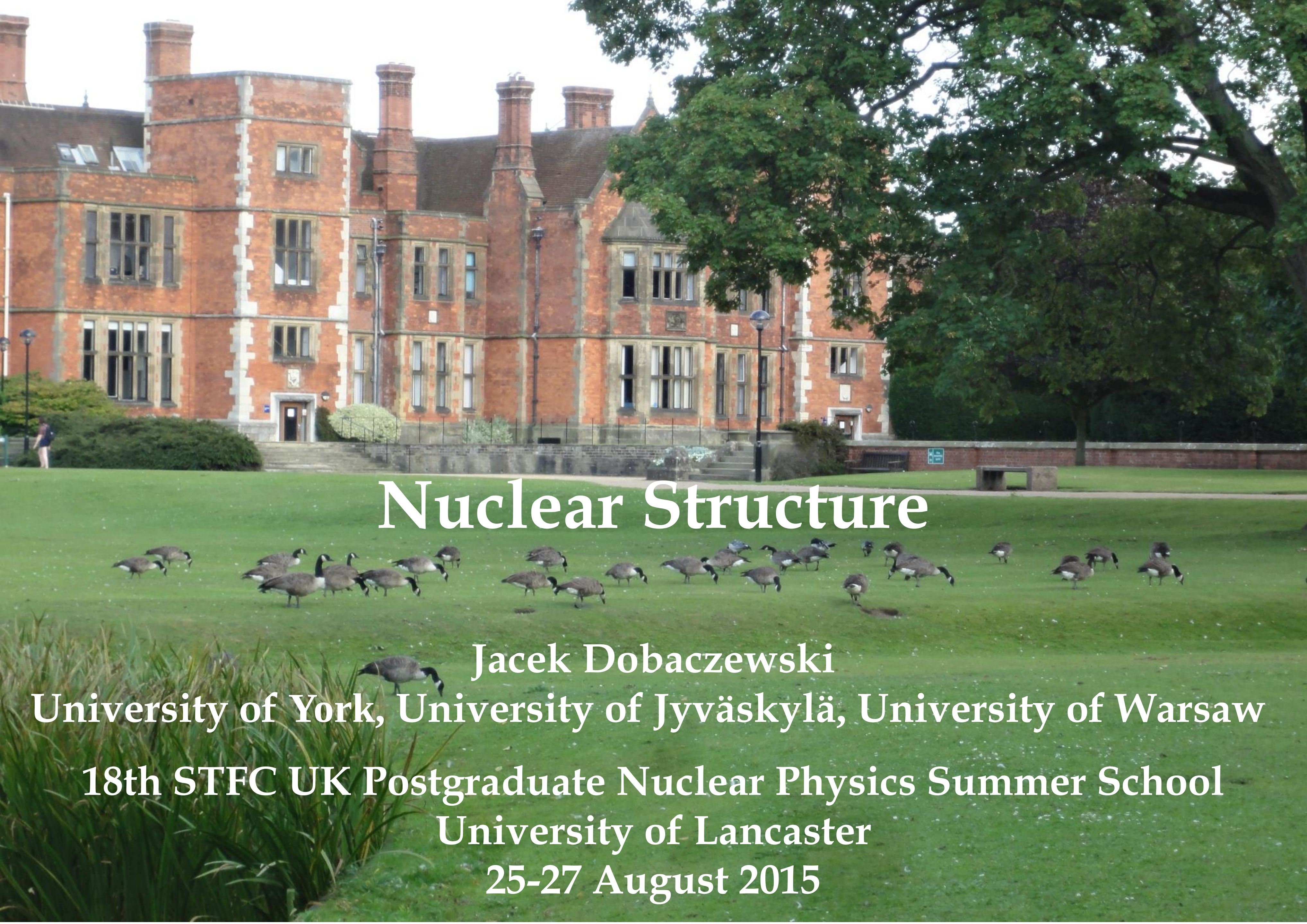

Nuclear Structure

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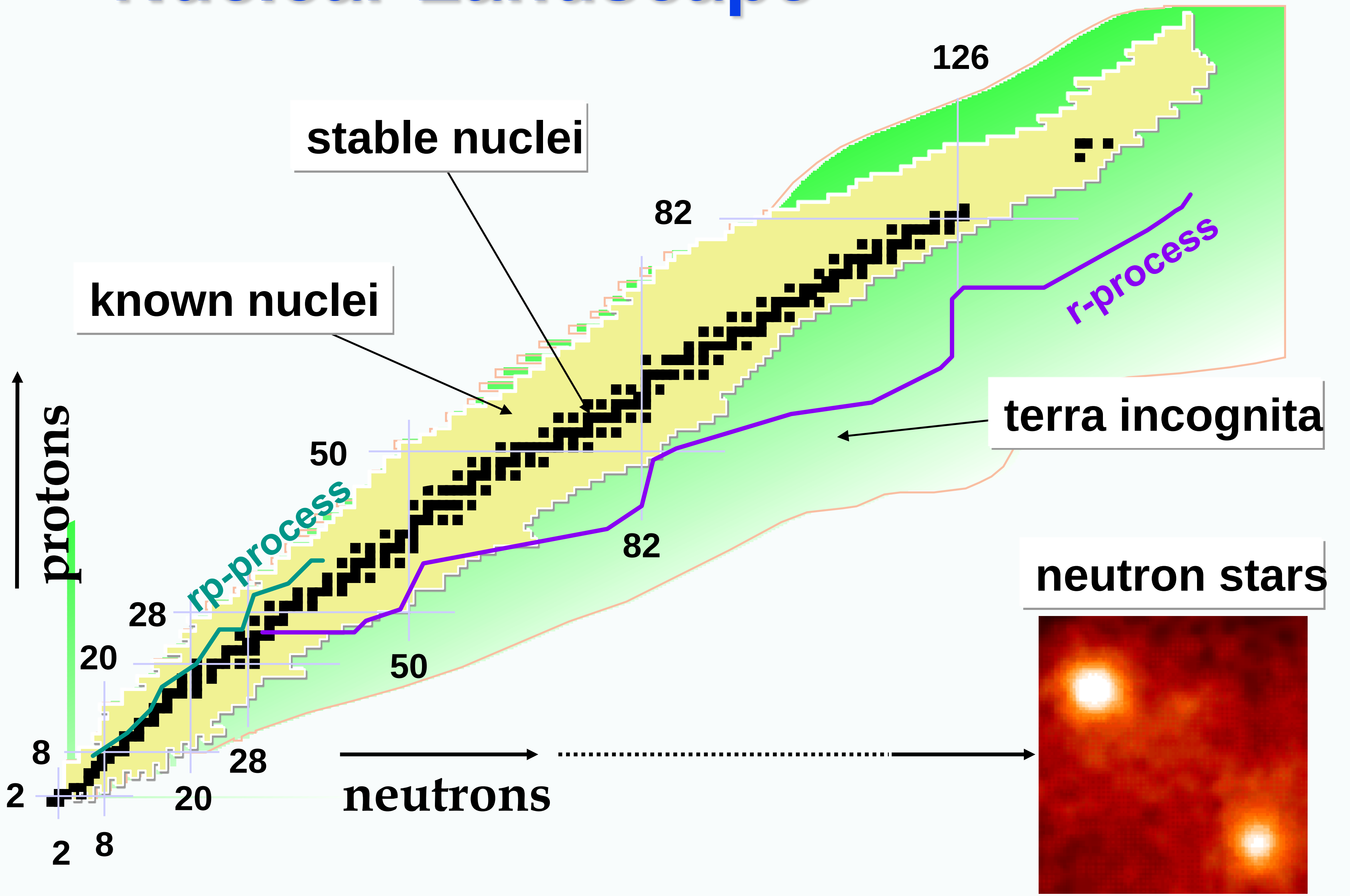
18th STFC UK Postgraduate Nuclear Physics Summer School
University of Lancaster
25-27 August 2015

