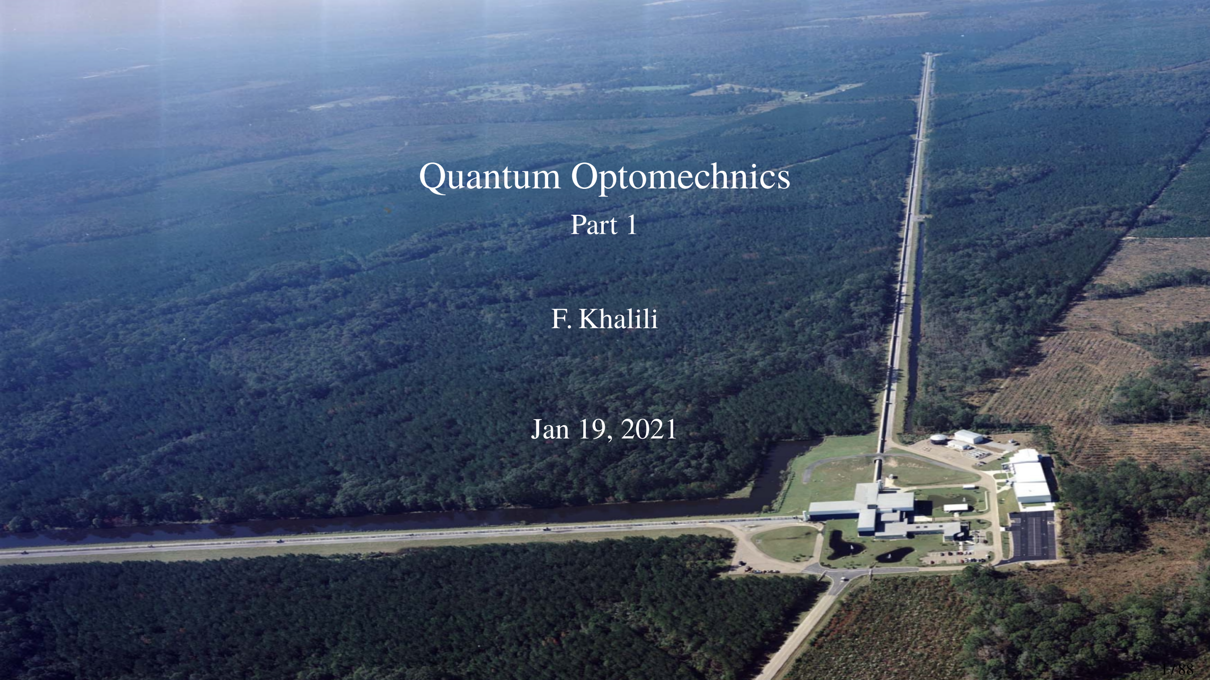

Quantum Optomechnics

Part 1

F. Khalili

Jan 19, 2021

Lecture 2:

- Quantum speedmeter
- Hamilitonian approach
- Optical spring
- Ground state optical cooling

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- Quantum speedmeter
- Hamilitonian approach
- Optical spring
- Ground state optical cooling

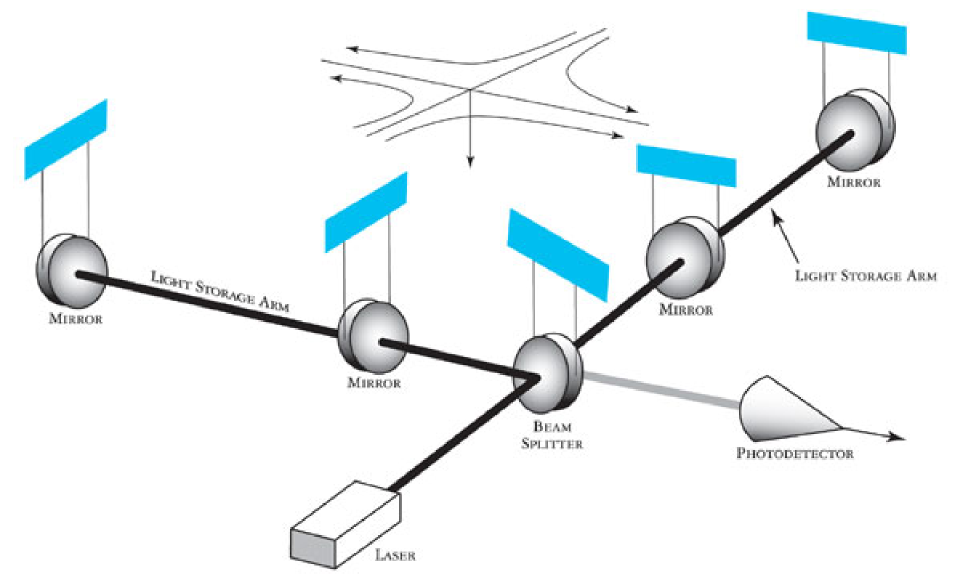
If time allows:

