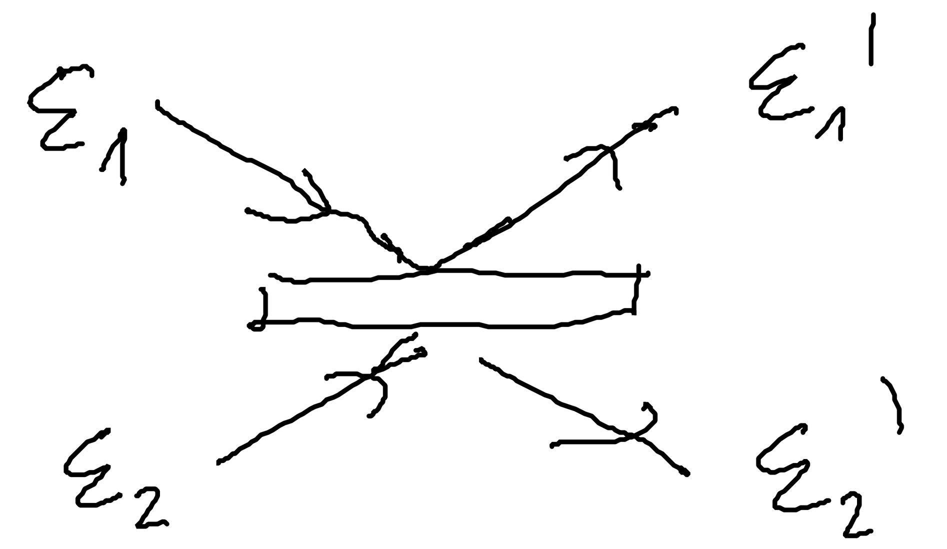
3. Linear Optical Networks

Mirrors, Beam splitters, Interferometers ... + Detectors

- Beamsplitter. (classically)



$$\begin{bmatrix}\mathcal{E}_1'\\ \mathcal{E}_2'\end{bmatrix} = \overbrace{\begin{bmatrix} r_1 & t_2 \\ t_1 & r_2\end{bmatrix}}^{B} \begin{bmatrix}\mathcal{E}_1\\ \mathcal{E}_2\end{bmatrix}$$

energy conservation : $|\mathcal{E}_1|^2 + |\mathcal{E}_2|^2 = |\mathcal{E}_1'|^2 + |\mathcal{E}_2'|^2$

$\Downarrow$

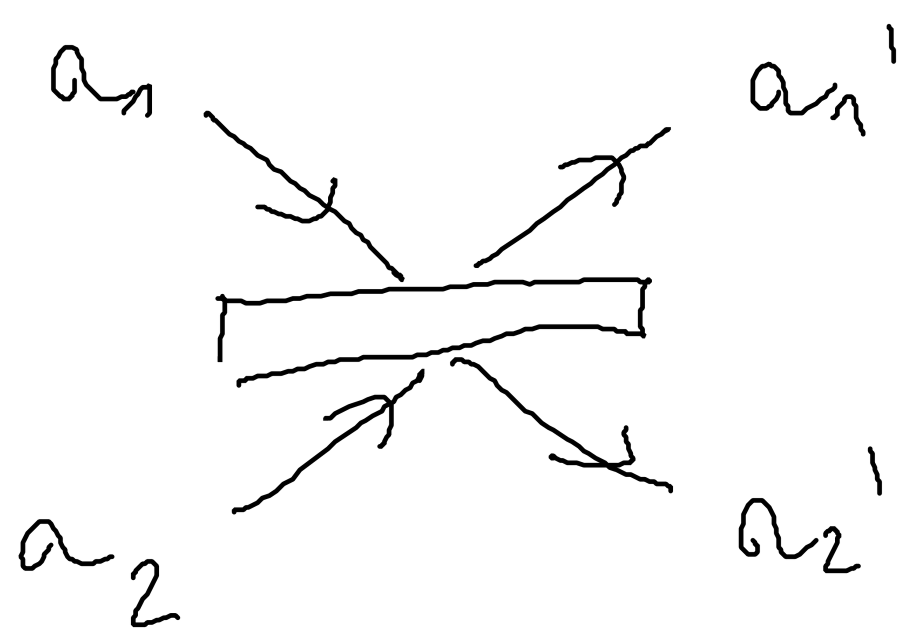
B - unitary

e.g. 50:50 beamsplitter $B_{50/50} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1\end{bmatrix}$

general

$$B = \frac{1}{\sqrt{2}}\begin{bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi\end{bmatrix}$$

