2. Single photon states

General Fock states $|n_1, n_2, \ldots\rangle = \frac{a_1^{+n_1}}{\sqrt{n_1!}} \frac{a_2^{+n_2}}{\sqrt{n_2!}} \cdots |0\rangle$

- given number of photons in different modes.

single photon states:

$|\psi\rangle = a_k^+ |0\rangle$ if we think of $\vec{a}(\vec{k}, \sigma)$ - corresponding to plane waves.

then: $a^+(k, \sigma)|0\rangle$ - single monochromatic photon in the whole space.

energy: $E = \hbar\omega = \hbar k c$

more realistic single photon states:

$$|\psi\rangle = \int d^3\vec{k} \sum_\sigma \tilde{\psi}(\vec{k}, \sigma) \hat{a}^+(\vec{k}, \sigma) |0\rangle$$

general single photon state

complex amplitudes

$\tilde{\Psi}(\vec{k}, \sigma)$ - photon wave function in momentum representation.

$$\langle\psi|\psi\rangle = 1 \Rightarrow \int d^3k \sum_\sigma |\tilde{\psi}(\vec{k}, \sigma)|^2 = 1$$

How to define a photon wave function in the position representation. $\vec{\psi}(\vec{r}, t) = ?$

{ look Bialynicki-Birula review: arxiv: quant-ph/0508202

$$\hat{\vec{\psi}} = \sqrt{\frac{\varepsilon_0}{2}} (\hat{\vec{E}} + i\sigma c \hat{\vec{B}}) = \vec{\psi}^{(+)} + \vec{\psi}^{(-)}$$

$\hat{a} e^{-i\omega t}$ ... $\hat{a}^+ e^{i\omega t}$

$$\hat{\vec{E}} = \vec{E}^{(+)} + \vec{E}^{(-)}$$

positive freq. part

Define:

$$\vec{\Psi}_\sigma(\vec{r}, t) := \langle 0| \hat{\vec{\Psi}}_\sigma(\vec{r}, t) |\psi\rangle = \langle 0| \vec{\Psi}_\sigma^{(+)} |\psi\rangle =$$

$$\vec{\psi}_\sigma(\vec{r},t) = \langle 0 | \hat{\vec{\psi}}_\sigma^{(+)}(\vec{r},t) | \psi \rangle =$$

$$= \langle 0 | \int d^3\vec{k} \sqrt{\frac{\hbar \omega_k}{(2\pi)^3}} \; \vec{e}(\vec{k},$$