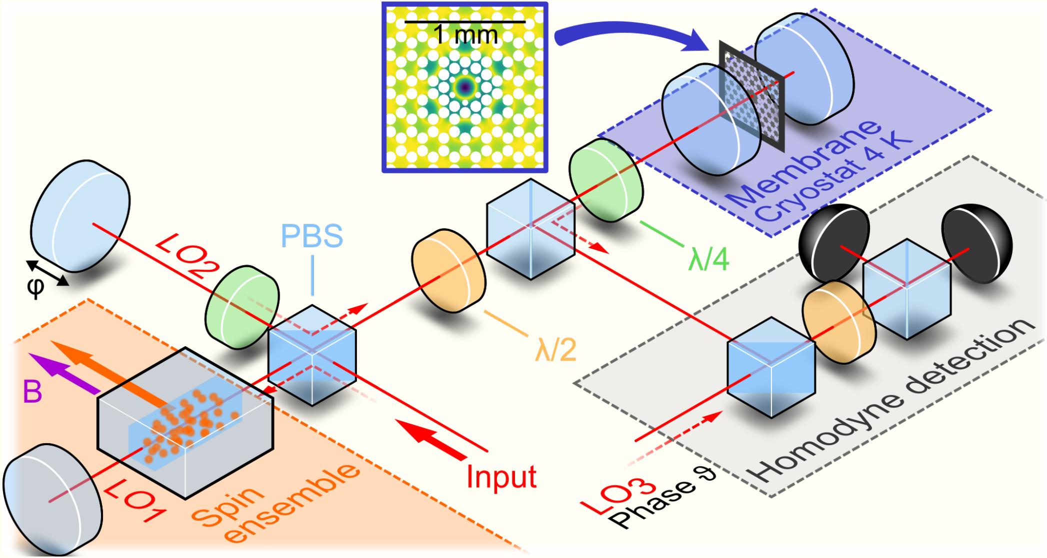
- Hybrid systems
- Non-stationary optomechanics

See more in:

- S. Danilishin and F. Khalili, *Quantum measurement theory in gravitational-wave detectors*, Living Reviews in Relativity **15**, 5 (2012)
- M. Aspelmeyer, T. Kippenberg, and F. Marquardt, *Cavity optomechanics*, Rev. Mod. Phys **86**, 1391 (2014)
- S. Danilishin, F. Khalili, and H. Miao, *Advanced quantum techniques for future gravitational-wave detectors*, Living Reviews in Relativity **22**:1, 2, (2019)
- F. Khalili and E. Zeuthen, *Quantum limits for stationary force sensing*, arXiv:2011.14716 (2020)

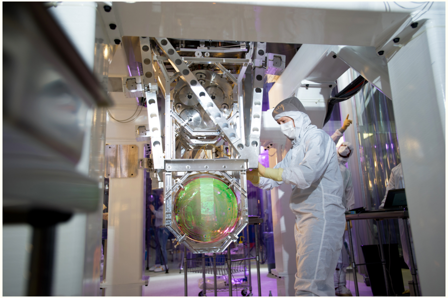
Quantum optomechanics





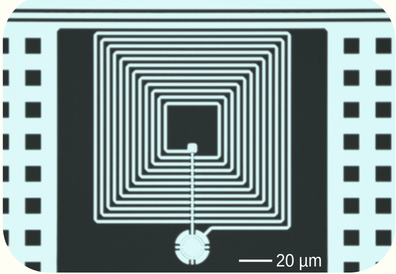
R.Thomas *et al*, arXiv:2003.11310 (2020)

Quantum optomechanics

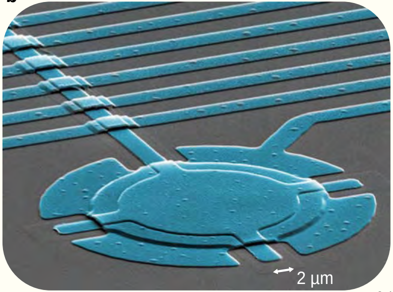


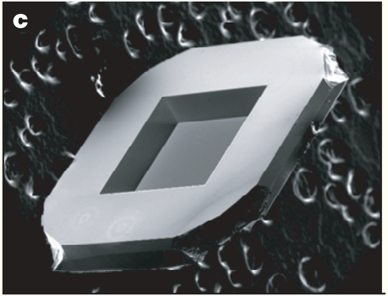
Macroscopic
(≡ consisting
of many-atoms)
mechanical
free masses
and resonators

