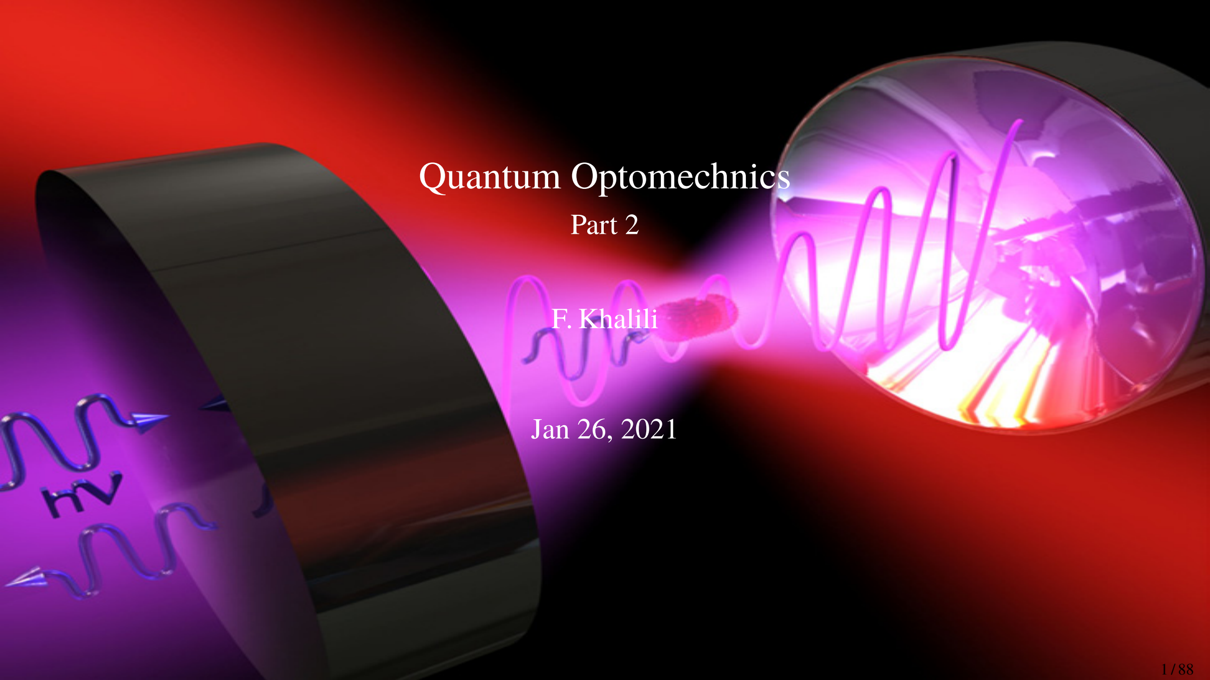


Quantum Optomechnics

Part 2

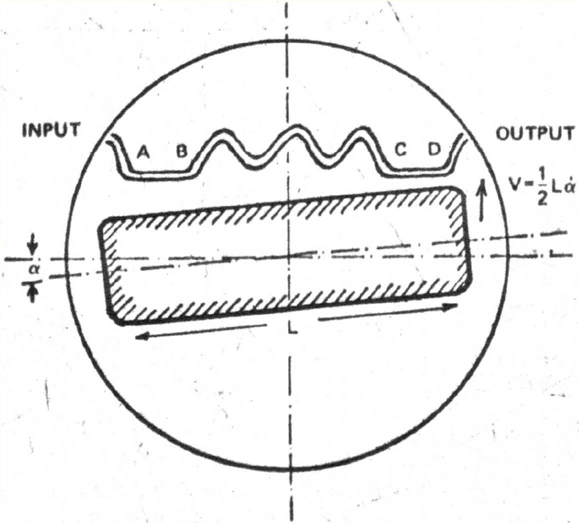
F. Khalili

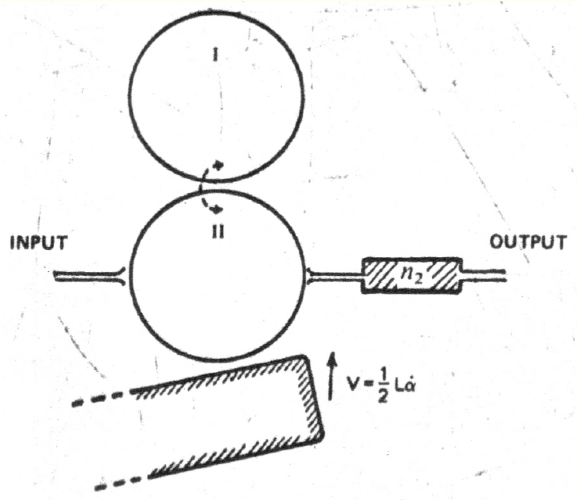
Jan 26, 2021

1. Quantum speedmeter
2. Optical cavity: Hamilitonian approach
3. Optical spring
4. Ground state optical cooling
5. Quantumnes of mechanical objects

Will not be considered:

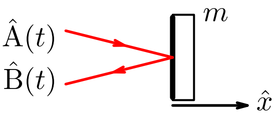
- Filter cavities
- Hybrid systems
- Non-stationary optomechanics
- Non-Gaussian optomechanics
- ...





V.B.Braginsky, F.Ya.Khalili, Phys. Lett. A **147**, 251 (1990)

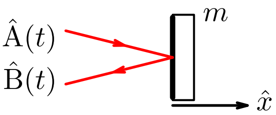
Position meter



$$\mathrm{B} = \mathrm{A}\,, \quad \hat{\mathrm{b}}_c(t) = \hat{\mathrm{a}}_c(t)\,, \quad \hat{\mathrm{b}}_s(t) = \hat{\mathrm{a}}_s(t) + 2k_p\mathrm{A}\hat{x}(t)$$

SQL!

Position meter



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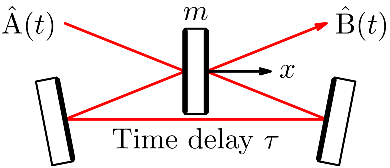
SQL!

