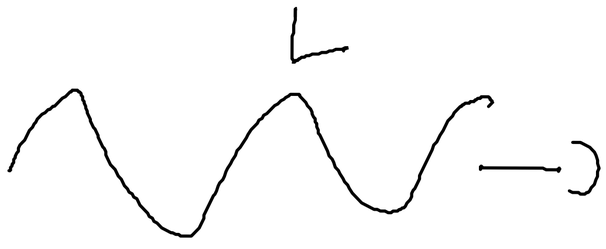
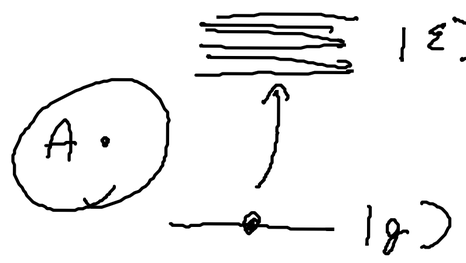
7. Photodetection theory

We try to model photoelectric effect





$$\mathcal{H} = \mathcal{H}_L \otimes \mathcal{H}_A$$

$$\hat{H} = \underbrace{\hat{H}_L + \hat{H}_A}_{\hat{H}_0} + \hat{H}_I$$

$$\hat{H}_I = -\hat{\vec{d}} \cdot \hat{\vec{E}}(\vec{r})$$

$$\begin{cases} \hat{\vec{d}} = \sum_{ij} \overbrace{\langle e_i | e\vec{q} | e_j \rangle}^{d_{ij}} |e_i\rangle\langle e_j| = e\vec{q} \\ \hat{L}_{basis\ in\ \mathcal{H}_A} \end{cases}$$

d_{ij} - dipole moment operator

$$H_A |e_i\rangle = \varepsilon_i |e_i\rangle$$

We work in the interaction picture (Dirac)

$$H_I^D = e^{\frac{iH_0 t}{\hbar}} H_I e^{-\frac{iH_0 t}{\hbar}} = -\hat{\vec{d}}(t) \cdot \hat{\vec{E}}(\vec{r}, t)$$

$$= \sum_{ij} e^{\frac{i(\varepsilon_i - \varepsilon_j)t}{\hbar}} d_{ij} |e_i\rangle\langle e_j|$$

Initially: $|\Psi(t_0)\rangle_{LA} = |\Psi\rangle_L \otimes |g\rangle_A$

$$|\Psi^D(t)\rangle_{LA} = \mathcal{T}\left[e^{-\frac{i}{\hbar}\int_{t_0}^{t} H_I^D(t)\,dt'} \right] |\Psi^D(t_0)\rangle_{LA} =$$

$$= \left(\mathbb{1} - \frac{i}{\hbar}\int_{t_0}^{t} dt_1\, \hat{H}_I^D(t_1) + \left(\frac{i}{\hbar}\right)^2 \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2\, \hat{H}_I^D(t_1)\hat{H}_I^D(t_2) + \dots \right) |\Psi^D(t_0)\rangle$$

Let $|\phi\rangle|\varepsilon\rangle$ be some state with $\langle\varepsilon|g\rangle = 0$ $\begin{cases} \text{detector} \\ \text{clicked} \end{cases}$

Probability amplit

$\begin{cases} |\varepsilon\rangle - \text{energy eigenstate} \\ H_A|\varepsilon\rangle = \varepsilon|\varepsilon\rangle \end{cases}$