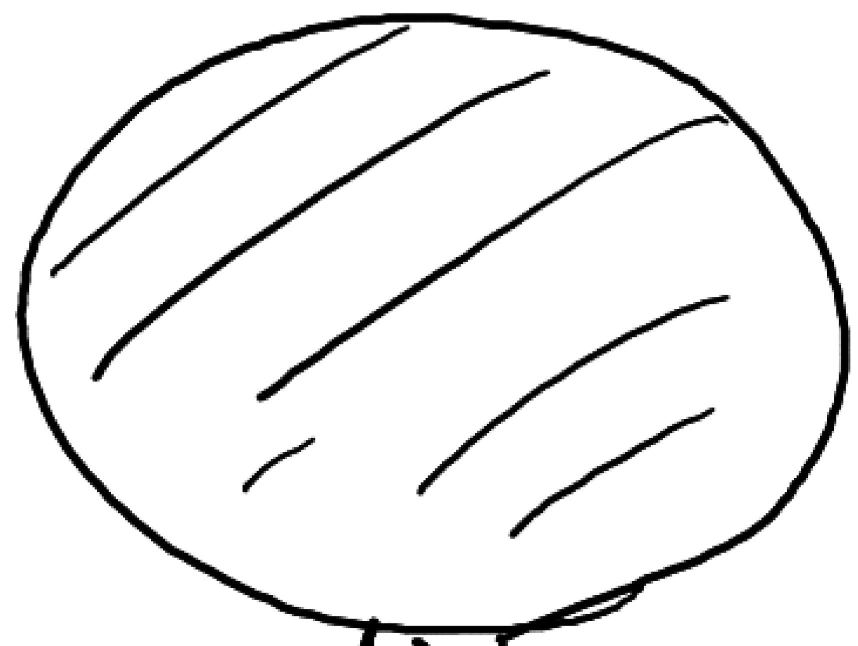
$$m_i = \langle\psi|\sigma_i|\psi\rangle, \qquad \frac{1}{2}\mathrm{Tr}(\sigma_i\sigma_j) = \delta_{ij}$$

ogólny stan mieszany qubitu:

Bloch ball

$$\rho = \sum_k q_k |\psi_k\rangle\langle\psi_k| = \frac{1}{2}\Big(\mathbb{1} + \underbrace{\Big(\sum_k q_k \vec{m}_k\Big)}_{\vec{m}}\cdot\vec{\sigma}\Big) \qquad |\vec{m}| \leq 1$$

$\rho = \frac{1}{2}(\mathbb{1} + \underbrace{\vec{\sigma}\circ\vec{m}}_{\text{has eigenvalues } \pm|\vec{m}|})$, if $|\vec{m}| > 1$, $\rho \not\geq 0$