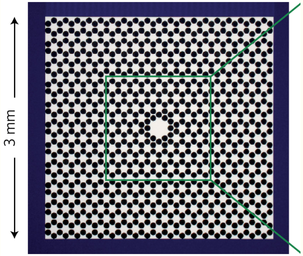
a



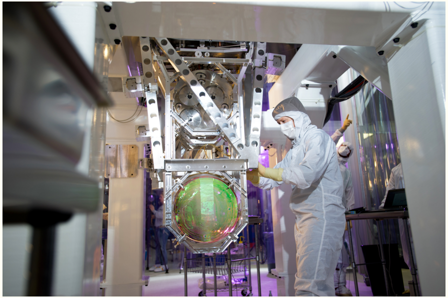
b

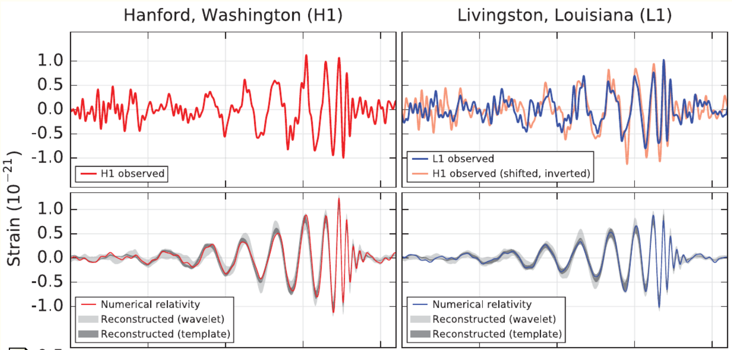






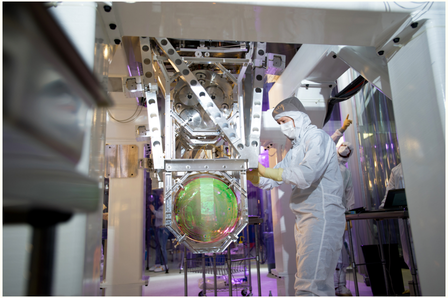
Quantum optomechanics

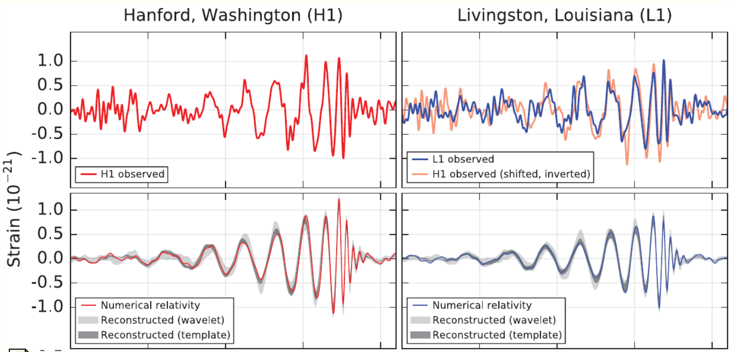




B.Abbott *et al*, Phys. Rev. Lett. **116** 061102 (2016)

Quantum optomechanics





B.Abbott *et al*, Phys. Rev. Lett. **116** 061102 (2016)

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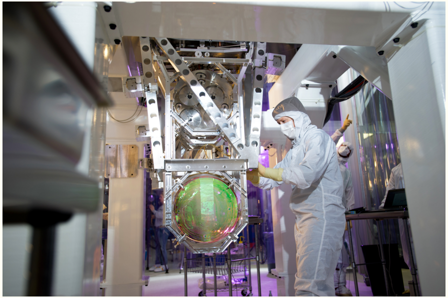
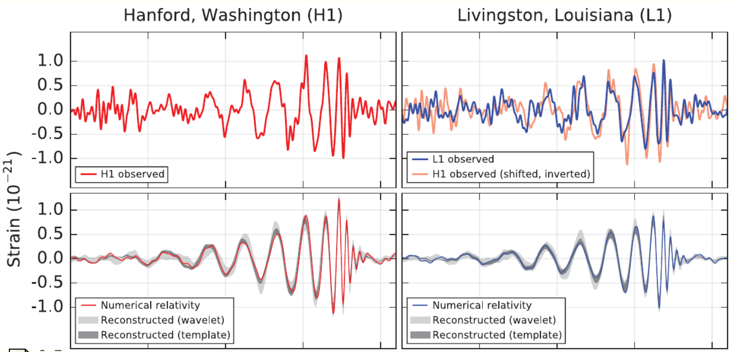
Quantum optomechanics





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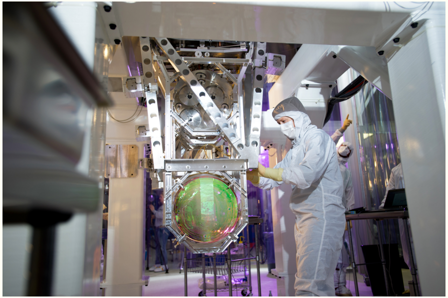

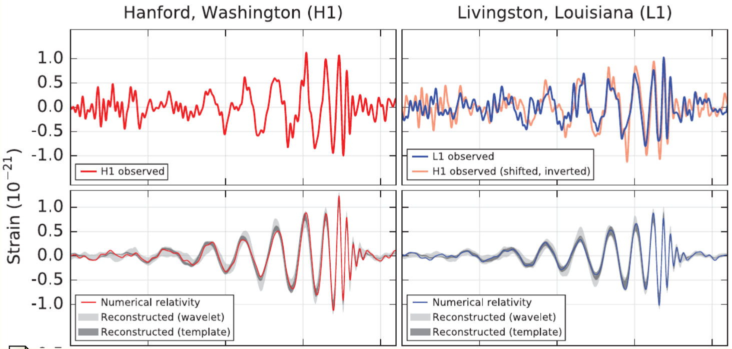

B.Abbott *et al*, Phys. Rev. Lett. **116** 061102 (2016)

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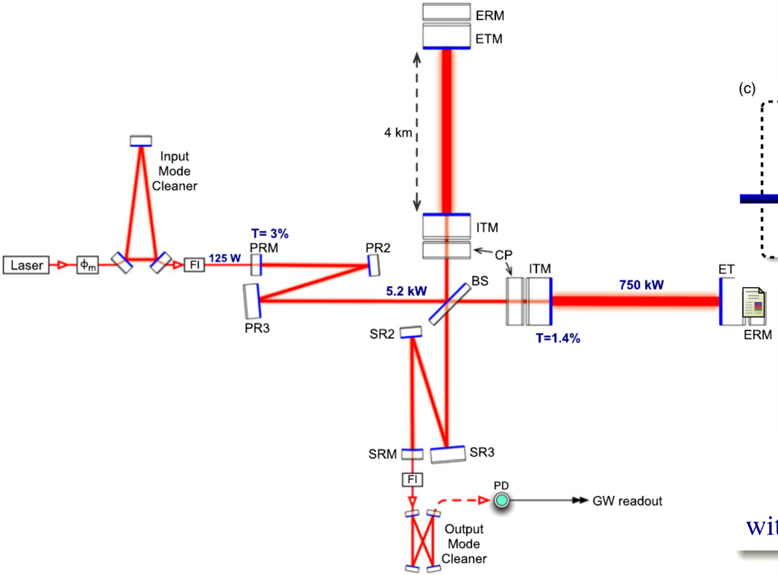
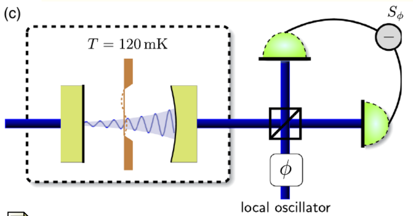
$$\Delta x_{\mathrm{SQL}} = \sqrt{\frac{\hbar T}{m}} = \sqrt{\frac{\hbar \times 10\,\mathrm{ms}}{40\,\mathrm{kg}}} \sim 10^{-19}\,\mathrm{m}$$