Speed meter



$$\mathrm{B} = \mathrm{A}\,, \quad \hat{b}_c(t) = \hat{a}_c(t)$$

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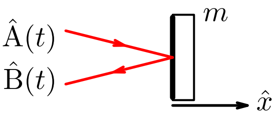
Position meter



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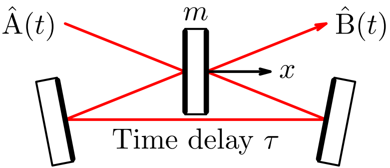

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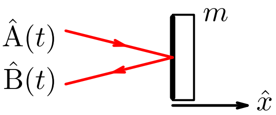
Position meter



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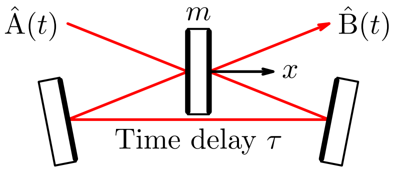
SQL!

Speed meter



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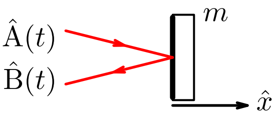
$$\hat{v}(t) = \frac{d\hat{x}(t)}{dt} \Leftrightarrow \hat{v}(\Omega) = -i\Omega\hat{x}(\Omega)$$

Homodyne detector: $i \propto \hat{\mathrm{b}}_c(t)\cos\zeta + \hat{\mathrm{b}}_s(t)\sin\zeta = \hat{\mathrm{a}}_c(t)\cos\zeta + [\hat{\mathrm{a}}_s(t) - 2k_p\mathrm{A}\tau\hat{v}(t)]\sin\zeta$

$$= -2k_p\mathrm{A}\tau\sin\zeta[\hat{v}_{\mathrm{fl}}(t) + \hat{v}(t)]$$

Position meter



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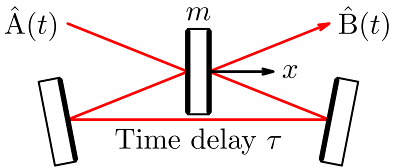
Speed meter



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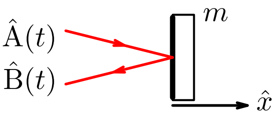
The measurement noise: $\hat{v}_{\mathrm{fl}}(t) = \dfrac{d\hat{x}_{\mathrm{fl}}(t)}{dt} = -\dfrac{1}{2k_p\mathrm{A}\tau}[\hat{a}_c(t)\cot\zeta + \hat{a}_s(t)]$

Position meter



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SQL!

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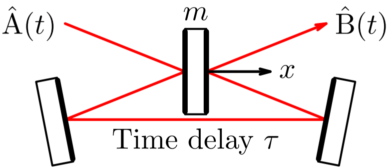

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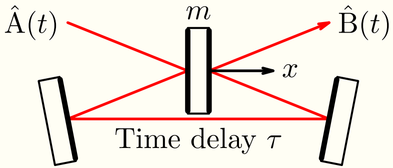
$= -2k_p A\tau\sin\zeta[\hat{v}_{fl}(t) + \hat{v}(t)]$

The measurement noise: $\hat{v}_{fl}(t) = \frac{d\hat{x}_{fl}(t)}{dt} = -\frac{1}{2k_p A\tau}[\hat{a}_c(t)\cot\zeta + \hat{a}_s(t)]$

The back action force: $\hat{F}_{fl}(t) = 2\hbar k_p A[\hat{a}_c(t+\tau) - \hat{a}_c(t)] \approx 2\hbar k_p A\tau\frac{d\hat{a}_c(t)}{dt}$

Speed meter



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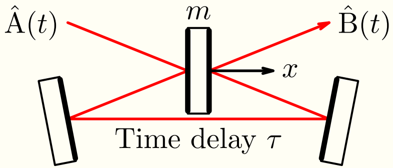
Speed meter



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Fourier picture :

$$\hat{x}_{\text{fl}}(\Omega) = \frac{\hat{v}_{\text{fl}}(\Omega)}{-i\Omega}, \quad \hat{v}_{\text{fl}}(\Omega) = -\frac{1}{2k_p A\tau}[\hat{a}_c(\Omega)\cot\zeta + \hat{a}_s(\Omega)]$$

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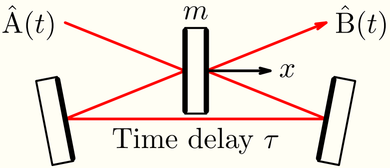
Speed meter



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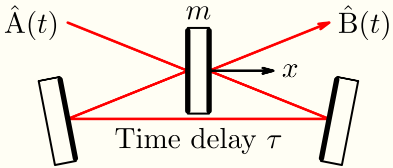
Quantum noise spectral densities :

$$S_x(\Omega) = \frac{S_v}{\Omega^2} = \frac{\hbar c^2}{16\omega_p I_0 \Omega^2\tau^2 \sin^2\zeta}, \quad S_F(\Omega) = \Omega^2 S_p = \frac{4\hbar\omega_p I_0 \Omega^2\tau^2}{c^2}, \quad S_{xF} = -S_{vp} = \frac{\hbar}{2}\cot\phi$$



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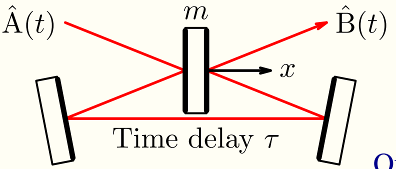
$$\boxed{S_x(\Omega)S_F(\Omega) - S_{xF}^2 = S_vS_p - S_{vp}^2 = \frac{\hbar^2}{4}}$$

Speed meter



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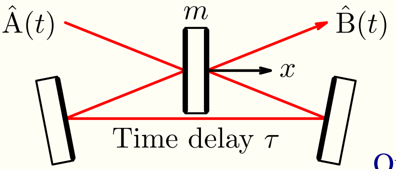

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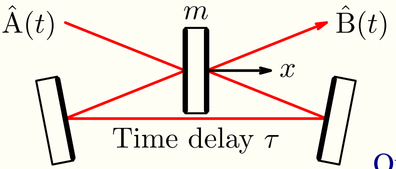
Sum quantum noise optimization : $\left[\hat{x}_{\text{sum}}(\Omega) = \hat{x}_{\text{fl}}(\Omega) + \chi(\Omega)\hat{F}_{\text{fl}}(\Omega)\right]$

$$S_{\text{sum}}(\Omega) = S_x(\Omega) + \frac{2S_{xF}}{-m\Omega^2} + \frac{S_F(\Omega)}{m^2\Omega^4} = \frac{1}{\Omega^2}\underbrace{\left(S_v + \frac{2S_{vp}}{m} + \frac{S_p}{m^2}\right)}_{\text{DC!}}$$