Introduction

Nuclear Landscape



Nuclear Landscape

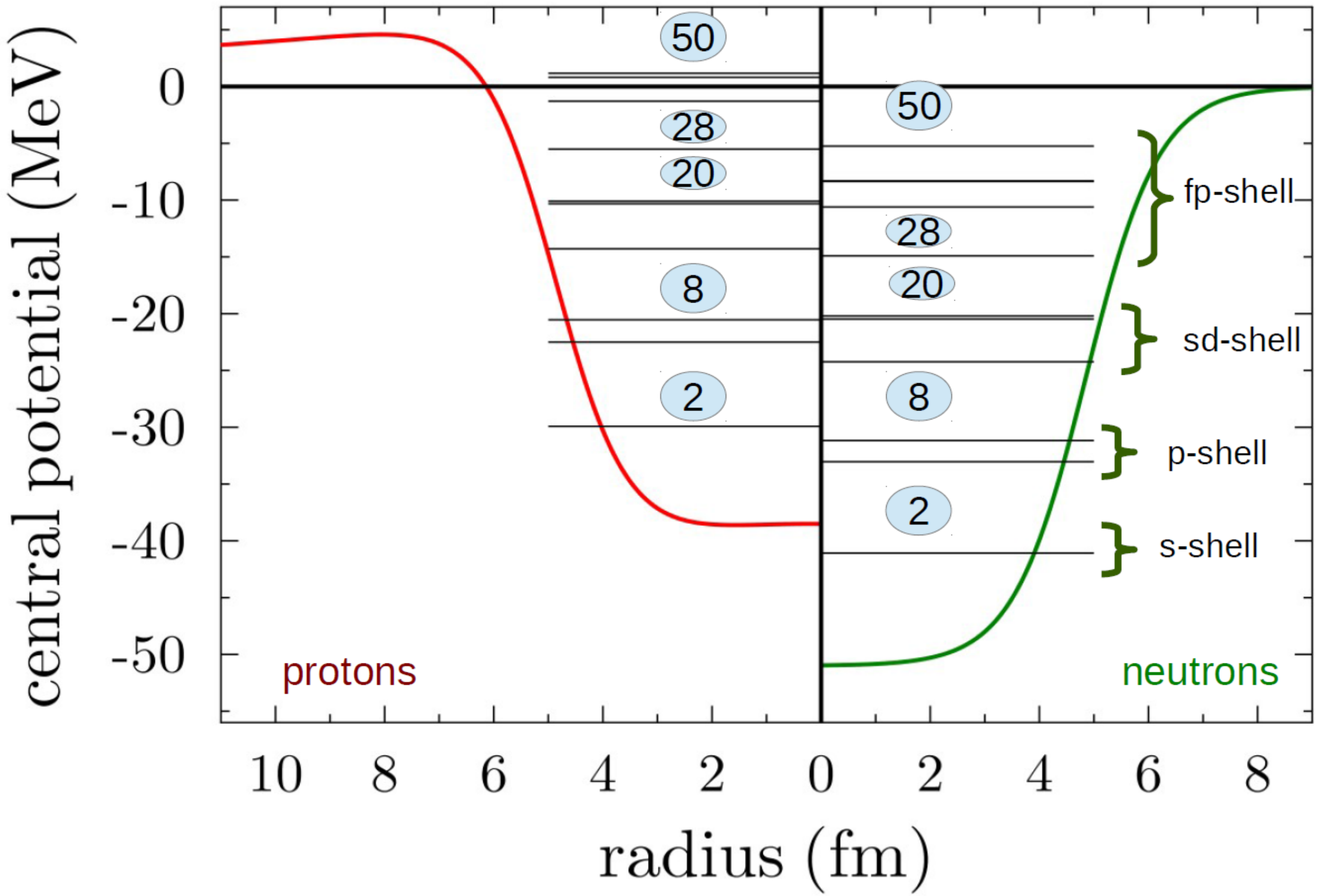
Lessons learnt

There are only less than 300 stable nuclides in nature (black squares), while already about 3000 other ones have been synthesized and studied in nuclear structure laboratories (yellow zone). However, the nuclear landscape extends further away into uncharted territories (green zone), where probably double of that await discovery. Properties of these exotic systems cannot be at present reliably derived from theoretical models, because our knowledge of basic ingredients thereof is still quite rudimentary. Derivations form first principles allow us already now to recognize general features of nuclear forces, energy-density functionals, or shell-model interactions, however, plenty of these features require careful adjustment to precise nuclear data. Such adjustments, especially when performed for exotic, extreme systems, provide invaluable information, and then in turn allow for more reliable extrapolations.

Phenomenological nuclear mean field

56Ni, Woods-Saxon potential with Bohr-Mottelson parameterization

(Parameterization from A. Bohr and B. Mottelson, Nuclear Structure, Vol. I (1969))



Slide by Markus Kortelainen

Contents:

1. Quarks, gluons, mesons, nucleons, and nuclei
   - QCD fundamentals
   - Composite particles
   - NN interactions
2. Spontaneous symmetry breaking
   - Parity – NH_3 molecule
   - Rotational – deuteron & nuclei
3. Effective fields in nuclear systems
4. Physics of exotic nuclei
   - Matter distributions
   - Shell structure
   - Pairing

Home page: http://www.fuw.edu.pl/~dobaczew/

Fundamentals

Quantum electrodynamics (QED)

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \bar{\psi}_e\gamma^\mu[\partial_\mu + ieA_\mu]\psi_e - m_e\bar{\psi}_e\psi_e$$

- Space-time index μ=0,1,2,3
- Units: $\hbar = c = 1$, $e = \sqrt{4\pi\alpha}$, $1/\alpha \simeq 137$
- Tensor of the electromagnetic field: $F^{\mu\nu}=\partial^\mu A^\nu-\partial^\nu A^\mu$ (contains electric and magnetic fields)
- Vector potential of the photon field: A^μ
- Dirac four-spinor of the electron field: ψ_e
- Elementary charge: e (electron charge is q=$-e$)
- Electron mass: m_e
- Dirac 4×4 matrices γ^μ:

$$\gamma^0=-i\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},\ \vec{\gamma}=-i\begin{pmatrix}0 & \vec{\sigma}\\ -\vec{\sigma} & 0\end{pmatrix},\ \gamma_5=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

- Pauli 2×2 matrices $\vec{\sigma}$:

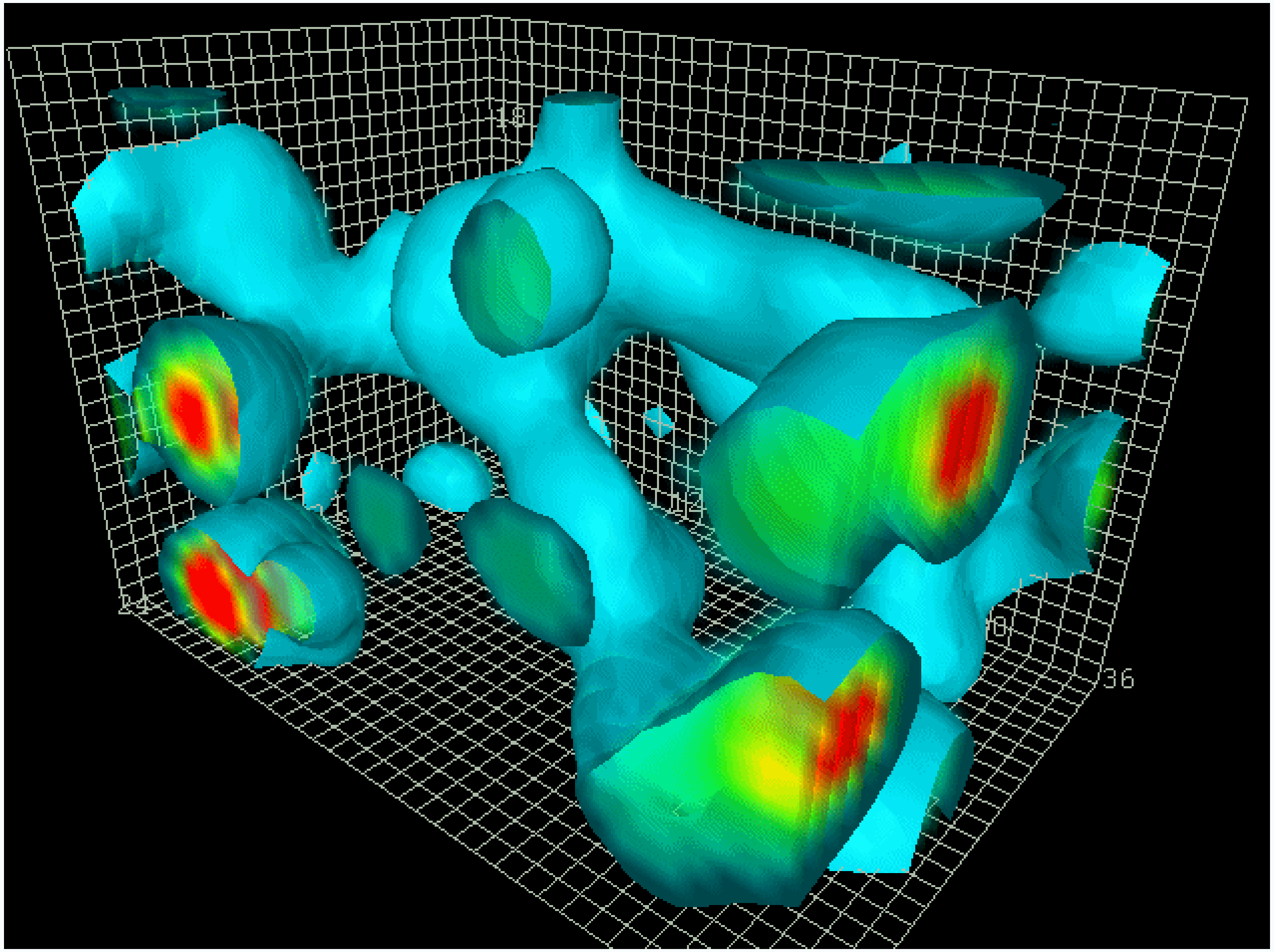
$$\sigma_1=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},\ \sigma_2=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix},\ \sigma_3=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

Quantum chromodynamics (QCD)

$$\mathcal{L} = -\frac{1}{4}F^{\alpha}_{\mu\nu}F^{\mu\nu}_{\alpha} - \sum_n \bar{\psi}_n \gamma^{\mu}[\partial_{\mu} - igA^{\alpha}_{\mu}t_{\alpha}]\psi_n - \sum_n m_n \bar{\psi}_n \psi_n$$

- Color index α=1,. . . ,8
- Flavor index n=1,. . . ,6 (or n=1,2 for nuclei)
- Tensors of the color fields: $F^{\alpha}_{\mu\nu}{=}\partial_{\mu}A^{\alpha}_{\nu} - \partial_{\nu}A^{\alpha}_{\mu} + C^{\alpha}_{\beta\gamma}A^{\beta}_{\mu}A^{\gamma}_{\nu}$ (contains gluon fields)
- Vector potentials of the gluon fields: A^{μ}_{α}
- Dirac four-spinors of the quark fields: ψ_n
- Color charge (strong coupling constant): g
- Quark masses: m_n
- Generators of the SU(3) color group: 3×3 matrices t_{α}

The QCD vacuum



Derek B. Leinweber

http://hermes.physics.adelaide.edu.au/theory/staff/leinweber/VisualQCD/QCDvacuum/welcome.html