N.S.Kampel *et al*, Phys. Rev. X **7**, 021008 (2017)

Light: the ONLY way
to probe mechanical motion
with the quantum-grade precision

J.Aasi *et al*, Class. Quantum Grav. **32**, 074001 (2015)

Characteristic inequalities

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Quantization of the running wave

$$\hat{\vec{E}}(\vec{r},t) = \sum_{\sigma} \int d^3\vec{k} \sqrt{\frac{\hbar\omega_k}{2(2\pi)^3\epsilon_o}}\, \vec{e}(\vec{k},\sigma)\hat{a}(\vec{k},\sigma)e^{i(\vec{k}\vec{r}-\omega_k t)} + \text{h.c.}, \qquad \left[\hat{a}(\vec{k},\sigma), \hat{a}^{\dagger}(\vec{k}',\sigma')\right] = \delta_{\sigma\sigma'}\delta^{(3)}(\vec{k}-\vec{k}')$$

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An equivalent form:

$$\hat{A}(t) = \left[\int_{-\omega_p}^{\infty} \sqrt{\frac{\omega_p+\Omega}{2\omega_p}}\,\hat{a}(\omega_p+\Omega)e^{-i(\omega+\Omega)t}\,\frac{d\Omega}{2\pi} + \int_{-\infty}^{\omega_p} \sqrt{\frac{\omega_p-\Omega}{2\omega_p}}\,\hat{a}^\dagger(\omega_p-\Omega)e^{i(\omega_p-\Omega)t}\,\frac{d\Omega}{2\pi}\right]$$

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$$|\Omega| \ll \omega_p \;\Rightarrow\; \hat{\mathrm{A}}(t) \approx \frac{1}{\sqrt{2}}\int_{-\infty}^\infty \left[\hat{\mathrm{a}}(\omega_p+\Omega)e^{-i\omega_p t} + \hat{\mathrm{a}}^\dagger(\omega_p-\Omega)e^{i\omega_p t}\right]\frac{d\omega}{2\pi}$$

Two-photon quadratures

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$$\hat{\mathrm{a}}_c(t) \to \mathrm{A}_c + \hat{\mathrm{a}}_c(t)\,, \qquad \hat{\mathrm{a}}_s(t) \to \mathrm{A}_s + \hat{\mathrm{a}}_s(t)\,, \qquad \langle\hat{\mathrm{a}}_{c,s}(t)\rangle = 0$$

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Just convention (choice of the time reference): $\mathrm{A}_s = 0 \;\Rightarrow$

$$\boxed{\hat{\mathrm{A}}(t) = [\mathrm{A} + \hat{\mathrm{a}}_{c,s}(t)]\cos\omega_p t + \hat{\mathrm{a}}_s(t)\sin\omega_p t}$$

Strong pump field

$$\hat{A}(t) = [A + \hat{a}_c(t)] \cos \omega_p t + \hat{a}_s(t) \sin \omega_p t$$

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Amplitude/Power noise

$$\hat{I}(t) = \hbar\omega_p \overline{\hat{\mathrm{A}}^2(t)} = \frac{\hbar\omega_p}{2} \left\{ [\mathrm{A} + \hat{\mathrm{a}}_c(t)]^2 + \hat{\mathrm{a}}_s^2(t) \right\} = I_0 + \delta\hat{I}(t)$$

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Amplitude/Power noise

$$\hat{I}(t) = \hbar\omega_p \overline{\hat{A}^2(t)} = \frac{\hbar\omega_p}{2} \left\{ [A + \hat{a}_c(t)]^2 + \hat{a}_s^2(t) \right\} = I_0 + \delta\hat{I}(t)$$

The mean power: $I_0 = \dfrac{\hbar\omega_p A^2}{2}$

The noise: $\delta\hat{I}(t) = \dfrac{\hbar\omega_p}{2} \left[2A\hat{a}_c(t) + \hat{a}_c^2(t) + \hat{a}_s^2(t) \right] = \hbar\omega_p A \hat{a}_c(t) + O(\hat{a}_{c,s}^2)$

Strong pump field

$$\hat{A}(t) = [A + \hat{a}_c(t)] \cos \omega_p t + \hat{a}_s(t) \sin \omega_p t$$

$$\frac{\hat{a}^2_{c,s}}{A^2} \sim \frac{1}{N} \ll 1$$

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Phase noise

$$\hat{A}(t) = [A + \hat{a}_c(t)] \cos \omega_p t + \hat{a}_s(t) \sin \omega_p t$$

$$= [A + \hat{a}_c(t)] \cos[\omega_p t + \hat{\phi}(t)] + \mathcal{O}(\hat{a}^2_{c,s})$$

$$\hat{\phi}(t) = -\frac{\hat{a}_s(t)}{A}$$

Spectral densities

$\hat{a}_{c,s}$ are random functions $\Rightarrow$
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Noise properties

$$[\hat{X}, \hat{Y}] = iC \;\Rightarrow\; \Delta^2_{XX}\Delta^2_{YY} - \Delta^2_{XY} \geqslant \frac{C^2}{4}$$

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Gaussian minimum-uncertainty states:

$$S_{cc} = \frac{1}{2}(\cosh 2r + \sinh 2r \cos 2\theta)\,, \quad S_{ss} = \frac{1}{2}(\cosh 2r - \sinh 2r \cos 2\theta)\,, \quad S_{cs} = \frac{1}{2}\sinh 2r \sin 2\theta$$