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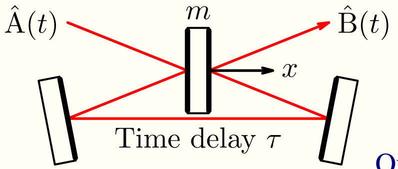
Speed meter



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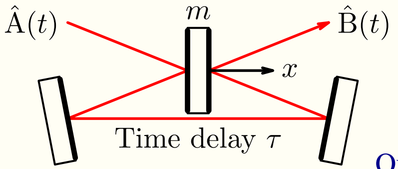
$$S_{vp} = -\frac{S_p}{m} \quad\Rightarrow\quad S_{\rm sum}(\Omega) = \frac{\hbar^2}{4\Omega^2 S_p}$$

Speed meter



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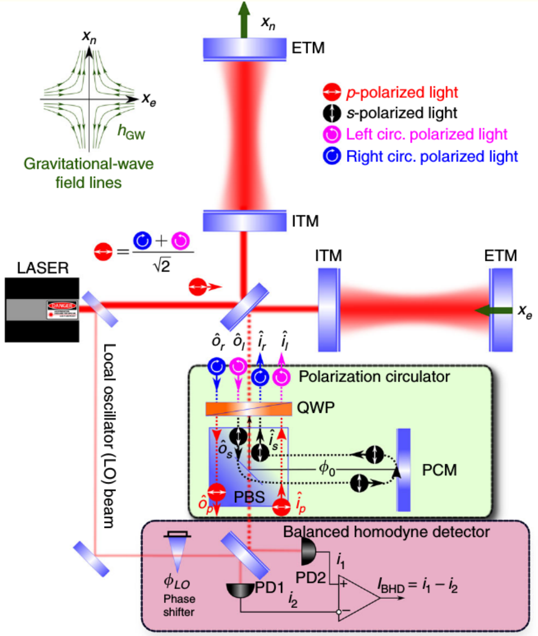
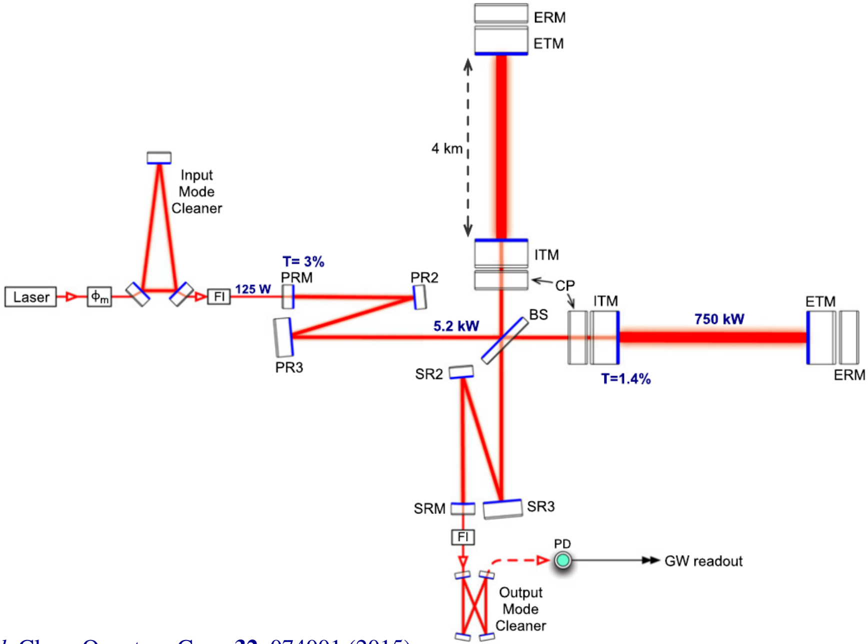
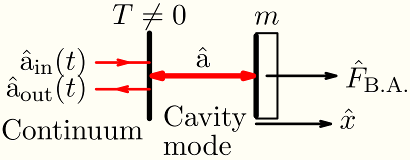
$$S_{vp} = -\frac{S_p}{m} \Rightarrow S_{sum}(\Omega) = \frac{\hbar^2}{4\Omega^2 S_p} \to 0 \quad \text{if} \quad S_p \propto I_0 \to \infty$$



S.L.Danilishin *et al*, Light: Science & Applications **7**, 11 (2018)



J.Aasi *et al*, Class. Quantum Grav. **32**, 074001 (2015)



$$[\hat{a}, \hat{a}^\dagger] = 1$$

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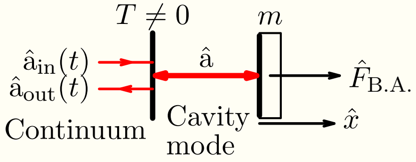

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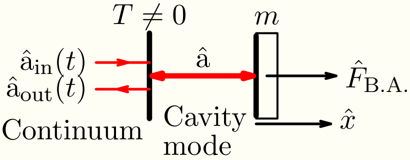
Closed cavity, $T = 0$ (for a while)

$$\hat{\mathcal{H}} = \hbar[(\omega_o - G\hat{x})\hat{a}^\dagger\hat{a} + A^* e^{i\omega_p t}\hat{a} + A e^{i\omega_p t}\hat{a}^\dagger] + \hat{\mathcal{H}}_m$$

e.g. $G = \omega_o / L$



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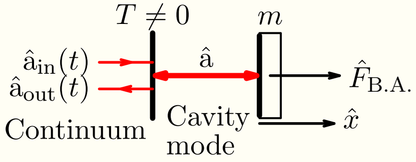

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Linearization:

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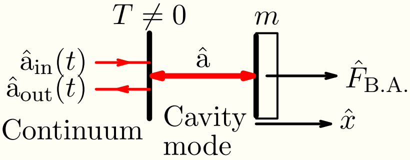

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$$\frac{d\hat{a}(t)}{dt} - i\delta\hat{a}(t) = ig\hat{x}(t),$$

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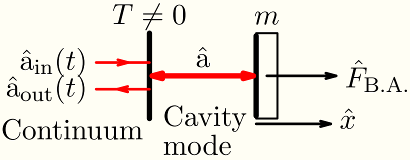

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Adding the continuum, by hand

$$\frac{d\hat{a}(t)}{dt} + (\gamma - i\delta)\hat{a}(t) = ig\hat{x}(t) + \sqrt{2\gamma}\,\hat{a}_{in}(t)$$

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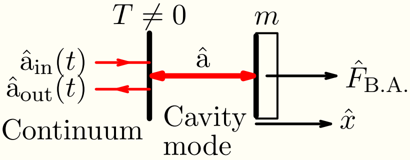

