Quantum Estimation and Measurement Theory

Problem set 10

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Problem 1 During the lecture we have derived the cost of the optimal Bayesian strategy of indentifying a completely unknown state of a qubit, $|\psi\rangle_{(\theta,\varphi)} = \cos(\theta/2)|0\rangle + \sin(\theta/2)\exp(\mathrm{i}\varphi)|1\rangle$, from its N copies The term completely unknown means that the states are drawn at random using a uniform prior distribution on the Bloch sphere. Assuming the cost function takes he form: $C(\psi,\tilde{\psi}) = 4\left(1-|\langle\psi|\tilde{\psi}\rangle|^2\right)$ we have proven that the minimal achievable cost reads:

$$C = 4\left(1-\frac{N+1}{N+2}\right). \tag{1}$$

We want to consder its asymptotic behaviour and compare it with the predictions od multi-parameter Cramer-Rao bound. Proceed as follows:

a) Show that in case $|\psi\rangle$, $|\tilde{\psi}\rangle$ are close to each other, the cost function in the lowest order expansion takes the form: $C(\psi,\tilde{\psi}) \approx \Delta^2\theta + \sin^2(\theta)\Delta^2\varphi$, where $\Delta^2\theta = (\theta-\tilde{\theta})^2$, $\Delta^2\varphi = (\varphi-\tilde{\varphi})^2$.

b) Compute the Quantum Fisher Information matrix for the problem of estimating θ, φ on state $|\psi\rangle_{(\theta,\varphi)}$ (this was done in one of previous problem sets)

c) Make use of the multi-parameter quantum Cramera-Rao bound to derive a bound on the estimation precision, treating as the effective cost the following weighted sum of variances:$\Delta^2\theta + \sin^2(\theta)\Delta^2\varphi$.

d) Compare the asymptotic behaviour of (1) with the bound arising from application of the multi-parameter Cramer-Rao bound. Try to interpret this observations.