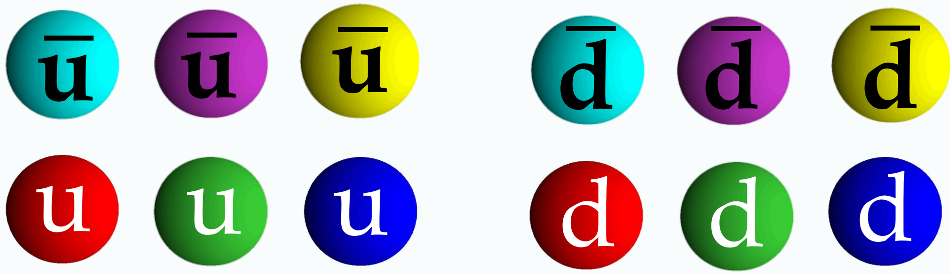
The QCD vacuum

The empty space is not empty at all! The vacuum is one of the solutions of the field equations which minimizes the energy for a state with baryon and lepton numbers equal to zero. Such a state may contain arbitrary numbers of particle-antiparticle pairs that can spontaneously appear in the empty space. On the average, there can be non-zero numbers of these pairs at any time and point in space. Therefore, the vacuum can be an enormously complicated state with a non-zero energy density.

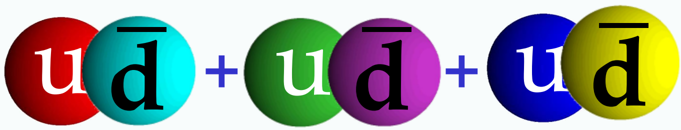
Lessons learnt

Main players in Nuclear Physics

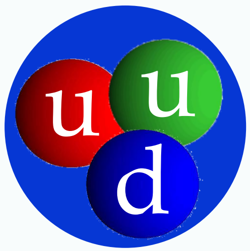
gluons



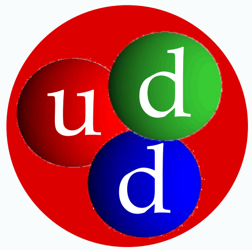
quarks



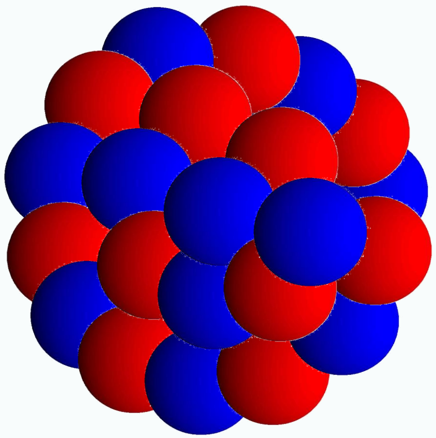
pions (π^+)