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$$\hat{F}_{\text{B.A.}}(t) = \hbar g[\hat{a}(t) + \hat{a}^\dagger(t)]$$

Fourier picture

$$\begin{pmatrix} \hat{a}^c_{\text{out}}(\Omega) \\ \hat{a}^s_{\text{out}}(\Omega) \end{pmatrix} = \mathbb{R}(\Omega) \begin{pmatrix} \hat{a}^s_{\text{in}}(\Omega) \\ \hat{a}^c_{\text{in}}(\Omega) \end{pmatrix} + \mathbf{G}(\Omega)\hat{x}(\Omega)$$

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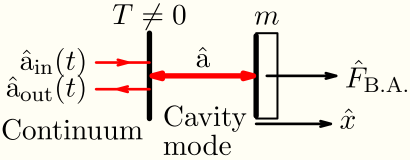

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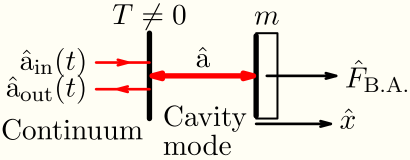
$\mathbb{R}(\Omega)$: a sophisticated 2×2 matrix

$\mathbf{G}(\Omega), \mathbf{F}(\Omega)$: sophisticated 2-comp. vectors



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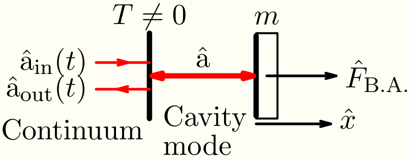

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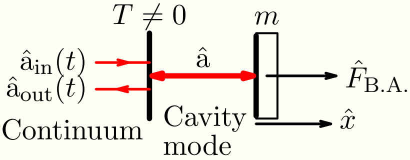
$$\delta = 0\,,\ |\Omega| \ll \gamma$$



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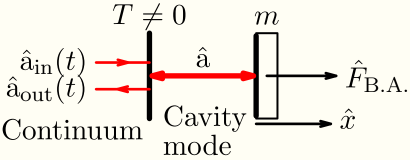

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Single mirror

$$\hat{b}_c = \hat{a}_c\,, \quad \hat{b}_s = \hat{a}_s + 2k_p A\hat{x}\,, \quad \hat{F}_{fl} = 2\hbar k_p A\hat{a}_c$$

Fourier picture

$$\begin{pmatrix}\hat{a}^c_{out}(\Omega)\\ \hat{a}^s_{out}(\Omega)\end{pmatrix} = \mathbb{R}(\Omega)\begin{pmatrix}\hat{a}^s_{in}(\Omega)\\ \hat{a}^c_{in}(\Omega)\end{pmatrix} + \mathbf{G}(\Omega)\hat{x}(\Omega)$$

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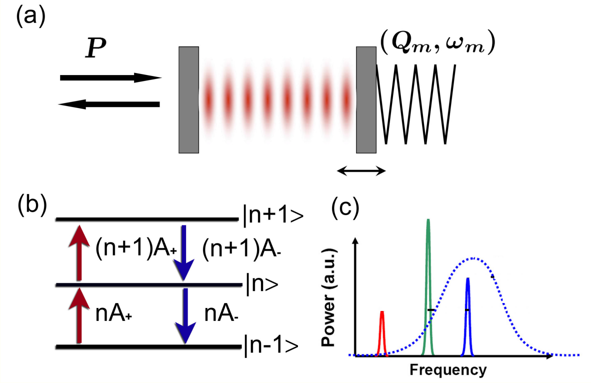


I.Wilson-Rae *et al*, Phys.Rev.Lett **99**, 093901 (2007)

Вестник
МОСКОВСКОГО УНИВЕРСИТЕТА
№ 1 — 1964

В. Б. БРАГИНСКИЙ, И. И. МИНАКОВА

ВЛИЯНИЕ СИСТЕМЫ ИЗМЕРЕНИЯ МАЛЫХ СМЕЩЕНИЙ НА ДИНАМИЧЕСКИЕ СВОЙСТВА МЕХАНИЧЕСКИХ КОЛЕБАТЕЛЬНЫХ СИСТЕМ

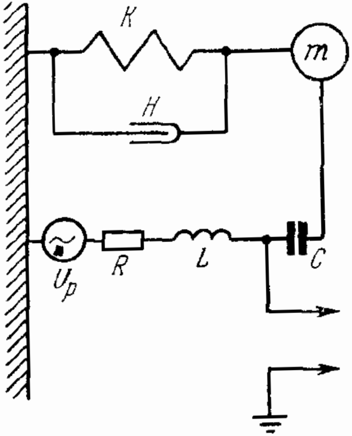


Рис. 1. Принципиальная схема преобразования механических колебаний в электрические.

Левый склон

$U \sim (в)$	1	3	5	7
T (сек)	11,0	11,4	12,2	13,0
θ	$+2{,}4\cdot10^{-3}$	$+4{,}2\cdot10^{-3}$	$+1{,}0\cdot10^{-3}$	$+2{,}4\cdot10^{-3}$

Правый склон

$U \sim (в)$	1	3	5	7
T (сек)	11,0	10,9	10,8	10,6
θ	$+10^{-4}$	$-1{,}4\cdot10^{-3}$	$-3{,}3\cdot10^{-3}$	$-2{,}1\cdot10^{-3}$

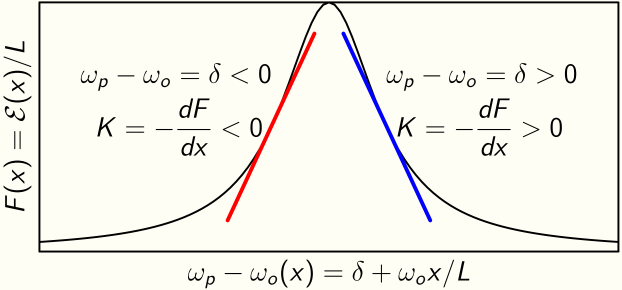
вносится в механическую колебательную систему с запаздыванием, и уравнение для малых колебаний крутильного маятника с учетом вносимой жесткости будет выглядеть так:

$$m_{экв}\ddot{y} + h\dot{y} + (K_{экв} \pm \Delta K_{\tau})\,y = 0.$$

Знак плюс соответствует правому склону, знак минус — левому склону резонансной кривой электрического контура. Нетрудно видеть, что запаздывание в положительной жесткости приводит к регенерации, запаздывание в отрицательной жесткости — к дегенерации. Условие самовозбуждения для правого склона при учете конечного τ имеет очень простой вид

