8. Interaction of a two-level atom with quantized e-m field.

We use dipole approximation:

$$\hat{H} = \hat{H}_A + \hat{H}_L + H_I$$

$$\hat{H}_A = \sum_i \underset{\underset{\hbar\nu_i}{\parallel}}{\varepsilon_i} |i\rangle\langle i| \;, \quad \hat{H}_L = \sum_K \hbar\omega_K \left(\hat{a}_K^\dagger \hat{a}_K + \frac{1}{2} \right), \quad \hat{H}_I = -\hat{\vec{d}} \cdot \hat{\vec{E}}(\vec{r}=0)$$

$$\hat{\vec{d}} = \sum_{ij} \vec{d}_{ij} |i\rangle\langle j| \;, \quad \hat{\vec{E}}(\vec{r}=0) = \sum_K \sqrt{\frac{\hbar\omega_K}{2\varepsilon_0 V}} \left(\hat{a}_K + \hat{a}_K^\dagger \right) \vec{e}_K$$

We move to interaction (Dirac) picture: $\hat{H}_0 = \hat{H}_A + \hat{H}_L$

$$\hat{H}_I^D(t) = e^{\frac{iH_0 t}{\hbar}} \hat{H}_I e^{-\frac{iH_0 t}{\hbar}} \;, \quad |\psi^D(t)\rangle = e^{\frac{iH_0 t}{\hbar}} |\psi(t)\rangle$$

$$i\hbar \frac{d|\psi^D(t)\rangle}{dt} = H_I^D(t) |\psi^D(t)\rangle$$

In our case:

$$\hat{H}_I^D(t) = -\sum_{ij,k} \vec{e}_k \cdot \vec{d}_{ij} \sqrt{\frac{\hbar\omega_k}{2\varepsilon_0 V}} e^{i(\nu_i - \nu_j)t} |i\rangle\langle j| \otimes \left(\hat{a}_K e^{-i\omega_K t} + \hat{a}_K^\dagger e^{i\omega_K t} \right)$$

If we have discretized optical frequencies (in optical cavity) and some ω_{K_0} i close to $\nu_i - \nu_j$ $(i=g,\ j=e)$

In this case we restrict to only two atomic states $|g\rangle, |e\rangle$ and a single optical frequency $\omega := \omega_K$.

We switch to standing wave mode:

$$\hat{\vec{E}} \approx \sqrt{\frac{\hbar\omega}{\varepsilon_0 V}} \left(\hat{a} e^{-i\omega t} + \hat{a}^\dagger e^{i\omega t} \right) \sin(kz)$$

$$H_I^D(t) = \sum_{i,j = g,e} \vec{e}_K \cdot \underline{\vec{d}_{ij}} \sqrt{\frac{\hbar\omega}{\varepsilon_0 V}} \sin kz \, |i\rangle\langle j| e^{i(\nu_i - \nu_j)t} \otimes \left(\hat{a} e^{-i\omega t} + \hat{a}^\dagger e^{i\omega t} \right)$$

$$\vec{d}_{ii} = 0$$

$\sigma_+ = |e\rangle\langle g|$, $\sigma_- = |g\rangle\langle e|$, $\nu = \nu_e - \nu_g$

$\lambda = \underline{\vec{e}_k \cdot \vec{d}_{eg}}$