Introduction to Quantization–Exam 25.06.2020

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Problem 0.1. *Let $V(x) > 1$ and $a(x,p) = p^2 + V(x)$. Using the semiclassical ($\hbar$-dependent) quantization $\mathrm{Op}_\hbar$, define r by*

$$\mathrm{Op}_\hbar(a)\mathrm{Op}_\hbar(a^{-1}) = \mathbb{1} + \mathrm{Op}_\hbar(r),$$

What is the (semiclassical) order of r? Compute the (semiclassical) principal and subprincipal symbol of r.

Problem 0.2. *Find the integral kernel of the operator*

$$\exp(-\sqrt{-\partial_x^2}).$$

Problem 0.3. *Find the Wick and anti-Wick symbol of the operator*

$$(\hat{x} + \hat{p})^2.$$

Problem 0.4. *Consider the operator*

$$B := \hat{p}a(\hat{x})\hat{p}.$$

What is its x,p and Weyl symbol?

Problem 0.5. *Consider the function*

$$f(x) := (x + \mathrm{i})^{-3}.$$

Let $\hat{f}$ be its Fourier transform. How many times is $\hat{f}$ differentiable?

Problem 0.6. *Let $\mathbb{R}^3 \ni x \mapsto V(x)$ be a smooth function such that $\lim_{|x|\to\infty} V(x) = \infty$. Δ denotes the Laplacian. What is the semiclassical prediction of the number of eigenvalues below $\mu \in \mathbb{R}$ of the Hamiltonian on $L^2(\mathbb{R}^3)$*

$$H_\hbar := \hbar^4\Delta^2 + V(x)$$

Problem 0.7. *Let* $\alpha \geq 0$. *What is the spectrum of the operator* $\mathrm{Op}(\mathrm{e}^{-\alpha x^2 - \alpha^{-1} p^2})$*?*

Problem 0.8. *Consider*

$$a(x,\xi) := (1 + \xi^4 \sin^2 x)^{\frac{1}{2}}.$$

Does a *belong to* S^m *for some* m*? If so, for which* m*?*

Problem 0.9. *Find the principal symbol (in the pseudodifferential sense) of*

$$A := -\partial_t^2 + (1+x^2)^{-1}\partial_x^2 + \partial_x.$$

Is this operator elliptic?