Quantum Optics 2019/2020, Problem set 6, 26.11.2020

Problem 1 We know that P-Glauber representation can be calculated using the following formula:

$$P(\alpha) = \mathrm{Tr}\left(\rho :\delta^{(2)}(\alpha - \hat{a}):\right),$$

where :: denotes normal ordering, and that it yields a diagonal coherent state representation of a quantum state:

$$\rho = \int \mathrm{d}^2\alpha\, P(\alpha)|\alpha\rangle\langle\alpha|.$$

Show that the P-Glauber representation can be alternatively calculated using the following formula

$$P(\alpha) = \frac{e^{|\alpha|^2}}{\pi^2}\int \mathrm{d}^2\beta\,\langle -\beta|\rho|\beta\rangle e^{|\beta|^2 + \beta^*\alpha - \beta\alpha^*}.$$

Problem 2 Prove that different quasi-probability distributions can be related with each other via a particular convolution with a Gaussian:

a) $W(\alpha) = \frac{2}{\pi}\int \mathrm{d}^2\gamma\, e^{-2|\gamma-\alpha|^2}P(\gamma)$

b) $Q(\alpha) = \frac{2}{\pi}\int \mathrm{d}^2\gamma\, e^{-2|\gamma-\alpha|^2}W(\gamma)$

c) $Q(\alpha) = \frac{1}{\pi}\int \mathrm{d}^2\gamma\, e^{-|\gamma-\alpha|^2}P(\gamma)$

In this sense W is more regular than P and Q is more regular than W as they are additionally "smeared".

Click to see the hint:

Problem 3 When moving back to q and p parametrization show that the Wigner function can be written as:

$$W(q,p) = \frac{1}{2\pi}\int \mathrm{d}w\, e^{-iwp}\langle q + \frac{w}{2}|\rho|q - \frac{w}{2}\rangle,$$

where $|q\rangle$ is the position operator eigenstate $\hat{q}|q\rangle = q|q\rangle$, $\langle q|q'\rangle = \delta(q-q')$. Remember about the normalization factor, when moving from α to q and p parametrization: $W(q,p) = \frac{1}{2}W(\alpha)$.

Show that marginals of $W(q,p)$ correspond to the true probability distributions od q and p respectively:

$$\int_{-\infty}^{\infty} \mathrm{d}p\, W(q,p) = \langle q|\rho|q\rangle, \quad \int_{-\infty}^{\infty} \mathrm{d}q\, W(q,p) = \langle p|\rho|p\rangle,$$

where $|q\rangle$, $|p\rangle$ are position and momentum eigenstates. This property makes the Wigner function the most natural choice for quasi-probabilty distributions.

Problem 4 Find and give a 3D plot (if possible) of the P-representation for the following states:

a) Coherent state $|\alpha_0\rangle$

b) Thermal state $\rho_T = (1 - e^{-\xi}) \sum_n e^{-n\xi} |n\rangle\langle n|$, $\xi = \hbar\omega/kT$.

c) Fock state $|n\rangle$

Click to see the answer

Problem 5 Find and give a 3D plot of the Wigner-Weyl representation for the following states:

a) Coherent state $|\alpha_0\rangle$

b) Thermal state $\rho_T = (1 - e^{-\xi}) \sum_n e^{-n\xi} |n\rangle\langle n|$, $\xi = \hbar\omega/kT$.

c) Fock state $|n\rangle$

d) Squeezed state $|r, 0\rangle = \hat{S}(r, 0)|0\rangle$, where $\hat{S}(r, 0) = e^{\frac{r}{2}(\hat{a}^2 - \hat{a}^{\dagger 2})}$ is the standard squeezing operator.

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Problem 6 Find and give a 3D plot of the Hussimi Q-representation for the following states:

a) Coherent state $|\alpha_0\rangle$

b) Thermal state $\rho_T = (1 - e^{-\xi}) \sum_n e^{-n\xi} |n\rangle\langle n|$, $\xi = \hbar\omega/kT$.

c) Fock state $|n\rangle$

d) Squeezed state $|r, 0\rangle = \hat{S}(r, 0)|0\rangle$.

Click to see the answer

Problem 7 Find and plot the Wigner-Weyl distribution for the following two states:

a) $\rho = \frac{1}{2}(|\alpha_0\rangle\langle\alpha_0| + |-\alpha_0\rangle\langle-\alpha_0|)$, where $|\alpha_0\rangle$ is a coherent state (choose $\alpha_0 = 0.1$, 1, 10 for the purpose of the plots)

b) $\rho = |\psi\rangle\langle\psi|$, $|\psi\rangle = \mathcal{N}\,(|\alpha_0\rangle + |-\alpha_0\rangle)$, where $\mathcal{N}$ is a normalization factor (which is not $1/\sqrt{2}!$). For simplicity assume α_0 is real, and take $\alpha_0 = 0.1$, 1, 10 for the purpose of producing the plots.

comment the differences.

Click to see the answer