- Beamspitter (quantum optics)



$$\begin{bmatrix}\hat{a}_1'\\ \hat{a}_2'\end{bmatrix} = B \begin{bmatrix}\hat{a}_1\\ \hat{a}_2\end{bmatrix}$$

(Heisenberg picture)

instead of computing:

$$\langle\psi| f(\vec{a}, \vec{a}^{\dagger}) |\psi\rangle \xrightarrow[\text{of beamsplitter}]{\text{presence}} \langle\psi| f(\vec{a}', \vec{a}'^{\dagger}) |\psi\rangle$$

$\hat{a}_j$ - input modes $\quad \hat{a}_j'$ - output modes

Evolution $\equiv$ transformation of modes.

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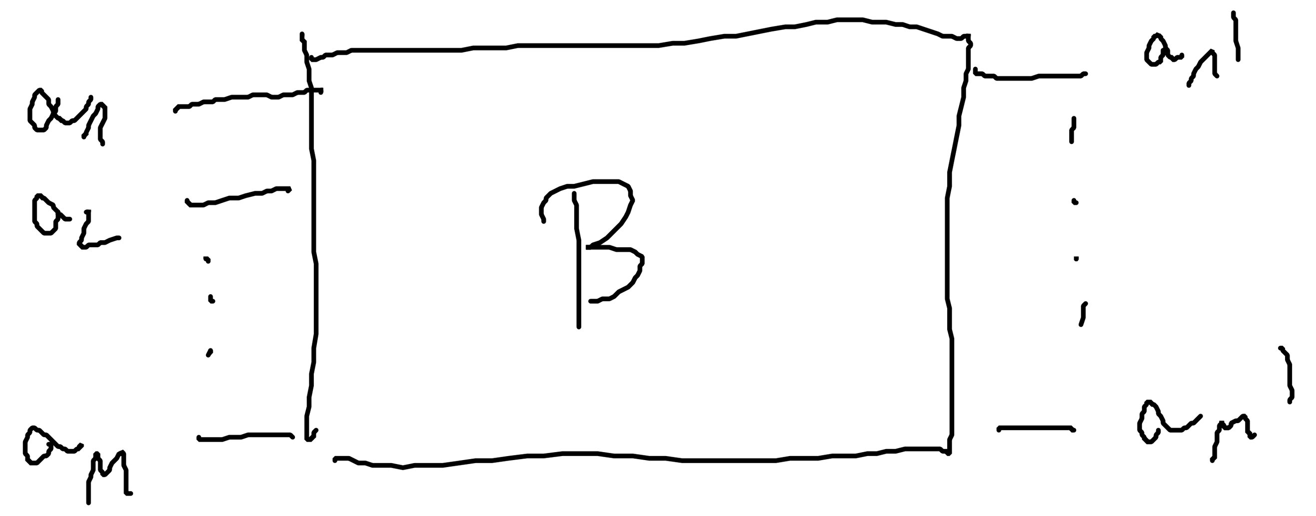
In general: (linear optical transformation (passive))



$$\vec{a}' = B\,\vec{a}$$

We want to describe it as a unitary transformation in the Hilbert space:

$$\vec{a}' = B\,\vec{a} = \begin{bmatrix} \hat{U}_B^\dagger \hat{a}_i \hat{U}_B \\ \vdots \\ U_B^\dagger \hat{a}_M U_B \end{bmatrix} = \hat{U}_B^\dagger \vec{\hat{a}}\, \hat{U}_B$$

If we find U_B we would be able to see things in Schrödinger picture:

$$|\psi'\rangle = \underline{U_B|\psi\rangle} \quad : \quad \langle\psi|\, f(\vec{a}'^\dagger, \vec{a}')\,|\psi\rangle = \langle\psi'|\, f(\vec{\hat{a}}^\dagger, \vec{a})\,|\psi'\rangle$$