Higher order coherence functions (mostly second order)

Fact

For a multimode coherent state $|\psi\rangle = \bigotimes_k |\alpha_k\rangle_k =: |\vec{\alpha}\rangle$ $|\{\alpha\}\rangle$

$$g^{(m)}(r_1, \dots, r_m, t_1, \dots t_m) = 1$$

Proof

$$g^{(m)}(r_1, \dots r_m, t_1 \dots t_m) = \frac{G^{(m)}(r_1 \dots r_m, t_1 \dots t_m; r_1 \dots r_m, \dots t_1 \dots t_m)}{G^{(1)}(r_1, t_1; r_1, t_1) \dots G^{(1)}(r_m, t_m; r_m, t_m)} =$$

$$= \frac{\langle : \hat{I}(r_1, t_1) \cdot \dots \cdot \hat{I}(r_m, t_m) : \rangle}{\langle \hat{I}(r_1, t_1) \rangle \cdot \dots \cdot \langle \hat{I}(r_m, t_m) \rangle}$$

$$\hat{I}(r,t) = \hat{E}^{(-)}(r,t) \hat{E}^{(+)}(r,t)$$

$$E^{(+)}(\vec{r}, t) = \sum_k \overbrace{\sqrt{\frac{\hbar \omega_k}{2\epsilon_0 V}}}^{A_k} \hat{a}_k e^{i(\vec{k}\vec{r} - \omega_k t)}$$

$$\langle \psi | \hat{I}(r,t) | \psi \rangle \underset{\hat{a}_k^\dagger \to \alpha_k^*}{\overset{\hat{a}_k \to \alpha_k}{=}} \left| \sum_k A_k \alpha_k e^{i(\vec{k}\vec{r} - \omega t)} \right|^2$$

$$\langle \psi | : \hat{I}(r_1, t_1) \cdot \dots \cdot \hat{I}(r_m, t_m) : | \psi \rangle = \prod_{i=1}^{m} \left| \sum_k A_k \alpha_k e^{i(\vec{k}\vec{r}_i - \omega_k t_i)} \right|^2$$

$=$

Comment: If we have P representation:

$$\rho = \int d^2\vec{\alpha}\, P(\vec{\alpha}) |\vec{\alpha}\rangle\langle\vec{\alpha}|$$

$$G^{(m)}(\dots) = \int d^2\vec{\alpha}\, P(\vec{\alpha})\, G^{(m)}_{|\vec{\alpha}\rangle}(\dots) =$$

$$= \int d^2\vec{\alpha}\, P(\vec{\alpha})\, I_{\vec{\alpha}}(r_1, t_1) \cdot \dots \cdot I_{\vec{\alpha}}(r_m, t_m)$$

$$I_{\vec{\alpha}}(\vec{r}, t) = \left| \sum_k A_k \alpha_k e^{i(\vec{k}\vec{r} - \omega t)} \right|^2$$

Second order temporal coherence function

$$g^{(2)}(\vec{r}, \vec{r}, t, t+\tau)$$

Consider single plane wave case, $\hat{E}^{(+)} \sim \hat{a} e^{i(\vec{k}\vec{r} - \omega t)}$

$$g^{(2)}(0) := g^{(2)}(\vec{r}, \vec{r}, t, t) = \frac{\langle \hat{a}^{+2} \hat{a}^2 \rangle}{\langle \hat{a}^+ \hat{a} \rangle^2} =$$

$$= \frac{\langle (\hat{a}^+ \hat{a})^2 \rangle - \langle \hat{a}^+ \hat{a} \rangle}{\langle \hat{a}^+ \hat{a} \rangle^2} = \frac{\langle \hat{n}^2 \rangle - \langle \hat{n} \rangle}{\langle \hat{n} \rangle^2} -$$