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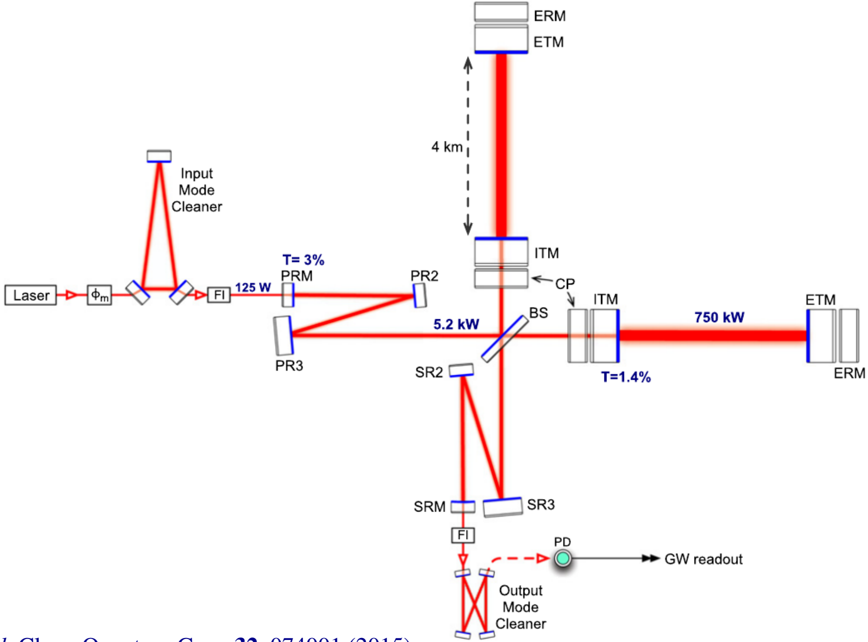
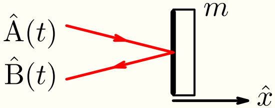
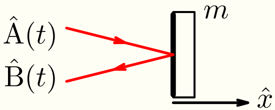


J.Aasi *et al*, Class. Quantum Grav. **32**, 074001 (2015)


$\hat{A}(t)$
$\hat{B}(t)$
m
$\hat{x}$



Input/output relations

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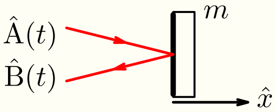

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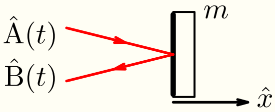

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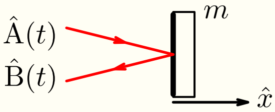

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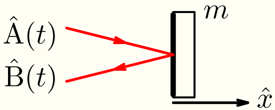

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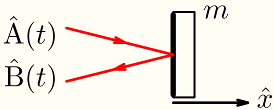
$$B = A\,, \quad \hat{b}_c(t) = \hat{a}_c(t)\,, \quad \hat{b}_s(t) = \hat{a}_s(t) + 2k_p A\hat{x}(t)$$

Measurement a.k.a. Imprecision a.k.a. Shot noise

Homodyne detector: $i \propto \overline{\hat{B}(t)\cos(\omega_p t - \zeta)} \propto [B + \hat{b}_c(t)]\cos\zeta + \hat{b}_s(t)\sin\zeta$



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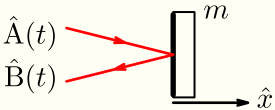
$$\text{Omitting the DC part:} \quad = \hat{\mathrm{a}}_c(t)\cos\zeta + [\hat{\mathrm{a}}_s(t) + 2k_p\mathrm{A}\hat{x}(t)]\sin\zeta = 2k_p\mathrm{A}\sin\zeta \times [\hat{x}_{\mathrm{fl}}(t) + \hat{x}(t)]$$



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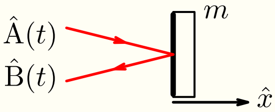

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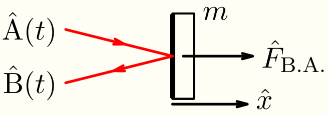
$$\text{Vacuum input state:} \quad S_x = \frac{\hbar c^2}{16\omega_p I_0 \sin^2\zeta}$$



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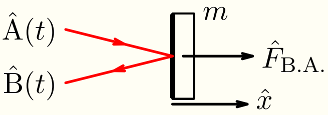

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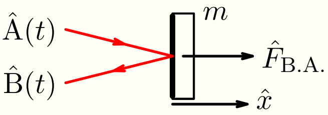
Back action a.k.a. Radiation-pressure noise

Radiation pressure force: $\hat{F}_{\text{R.P.}}(t) = \dfrac{2\hat{I}(t)}{c}$



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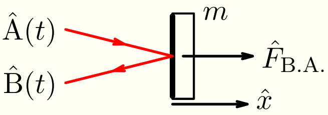
The AC part: $\hat{F}_{fl}(t) = \frac{2\delta\hat{I}(t)}{c} = 2\hbar k_p A\hat{a}_c(t)$



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$$\hat{x}_{\text{fl}}(t) = \frac{1}{2k_p\text{A}}[\hat{a}_c(t)\cot\zeta + \hat{a}_s(t)] \qquad S_x = \frac{\hbar c^2}{16\omega_p I_0 \sin^2\zeta}$$

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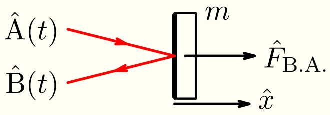
The spectral density: $S_F = \dfrac{4\hbar\omega_p I_0}{c^2}$



Measurement noise

$$\hat{x}_{fl}(t) = \frac{1}{2k_p A}[\hat{a}_c(t)\cot\zeta + \hat{a}_s(t)] \qquad S_x = \frac{\hbar c^2}{16\omega_p I_0 \sin^2\zeta}$$