$$h = \Delta K\cdot\tau.$$

Radiation pressure force in a detuned cavity



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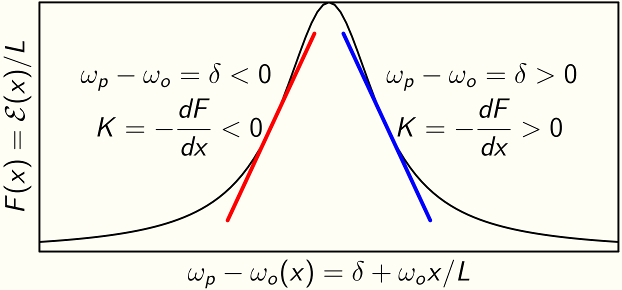
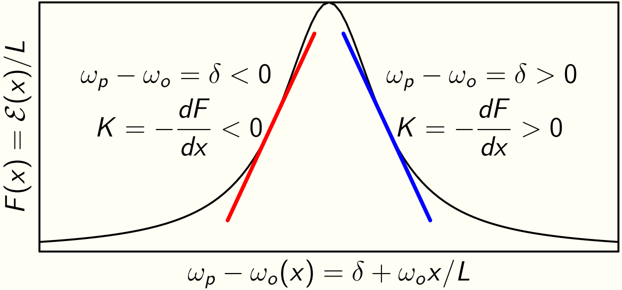


$$\mathcal{E}(x) = \frac{\gamma^2}{\gamma^2 + (\delta + \omega_o x/L)^2} \times \mathcal{E}_0$$

Radiation pressure force in a detuned cavity



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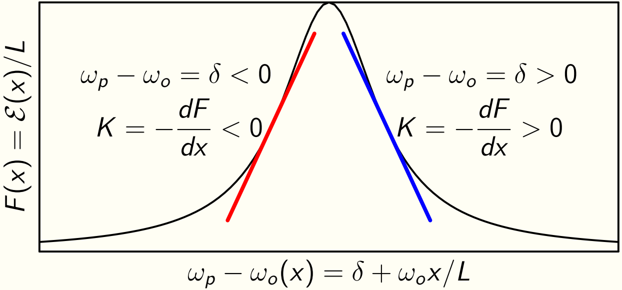

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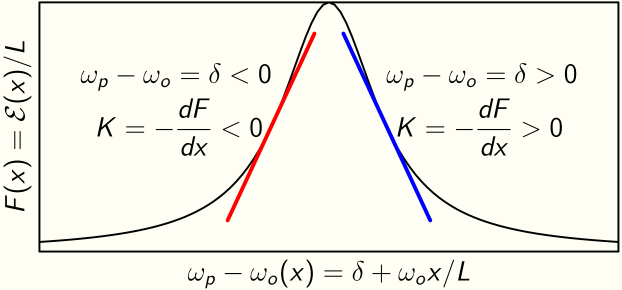

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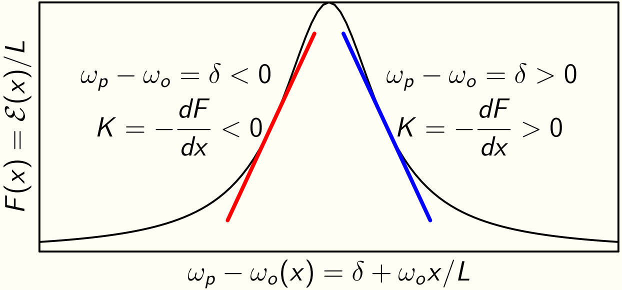

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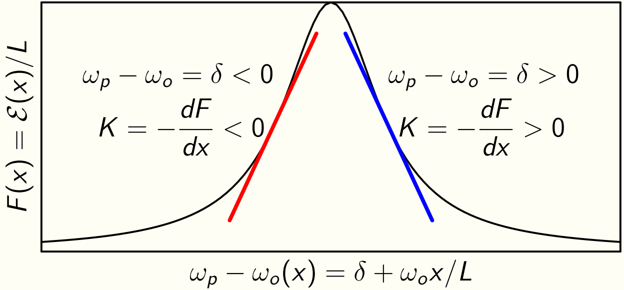

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Exact equations:

$$K(\Omega) = \frac{2\omega_o\mathcal{E}\delta/L^2}{(\gamma - i\Omega)^2 + \delta^2} = K_0(\Omega) - i\Omega H(\Omega)$$

$$K_0(\Omega) = \frac{2\omega_o\mathcal{E}\delta/L^2}{|(\gamma - i\Omega)^2 + \delta^2|^2}\left[\gamma^2 + \delta^2 - \Omega^2\right]$$

$$H(\Omega) = -\frac{4\omega_o\mathcal{E}\gamma\delta/L^2}{|(\gamma - i\Omega)^2 + \delta^2|^2}$$

Modification of the probe dynamics

$$S^{h}_{\mathrm{SQL}}(\Omega) \propto \hbar |\chi^{-1}_{\mathrm{dressed}}(\Omega)| \qquad \chi^{-1}_{\mathrm{dressed}}(\Omega) = \chi^{-1}(\Omega) + K(\Omega)$$

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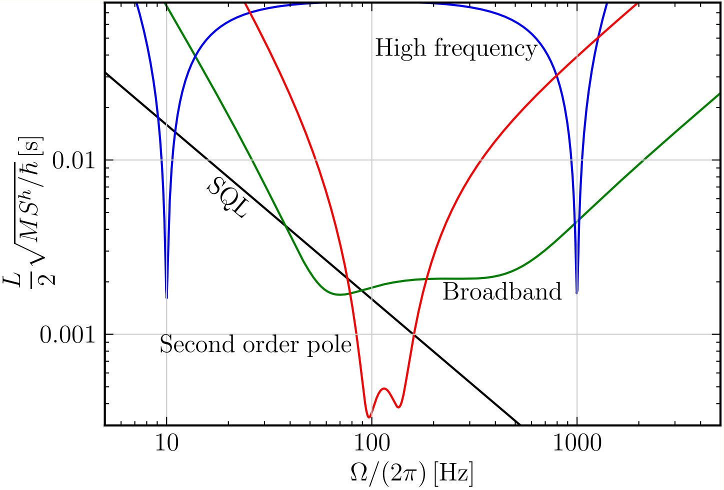
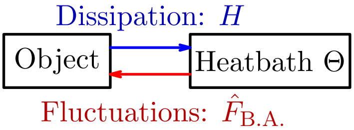


