4. Classical & non-classical states of light

Quantum E-M field from classical sources

Lagrangian: $\mathcal{L} = -\frac{1}{4\mu_0} F^{\mu\nu}F_{\mu\nu} - \underbrace{j^\mu}_{\text{classical}} A_\mu \qquad j^\mu = [\rho c, \vec{j}]$

Equation of motion: $\Box A^\mu = \mu_0 j^\mu$

Note: $\Pi^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_t A^\mu)}$ – the same as in no-sources case

The only difference in quantization procedure:

$$\hat{\mathcal{H}} = \sum_\mu \Pi^\mu \partial_t A_\mu - \mathcal{L} = \hat{\mathcal{H}}_0 + j^\mu \hat{A}_\mu$$

$$\hat{H} = \int d^3r\, \hat{\mathcal{H}} = \hat{H}_0 + \underbrace{\int d^3\vec{r}\, j^\mu \hat{A}_\mu}_{H_{int}}$$

{ In Coulomb gauge $A^0 = \frac{\varphi}{c}$ is still classical and is determined by ρ – classical charge density.
{ In what follows we set $\rho = 0$

$$H_{int} = \int d^3\vec{r}\, \vec{j} \cdot \vec{A}.$$

In no-sources case we worked in the Heisenberg picture and found the solution: (for discrete modes)

$$\hat{\vec{A}}(\vec{r},t) = i\mathcal{A} \sum_k \frac{1}{\sqrt{\omega_k}} \vec{e}_k \left(\hat{a}_k e^{i(\vec{k}\vec{r} - \omega_k t)} - \hat{a}_k^+ e^{-i(\vec{k}\vec{r} - \omega_k t)} \right)$$

$$\mathcal{A} = \sqrt{\frac{\hbar}{2\varepsilon_0 V}}$$

(To see how presence of $\vec{j}$ changes the quantum states of E-M field we use the interaction picture (Dirac picture)

$$\hat{\vec{A}}(t) = e^{\frac{iH_0 t}{\hbar}} \hat{\vec{A}}(0) e^{-\frac{iH_0 t}{\hbar}} \qquad |\psi(t)\rangle_D := e^{\frac{iH_0 t}{\hbar}} |\psi(t)\rangle_S$$

$$\frac{d|\psi(t)\rangle_D}{dt} = -\frac{i}{\hbar} \hat{H}_{int}(t) |\psi(t)\rangle_D$$