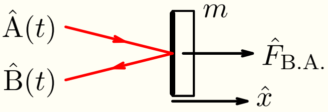
Back action a.k.a. Radiation-pressure noise

Radiation pressure force: $\hat{F}_{R.P.}(t) = \dfrac{2\hat{I}(t)}{c}$

The AC part: $\hat{F}_{fl}(t) = \dfrac{2\delta\hat{I}(t)}{c} = 2\hbar k_p A\hat{a}_c(t)$

The spectral density: $S_F = \dfrac{4\hbar\omega_p I_0}{c^2}$

The cross-correlation: $S_{xF} = \dfrac{\hbar}{2}\cot\zeta$



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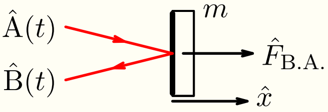
Uncertainty relation

$$S_x S_F - S_{xF}^2 = \frac{\hbar^2}{4}$$

(universal!)



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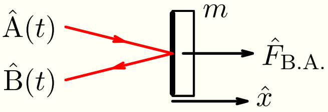
Sum quantum noise

Mechanical equation of motion:

$$\hat{x}(\Omega) = \chi(\Omega)\hat{F}_{\text{fl}} + x_{\text{signal}}(\Omega)$$

E.g. free mass: $\chi(\Omega) = -\frac{1}{m\Omega^2}$



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$$x_{\text{fl}}(\Omega) + \hat{x}(\Omega) = \hat{x}_{\text{sum}}(\Omega) + x_{\text{signal}}(\Omega)$$

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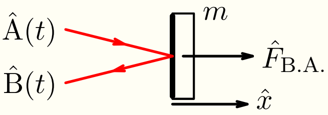

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The output signal:

$$x_{\rm fl}(\Omega) + \hat{x}(\Omega) = \hat{x}_{\rm sum}(\Omega) + x_{\rm signal}(\Omega)$$

$$\hat{x}_{\rm sum}(\Omega) = x_{\rm fl}(\Omega) + \chi(\Omega)\hat{F}_{\rm fl}(\Omega)$$

Sum quantum noise spectral density:

$$S_{\rm sum}(\Omega) = S_x + 2\chi(\Omega)S_{xF} + \chi^2(\Omega)S_F$$

SQL

$$\left(\text{Just an example:}\quad S_x = \frac{\hbar c^2}{16\omega_p I_0 \sin^2\zeta}, \quad S_F = \frac{4\hbar\omega_p I_0}{c^2}, \quad S_{xF} = \frac{\hbar}{2}\cot\zeta\right)$$

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$$S_{\rm sum}(\Omega) = \frac{\hbar^2/4 + S_{xF}^2}{S_F} + 2\chi(\Omega)S_{xF} + \chi^2(\Omega)S_F$$

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Naive approach: minimization of the imprecision noise S_x :

$S_{xF} = 0 \;\Leftrightarrow\; \zeta = \pi/2$ (the phase quadrature measurement) $\Rightarrow$

$$S_{\rm sum}(\Omega) = \left(S_x = \frac{\hbar^2}{4S_F}\right) + \chi^2(\Omega)S_F$$

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$$S_{xF} = 0 \iff \zeta = \pi/2 \quad \text{(the phase quadrature measurement)} \implies$$

$$S_{\rm sum}(\Omega) = \left(S_x = \frac{\hbar^2}{4S_F}\right) + \chi^2(\Omega)S_F \geqslant S_{\rm SQL}(\Omega) = \hbar|\chi(\Omega)|$$

SQL

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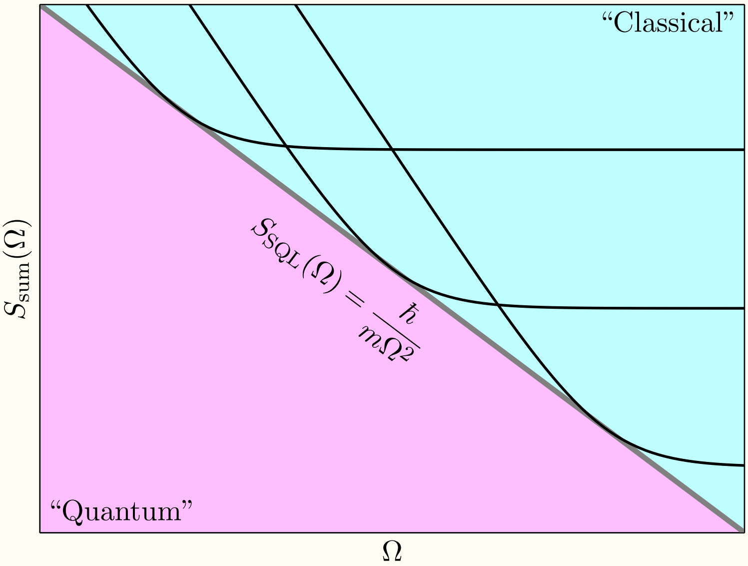
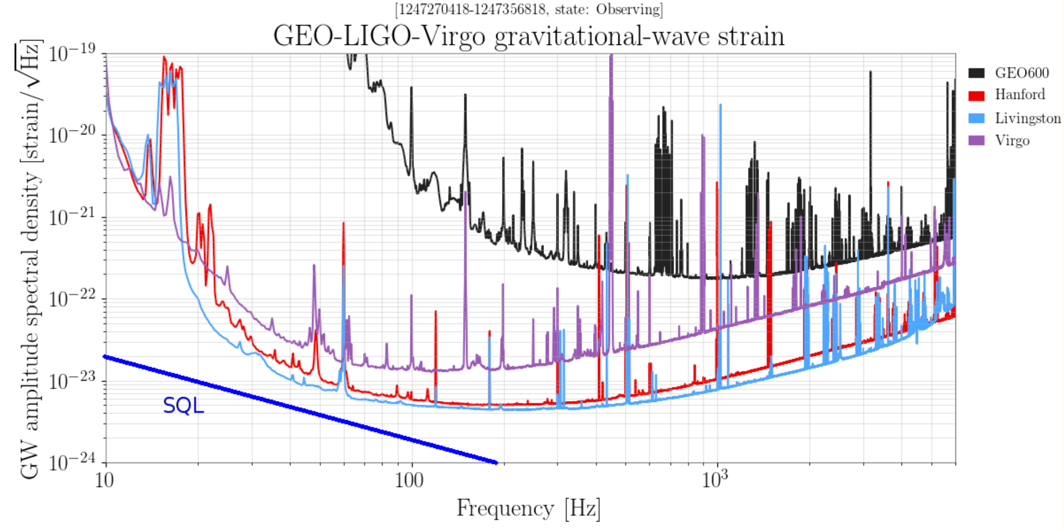
$$S_{\text{sum}}(\Omega) = S_x + \frac{S_F}{m^2\Omega^4} \geqslant \frac{\hbar}{m\Omega^2}$$