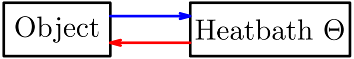
S. Danilishin and F. Khalili, Living Reviews in Relativity **15**, 5 (2012)

Fluctuation-Dissipation Theorem



Fluctuation-Dissipation Theorem

Dissipation: H



Fluctuations: $\hat{F}_{\mathrm{B.A.}}$

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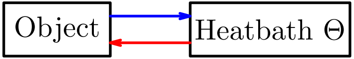

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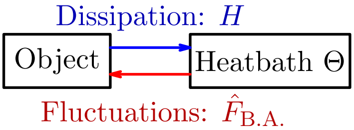

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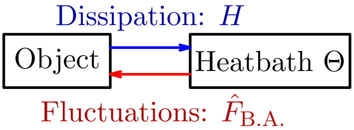

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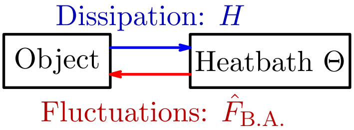

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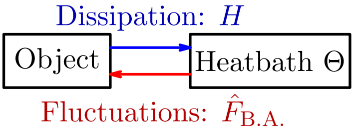

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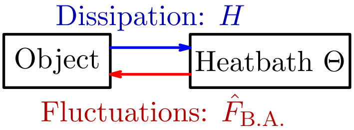

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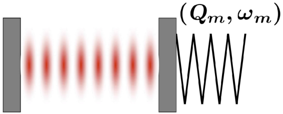

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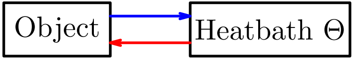

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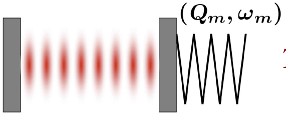

$$T \sim \frac{\hbar\omega_o}{\kappa_B} \sim 10^4 K?$$

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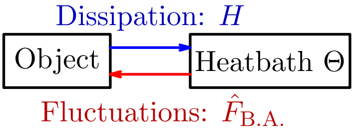

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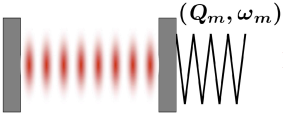

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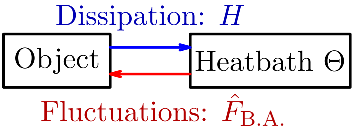

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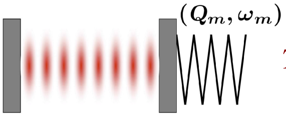

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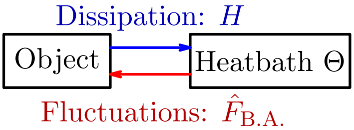

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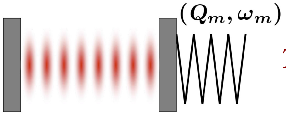

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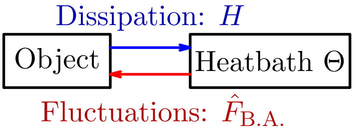

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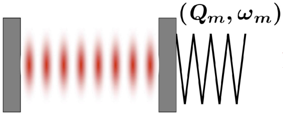

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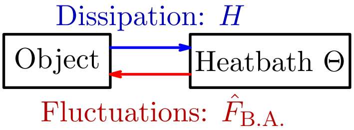

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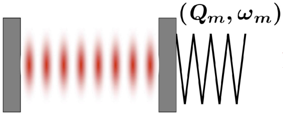

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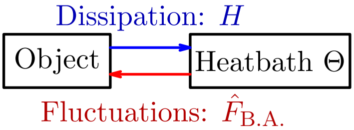
$$|\Omega| \gg \gamma \Rightarrow \Theta_{\mathrm{R.P.}}(\Omega) = \hbar\left(|\Omega| + \frac{\gamma^2}{2|\Omega|}\right)$$

$\approx \hbar|\Omega|$: vacuum noise!

Fluctuation-Dissipation Theorem



$$S_{\rm FDT} = 2\Theta H$$

$$\Theta = \frac{\hbar|\Omega|}{2}\coth\frac{\hbar|\Omega|}{2\kappa_B} = \begin{cases} \hbar|\Omega|/2 & \kappa_B T \ll \hbar|\Omega| \\ \kappa_B T & \kappa_B T \gg \hbar|\Omega| \end{cases}$$

Two heatbaths

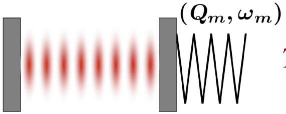
$$2\kappa_B\Theta_1 H_1 + 2\kappa_B\Theta_2 H_2 = 2\kappa_B\Theta_{\rm eff}(H_1 + H_2) \;\Rightarrow$$

$$\Theta_{\rm eff} = \frac{\Theta_1 H_1 + \Theta_2 H_2}{H_1 + H_2}$$

$$H_2 \gg H_1, \quad \Theta_2 \ll \Theta_1 \;\Rightarrow$$

$$\Theta_{\rm eff} \approx \Theta_1\frac{H_1}{H_2} + \Theta_2 \ll \Theta_1$$

Optical spring



$$T \sim \frac{\hbar\omega_o}{\kappa_B} \sim 10^4 K?$$

$$\delta < 0 \;\Rightarrow\; H_{\rm R.P.}(\Omega) = \frac{4\hbar g^2\gamma|\delta|}{|(\gamma - i\Omega)^2 + \delta^2|^2}$$

$$S_F(\Omega) = \frac{2\hbar^2 g^2\gamma(\gamma^2 + \delta^2 + \Omega^2)}{|(\gamma - i\Omega)^2 + \delta^2|^2}$$

$$\Theta_{\rm R.P.}(\Omega) = \frac{S_F(\Omega)}{2H(\Omega)} = \frac{\hbar}{2|\delta|}(\gamma^2 + \delta^2 + \Omega^2)$$

