Quantum Optics 2020/2021, Problem set 9, 12.01.2021

Problem 1 Consider a two level atom ($|g\rangle$, $|e\rangle$) interacting with a single mode classical electromangetic field that oscillates with frequency ω: $\vec{E}(t) = E\vec{e}_k \cos(\omega t)$. In the rotating wave approximation the Hamiltonian for this semi-classical model is given by:

$$H = \frac{1}{2}\hbar\nu\hat{\sigma}_z + \chi(\hat{\sigma}_+ e^{-i\omega t} + \hat{\sigma}_- e^{i\omega t})$$

where $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$, $\sigma_+ = |e\rangle\langle g|$, $\sigma_- = |g\rangle\langle e|$. Parameter $\chi = E\vec{e}_k \cdot \vec{d}_{eg}/2$ represents the interaction strength between the atom and the field, where d_{eg} is the matrix element of the dipole moment vector between states $|g\rangle$ and $|e\rangle$.

Assuming the atom is initially in state

$$|\psi(0)\rangle = c_g(0)|g\rangle + c_e(0)|e\rangle$$

find the state of the atom $|\psi(t)\rangle$ at time t. Discuss similarity and differences with the fully quantum Jaynes-Cummings model—what should be the initial state in the fully quantum Jaynes-Cummings model to mimic the semi-classical behaviour.

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Problem 2 Consider the Jaynes-Cummings model of a two-level atom interacting with a single mode quantized electromagnetic field, where the Hamiltonian is given by:

$$H = \frac{1}{2}\hbar\nu\hat{\sigma}_z + \hbar\omega\hat{a}^\dagger\hat{a} + \hbar\lambda(\hat{\sigma}_+\hat{a} + \hat{\sigma}_-\hat{a}^\dagger),$$

$\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$, $\sigma_+ = |e\rangle\langle g|$, $\sigma_- = |g\rangle\langle e|$, and λ is the interaction parameter.

Consider the following initial state of the system

$$|\psi(0)\rangle = |e\rangle \otimes |\alpha\rangle,$$

where $|\alpha\rangle$ is the coherent state of light.

a) Find the form of the state at time t

b) Write an expression for the inversion parameter $w(t)$.

c) Plot it for $t \in [0, 2\mathrm{s}]$, for exemplary parameters: $\lambda = 50\mathrm{Hz}$, $|\alpha| = 10$

d) Can you observe characteristic collapses and revivals in the plot. Can you estimate how the collapse time T_c (the characteristic time where the first collapse of Rabi oscillations happen) and revival time T_r (the time when Rabi oscillations come back...) scale as a function of system parameters.

e) Comment, whether the observed phenomena is expected to be seen in the semi-classical model where the light is described classically.

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Problem 3 The phenomena od strong interaction of atoms with electromagnetic field in the cavity can be used to entangle the atoms themselves (Nobel Prize 2014 for Serge Haroche). Consider the following idea. We have two atoms, with transition frequency perfectly matching the cavity frequency. Initially the cavity is in a vacuum state. We prepare the first atom in the excited state $|e\rangle$ and send it into the cavity so that it interacts via the Jaynes-Cummings Hamiltonian for some time t_1 and then leaves the cavity. After this we send another atom in state $|g\rangle$ and let it interact with the field inside the cavity for a time t_2. After that the second atom also leaves the cavity. Can you choose times t_1 and t_2 such that after this procedure the two atoms are entangled with each other and at the same time *not* entangled with the state of light in the cavity.

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