Quantum Estimation and Measurement Theory

Problem set 5

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Problem 1 Consider a Bayesian estimation problem, but with a different cost function than the mean squared error. In case when we want to estimate a phase (or some other angle-like parameter) $\theta \in [0, 2\pi]$, a more practical cost function is a function of the form $C(\theta, \tilde{\theta}) = 4\sin^2\left(\frac{\theta-\tilde{\theta}}{2}\right)$, which for small deviations between θ and $\tilde{\theta}$ is equivalent to the variance but respects that fact, that the 2π difference is not relevant. Average cost is then given by:

$$\bar{C} = \int \mathrm{d}\theta \mathrm{d}x \; 4\sin^2\left(\frac{\theta - \tilde{\theta}(x)}{2}\right) p(x|\theta)p(\theta). \tag{1}$$

Find the optimal Bayesian estimator for this cost function.

Problem 2 Analyze the conditions for saturation of the Bayesian Cramér-Rao inequality and check if the gaussian model consider during the lecture is the only one for which the inequality is actually saturated.