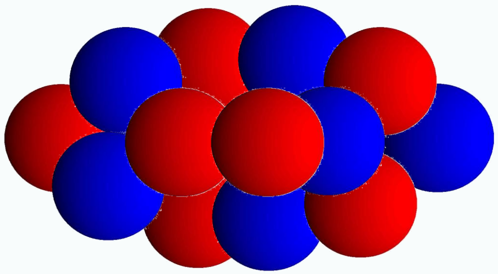
protons



neutrons



nuclei

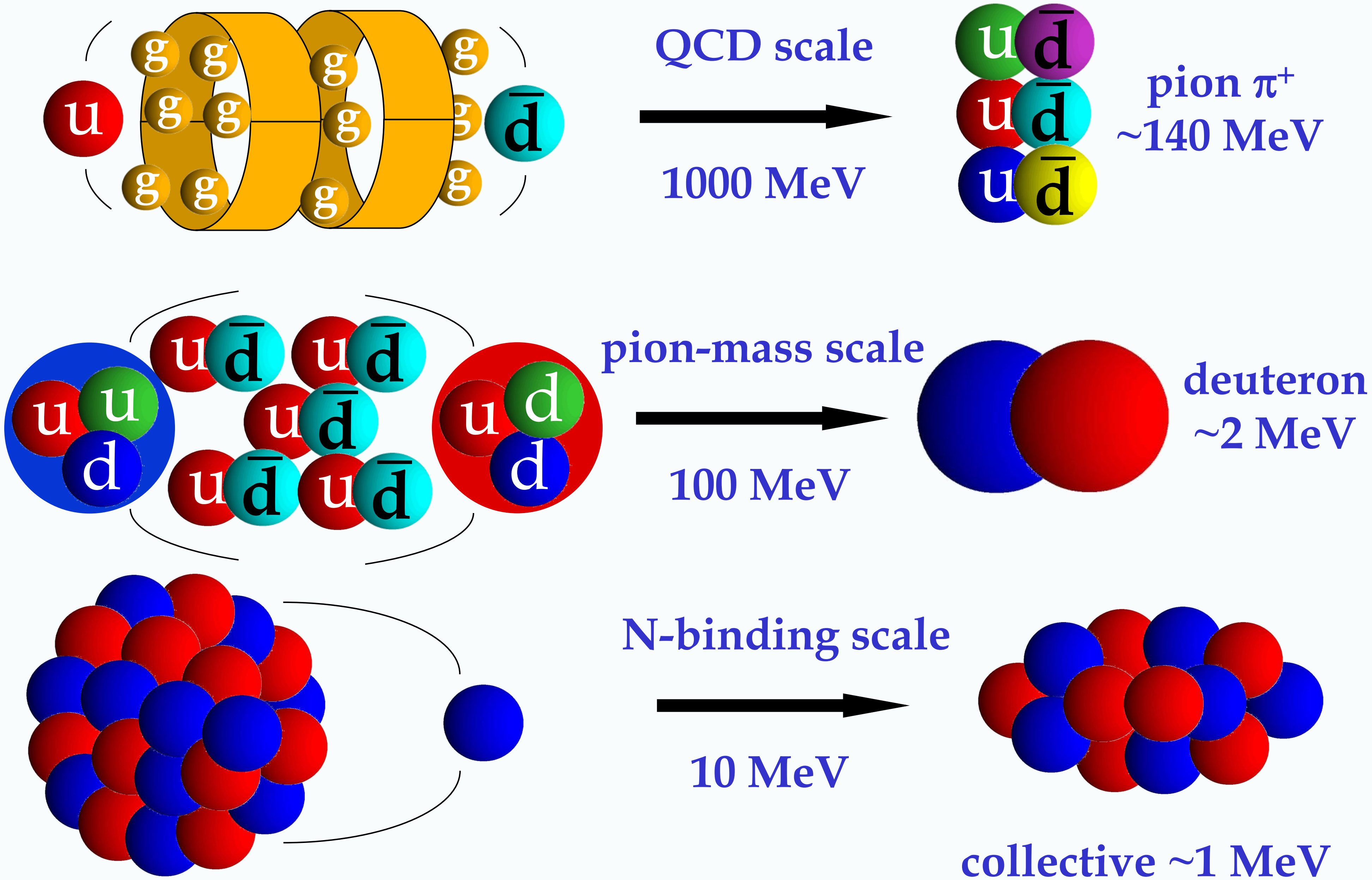


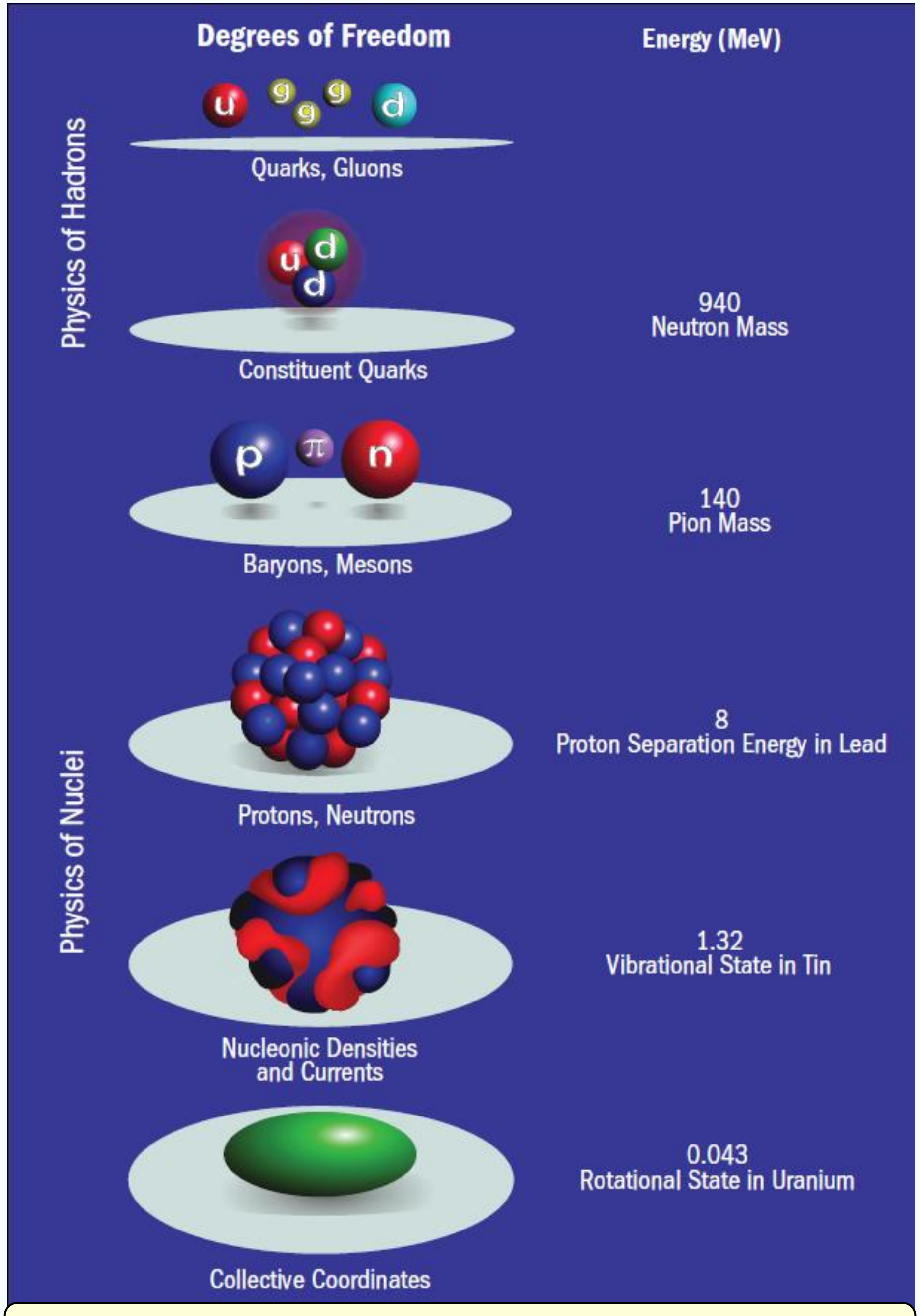
Main players in Nuclear Physics

Lessons learnt

Quarks and gluons are fundamental fields that appear in the QCD Lagrangian. All other elementary hadrons are composite objects that are solutions of the QCD for specific baryon numbers. Quarks and gluons belong to three and eight dimensional representations of the color SU(3) group; whereupon quarks are traditionally referred by the red, green, and blue (RGB) basic colors, while antiquarks by cyan, magenta, and yellow (CMY) complementary colors. Only color-SU(3) scalars (white composite particles) can propagate as free particles; all **colored fields are confined within white objects** and cannot be separated. White combinations of the quark-antiquark pairs are called mesons. Nucleons are white three-quark composite particles. Composite particles are known from experiment and cannot yet be fully calculated within the QCD. **Nuclei are composite particles built of nucleons**, of which the quark constituents are unresolved.

Scales of energy in Nuclear Physics



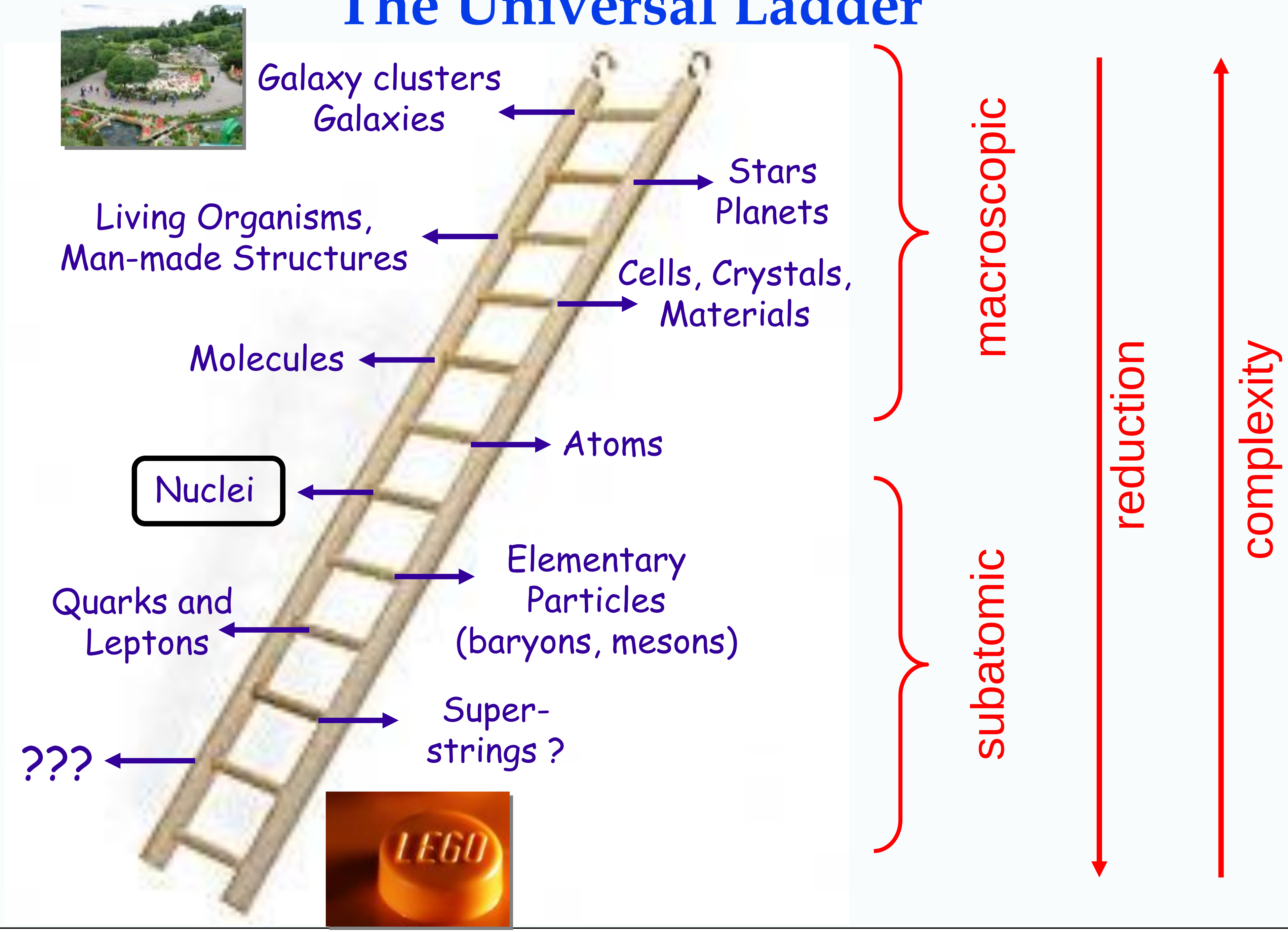


G.F. Bertsch, *et al.*, Scidac Review 6, 42 (2007)