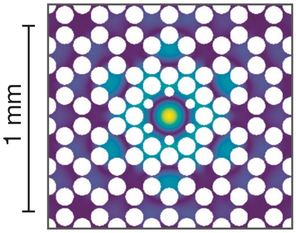


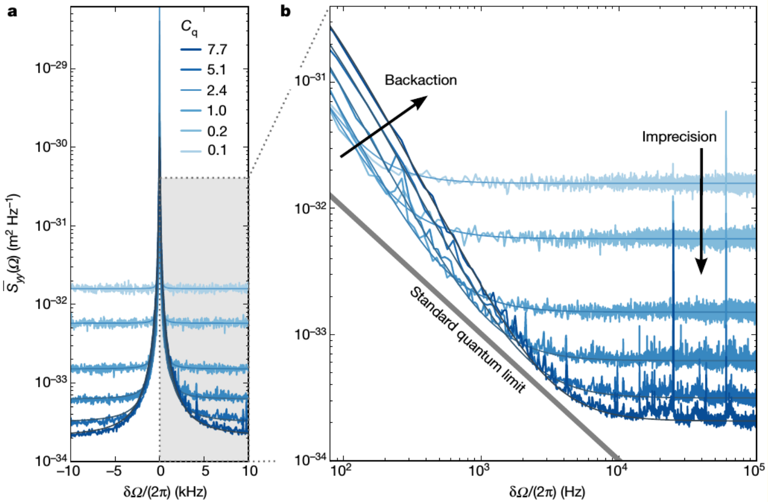
How far away is it from the reality?



https://www.gw-openscience.org/detector_status/day/20190716

How far away this from the reality?





M.Rossi *et al*, Nature **563**, 53 (2018)

Real optimization

$$S_x S_F - S_{xF}^2 = \frac{\hbar^2}{4} \quad (*) \qquad S_{\text{sum}}(\Omega) = S_x + 2\chi(\Omega) S_{xF} + \chi^2(\Omega) S_F \quad (**)$$

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$S_F \to \infty$ (that is $I_0 \to \infty$) $\Rightarrow$ $S_{\rm sum}(\Omega) \to 0$ No SQL!

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The problem:

$\chi(\Omega)$ depends on Ω. But S_x, S_{xF} does not.

Our simple example: $S_F = \frac{4\hbar\omega_p I_0}{c^2}\,,\; S_{xF} = \frac{\hbar}{2}\cot\zeta\,,\; \chi(\Omega) = -\frac{1}{m\Omega^2} \;\Rightarrow\; \frac{8\omega_p I_0}{mc^2}\tan\zeta = \Omega^2$

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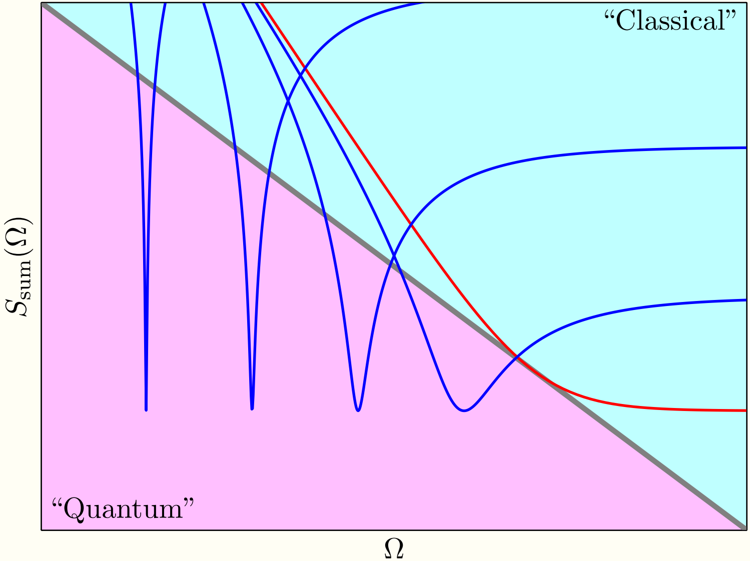

