Quantum Optics 2020/2021, Problem set 10, 22.01.2021, courtesy of Farit Khalili

Problem 1 - Squeezed input state Calculate the quantum noise spectral densities (S_x, S_F, and S_{xF}) of the optical position meter shown in Fig. 1, assuming the general squeezed state of the input light:

$$S_{cc} = \frac{1}{2}(\cosh 2r + \sinh 2r \cos 2\theta)\,, \quad S_{ss} = \frac{1}{2}(\cosh 2r - \sinh 2r \cos 2\theta)\,, \quad S_{cs} = \frac{1}{2}\sinh 2r \sin 2\theta\,.$$

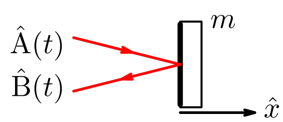

Figure 1: Simplest optical position meter

Click to see the answer:

Problem 2 - Multi-reflection meter Calculate the quantum noise spectral densities (S_x, S_F, and S_{xF}) of the multi-pass optical position meter shown in Fig. 2, assuming k reflections and the phase-squeezed input state [Eqs. () for $\theta = 0$, $r > 0$]. Estimate the optical power necessary to reach the SQL for the typical GW detectors parameters: $\omega_p = 2\pi c/1064\,\text{nm}$, $m \approx 100\,\text{kg}$, $\Omega \approx 10^3\,\text{s}^{-1}$.

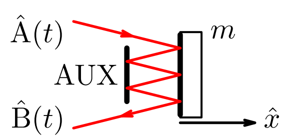

Figure 2: Optical position meter with multiple reflections. AUX: a fixed auxiliary mirror

Click to see the hint:

Click to see the answer:

Problem 3 - Damped probe object Calculate the optimized sum quantum noise spectral density $S_{\rm sum}$ for the case of a "damped" mechanical object with $\operatorname{Im}\chi \neq 0$. Compare the result with the Brownian (thermal) noise of the mechanical object.

Click to see the answer:

Problem 4 - "Quantum accelerometer" Similar to the quantum speedmeter, explore the sensitivity of the "Quantum accelerometer" device, with

$$S_x(\Omega) \propto \Omega^{-4}\,, \quad S_F(\Omega) \propto \Omega^{4}\,, \quad S_{xF} \propto \Omega^{0}\,. \tag{2}$$

Propose a possible optical scheme for this device.

Click to see the hint:

Problem 5 - "'Negative mass reference frame" Calculate the sum quantum noise spectral density $S_{\rm sum}$ for the setup which measures, using the same light, the sum $x + y$ of positions of a free mass m and a hypothetical negative free mass $-m$, see Fig. 3, assuming that the goal is to measure the signal displacement $x_{\rm signl}$ of the first object.

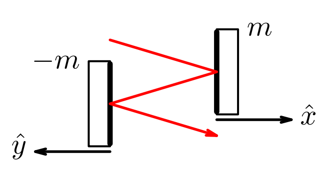

Figure 3: "Negative mass reference frame" meter

Click to see the hint: