Quantum Estimation and Measurement Theory

Problem set 12/13

return on 25.01.2019

Problem 1 Consider a Mach Zehnder interferometer which is fed with a the following two-mode N-photon state of light:

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|j = N/2, m = 0\rangle + |j = N/2, m = 1\rangle \right), \tag{1}$$

which was written using the notation where $|j, m\rangle$ corresponds to a $N = 2j$ photon state which is an eigen state of the $J_z = \frac{1}{2}(a^\dagger a - b^\dagger b)$ operator: $J_z|j, m\rangle = m|j, m\rangle$—in the standard photon number notation this state corresponds to $|j = N/2, m\rangle \equiv |n_a = N/2 + m, n_b = N/2 - m\rangle$, where n_a, n_b are photon numbers in mode a and b respectively. Make use of the general formula derived during the lecture for phase estimation precision in Mach-Zehnder interferometer expressed via expectation values and variances of appropriate J_i operators and try to find the scaling of precision in the limit of large N—everything you know from standard quantum mechanics about acting with J_i operators on $|j, m\rangle$ states works!. For which φ we get the highest sensitivity. Try to sketch the „Bloch ball" picture which would illustrate properties of this states and provide an intuitive explanation of its high sensitivity.

Problem 2 Try to think what input state to the Mach-Zhender interferometer would correspond to the N00N state case that we were considering when studying fundamental limits in quantum metrology (note that the NOON state we were considering was assumed to be *inside* the interferometer and now we are asking for the state the should be send *into* the interferometer). What would be the sensitivity of the protocol employing this state if we used the simple estimation scheme based on the measurement of the J_z observable (measurement performed after the second beam-splitter of the Mach-Zehnder interferometer)?