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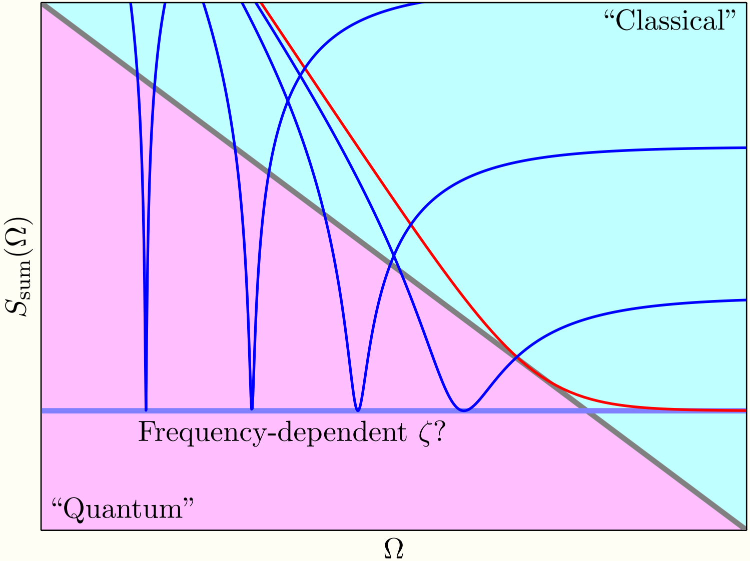

Broad-band beating the SQL

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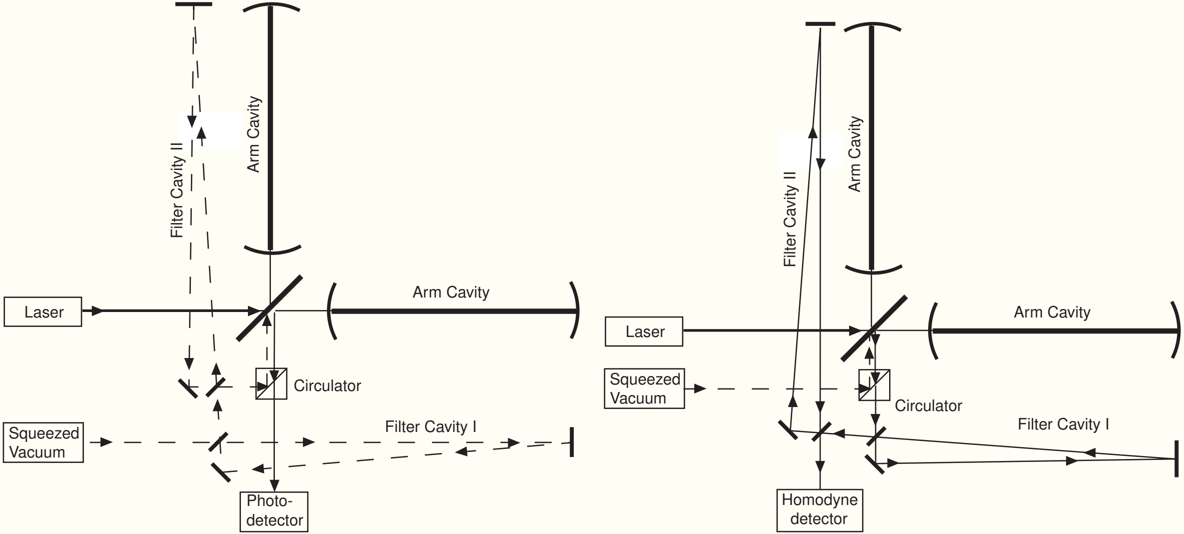
The problem:

$\chi(\Omega)$ depends on Ω. But S_x, S_{xF} do not. Or could they?

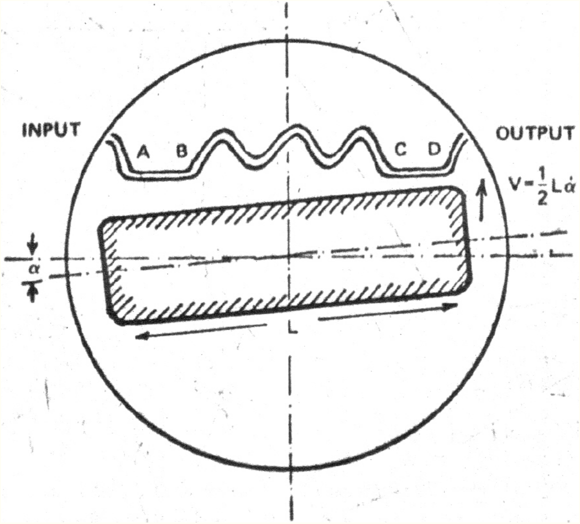
$$\frac{S_{xF}}{S_F} = -\chi(\Omega) \begin{cases} S_F \propto 1/\chi(\Omega) & \Rightarrow \text{Quantum speedmeter} \\ S_{xF} \propto \chi(\Omega) & \Rightarrow \text{Filter cavities} \end{cases}$$

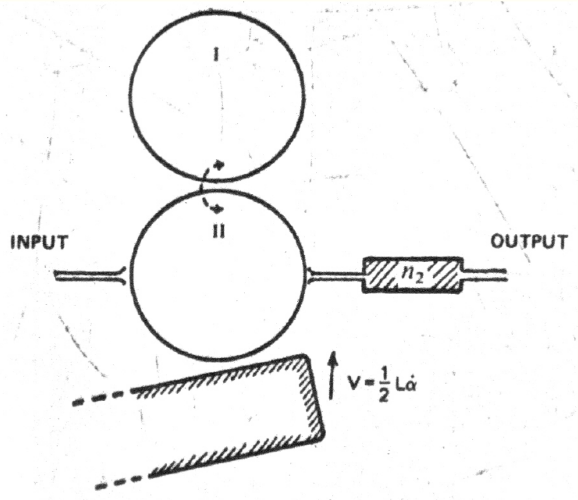
Filter cavities



H.J.Kimble *et al*, Phys.Rev.D **65**, 022002 (2001)

Quantum speedemter





V.B.Braginsky, F.Ya.Khalili, Phys. Lett. A **147**, 251 (1990)