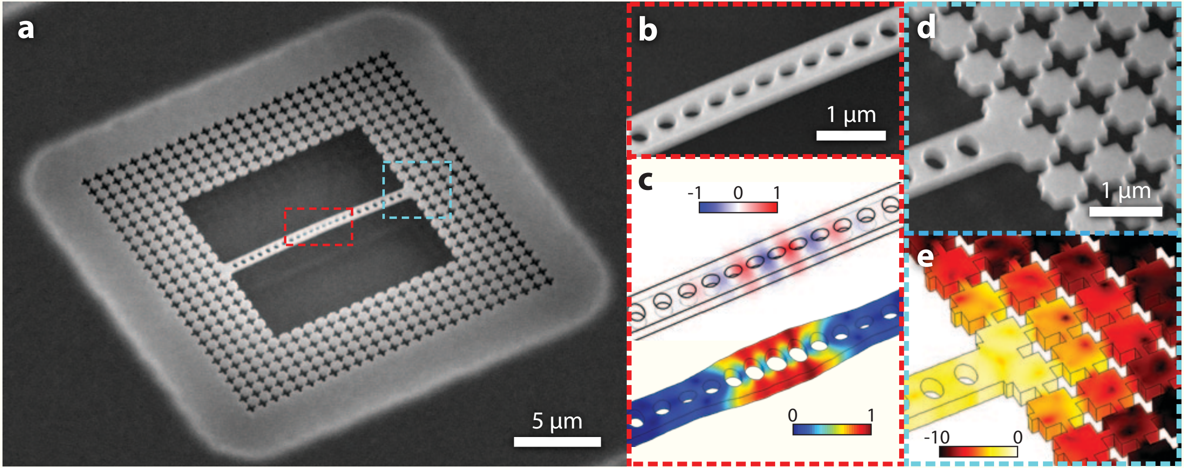
$$|\delta| = \sqrt{\Omega^2 + \gamma^2} \;\Rightarrow\; \Theta_{\rm R.P.}(\Omega) = \hbar\sqrt{\gamma^2 + \Omega^2}$$

$$|\Omega| \gg \gamma \;\Rightarrow\; \Theta_{\rm R.P.}(\Omega) = \hbar\left(|\Omega| + \frac{\gamma^2}{2|\Omega|}\right)$$

$\approx \hbar|\Omega|$: vacuum noise!

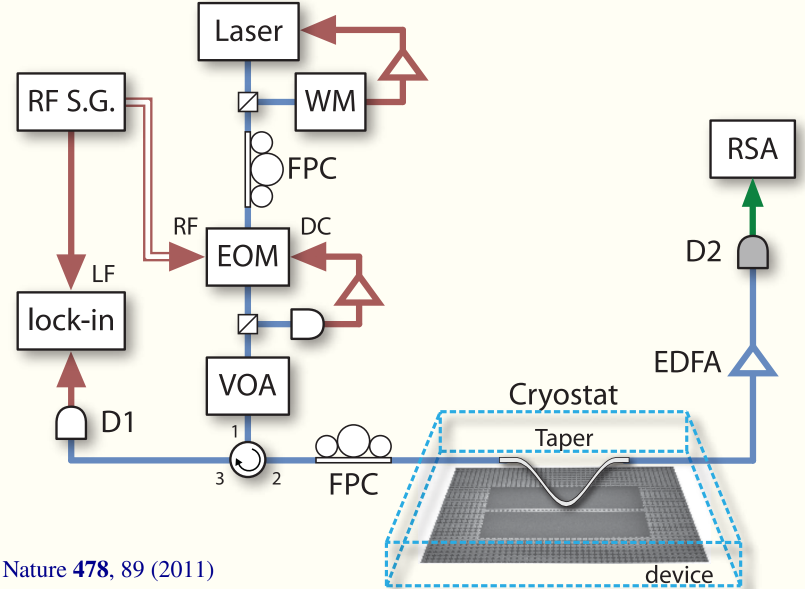
Maximum of $H_{\rm R.P.}$: $\Omega \approx \delta \approx \Omega_m$

Photonic ⊗ phononic crystal:



J. Chan *et al*, Nature **478**, 89 (2011)

Experiment



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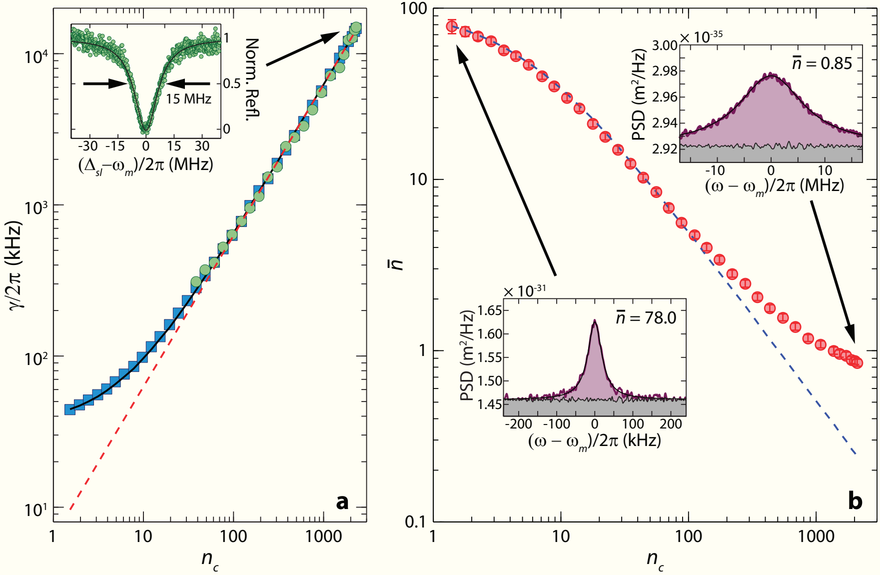

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Detection of a classical force

DE-quantization of the signal

The Hamiltonian: $\hat{\mathcal{H}} = \hat{\mathcal{H}}_{\text{probe}} - k\hat{x}\hat{y}_{\text{signal}} + \hat{\mathcal{H}}_y$

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$$k \to 0\,,\ \langle y \rangle \to \infty \quad \Rightarrow$$

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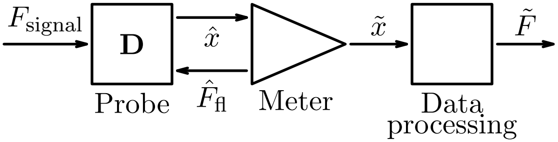

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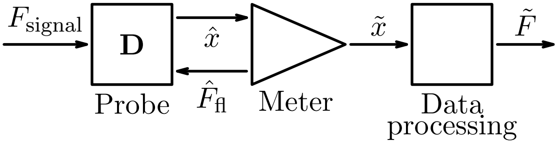

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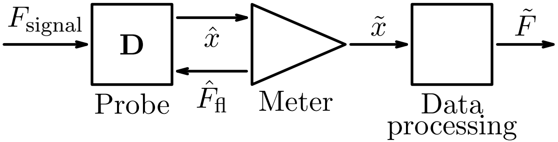

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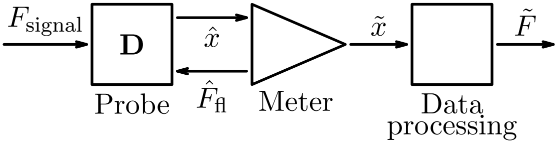

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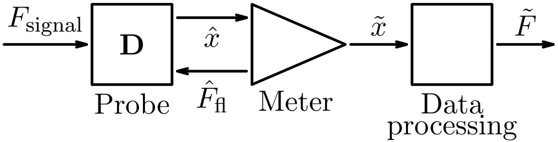

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$\hat{x}_{\rm fl}$, $\hat{F}_{\rm fl}$: optical operators in disguise!