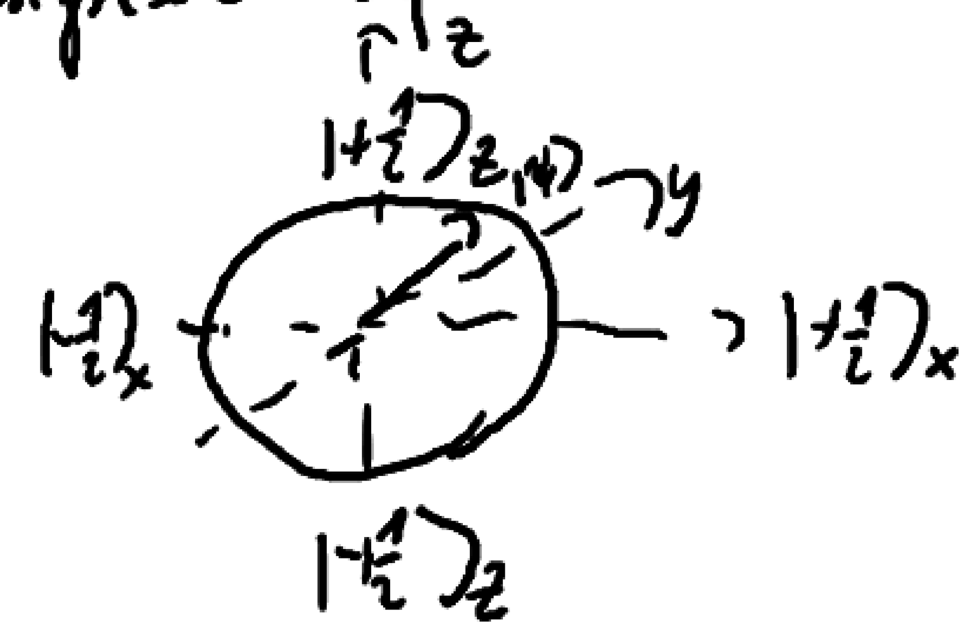


vs

qubit   bit

- physical examples of realization of a qubit

— spin $\frac{1}{2}$

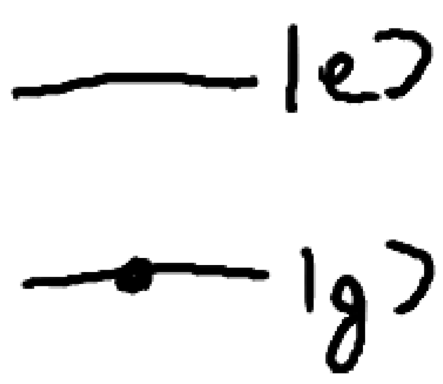


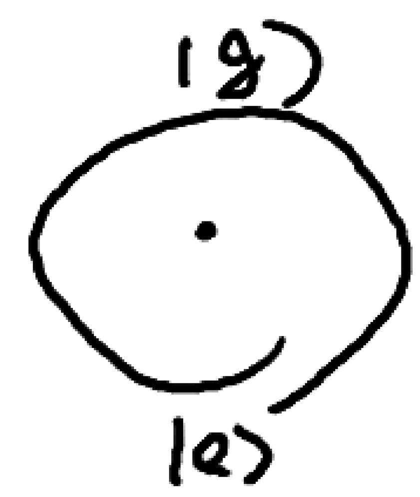
$S_i = \frac{\hbar}{2}\sigma_i$

$S_{\vec{m}} = \vec{S}\circ\vec{m} = \frac{\hbar}{2}\vec{\sigma}\circ\vec{m}$

$\vec{m}$ - direction of a well defined projection of the spin of a particle

— two-level atom





{ typically superposition decay towards $|g\rangle$ state via spontaneous emission

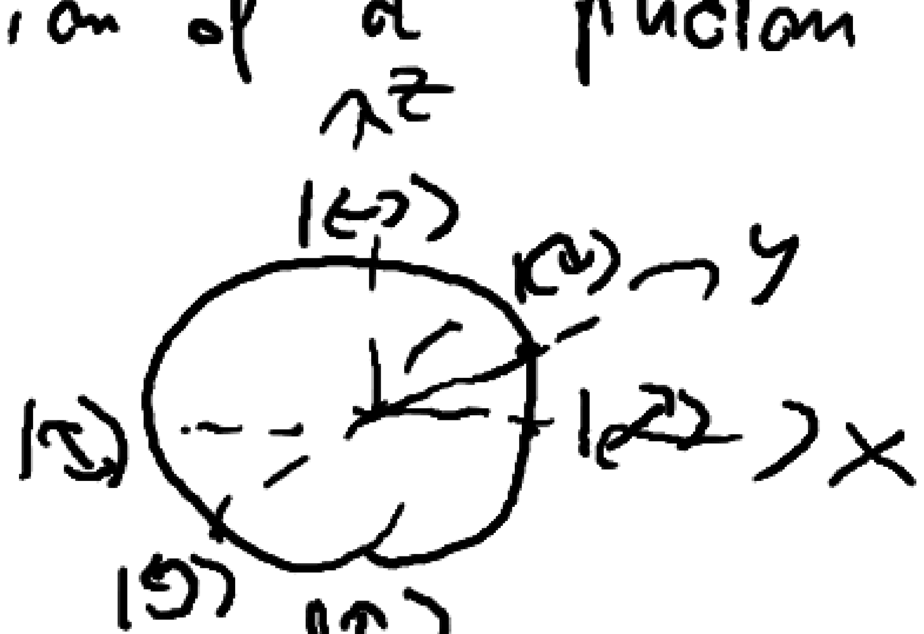
— photon travelling via two possible paths { double-slit experiment

$= |0\rangle$ $= |1\rangle$ $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$

φ — relative phase delay (difference of optical path length)

— polarization of a photon



$|\psi\rangle = \cos\frac{\theta}{2}|\leftrightarrow\rangle + e^{i\varphi}\sin\frac{\theta}{2}|\updownarrow\rangle$

φ — phase delay between horizontal & vertical polarization

- multiple q. systems

$\mathcal{H} = \mathcal{H}_1 \otimes \dots \otimes \mathcal{H}_N$ $\quad |\psi\rangle = \sum_{i_1, \dots, i_N} c_{i_1 \dots i_N} \underbrace{|i_1\rangle \otimes \dots \otimes |i_N\rangle}_{|i_1, \dots, i_N\rangle}$

a general pure state

$\dim \mathcal{H} = d \cdot N$

a general mixed state:

$$\rho = \sum_{\substack{i_1 \dots i_N \\ j_1 \dots j_N}} \rho^{i_1 \dots i_N}_{j_1 \dots j_N} |i_1\rangle\langle j_1| \otimes \dots \otimes |i_N\rangle\langle j_N|$$

- evolution of an isolated q. system

$|\psi(t)\rangle = \mathcal{U}_t |\psi(0)\rangle,$ $\quad$ for time independent H, $\mathcal{U}_t = e^{-\frac{iHt}{\hbar}}$

unitary operation

- evolution of a qubit

$|\psi'\rangle = \mathcal{U} |\psi\rangle$ $\qquad \mathcal{U} = e^{-\frac{i \vec{n} \cdot \vec{\sigma}}{2} \alpha}$

$SU(2)$ operation

rotation around axis $\vec{n}$ by an angle α