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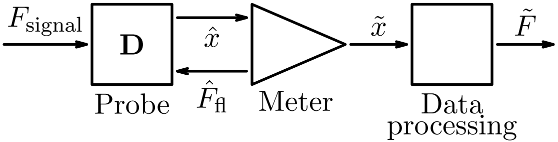

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All mechanical operators are gone!

The general idea:

- insert a really quantum (non-Gaussian) state into a mechanical object;
- wait as long as possible;
- extract the quantum state back;
- verify that it is intact.

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$$Q \sim 10^9 \;\Rightarrow\; \tau^* = Q/\Omega_m \to 10^3\,\mathrm{s}$$

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Coherent	–	–	–
Squeezed	+	–	?
non-Gaussian	+	+	+

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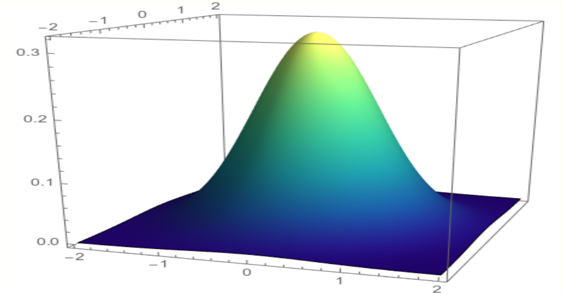
"An always positive Wigner function can serve as the hidden-variable probability distribution with respect to measurements corresponding to any linear combination of x and p".

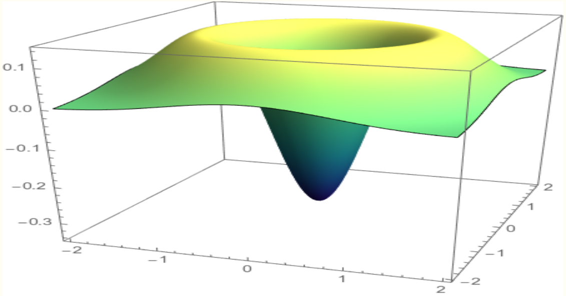
J. S. Bell, *Speakable and Unspeakable in Quantum Mechanics*, Cambridge Univ. Press, Cambridge, 1987

S. L. Braunstein, P. van Loock, RMP **77**, 513 (2005)

Non-Gaussian optomechanics

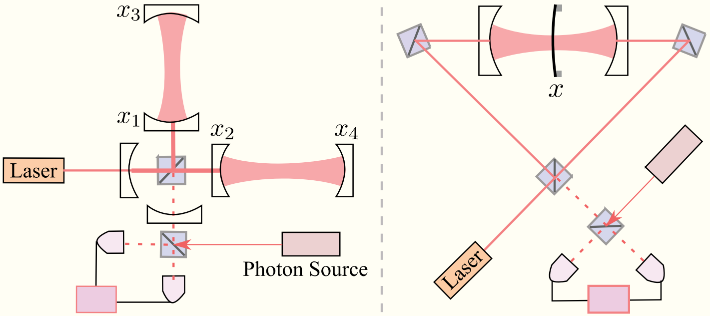


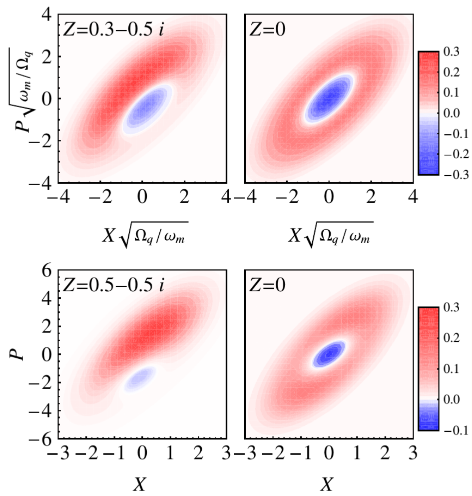
$|0\rangle$



$|1\rangle$

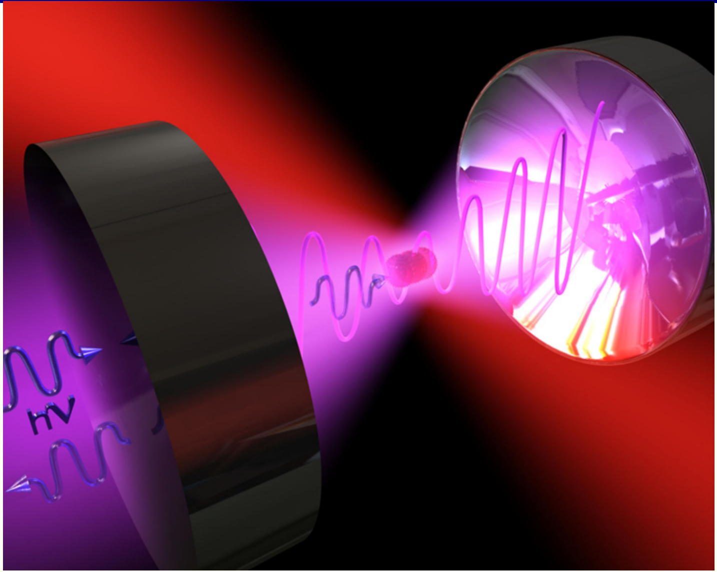
Non-Gaussian optomechanics





F. Khalili *et al*, Phys. Rev. Lett. **105**, 070403 (2010)

"Toward quantum superposition of living organisms"



O. Romero-Isart *et al*, New J. Phys. **12** 033015 (2010)