- An effective theory (ET) is a theory which "effectively" captures what is physically relevant in a given domain.
- The most appropriate description of particle interactions in the language of quantum field theory (QFT) depends on the energy at which the interactions are studied.
- Objective reductionism (Weinberg): the convergence of arrows of scientific explanation.
- Emergence (Anderson): "at each new level of complexity entirely new properties appear and the understanding of the new behaviors requires research which I think is as fundamental in its nature as any other".

Elena Castellani, physics/0101039

The Universal Ladder



Scales of energy in Nuclear

Lessons learnt

1000 MeV QCD scale: When two valence quarks are separated, they are connected by a tube of gluons and quark-antiquark pairs (the flux tube) that provides an interaction "potential" linearly growing with the distance. The same gluon-quark-antiquark soup binds the valence quarks into white composite particles and provides most of their mass.

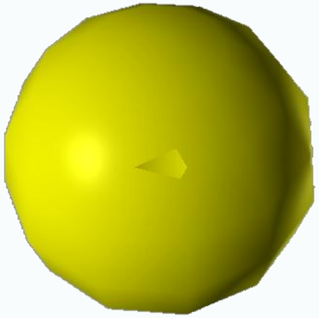
100 MeV pion-mass scale: When two nucleons are separated, they interact by exchanging one-, two-, ore several pions. Nuclei are bound as a result of pion exchanges within the background of the so-called chiral condensate.

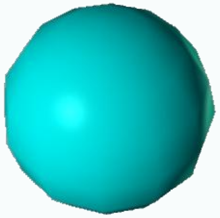
10 MeV N-binding scale: When a nucleon is separated from the nucleus, it interacts with the average field of all the remaining nucleons; its binding energy is a result of strong cancellation between the kinetic and interaction energies.

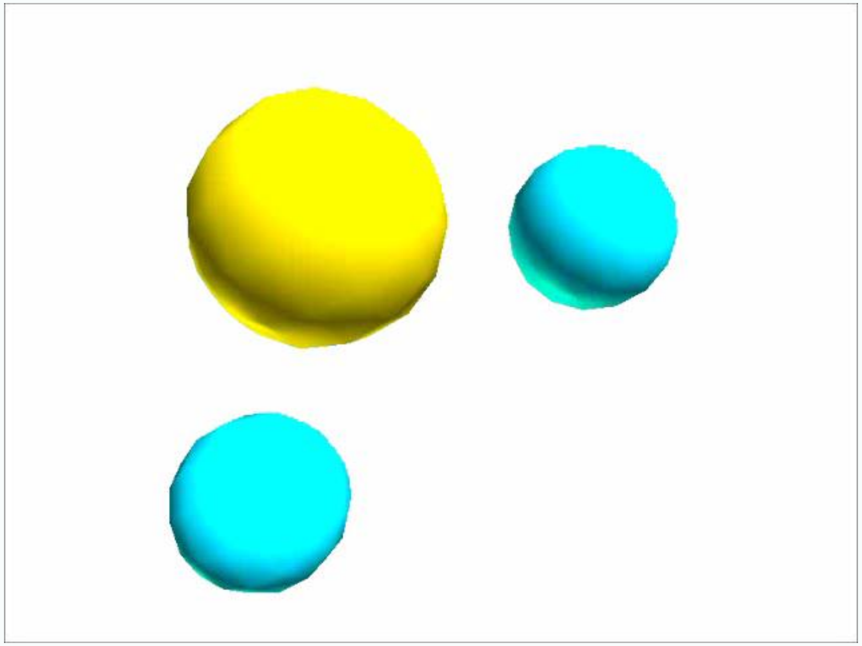
1 MeV collective scale: When the collective excitation of a nucleus is created, its energy is a coherent sum of small excitation energies of all or many constituent nucleons.

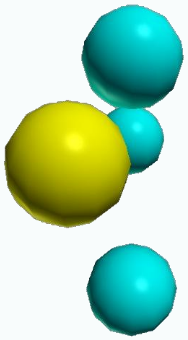
Spontaneous symmetry breaking

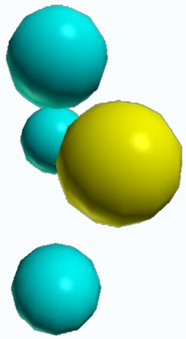
Ammonia molecule NH_3

Nitrogen atom 

Hydrogen atom 

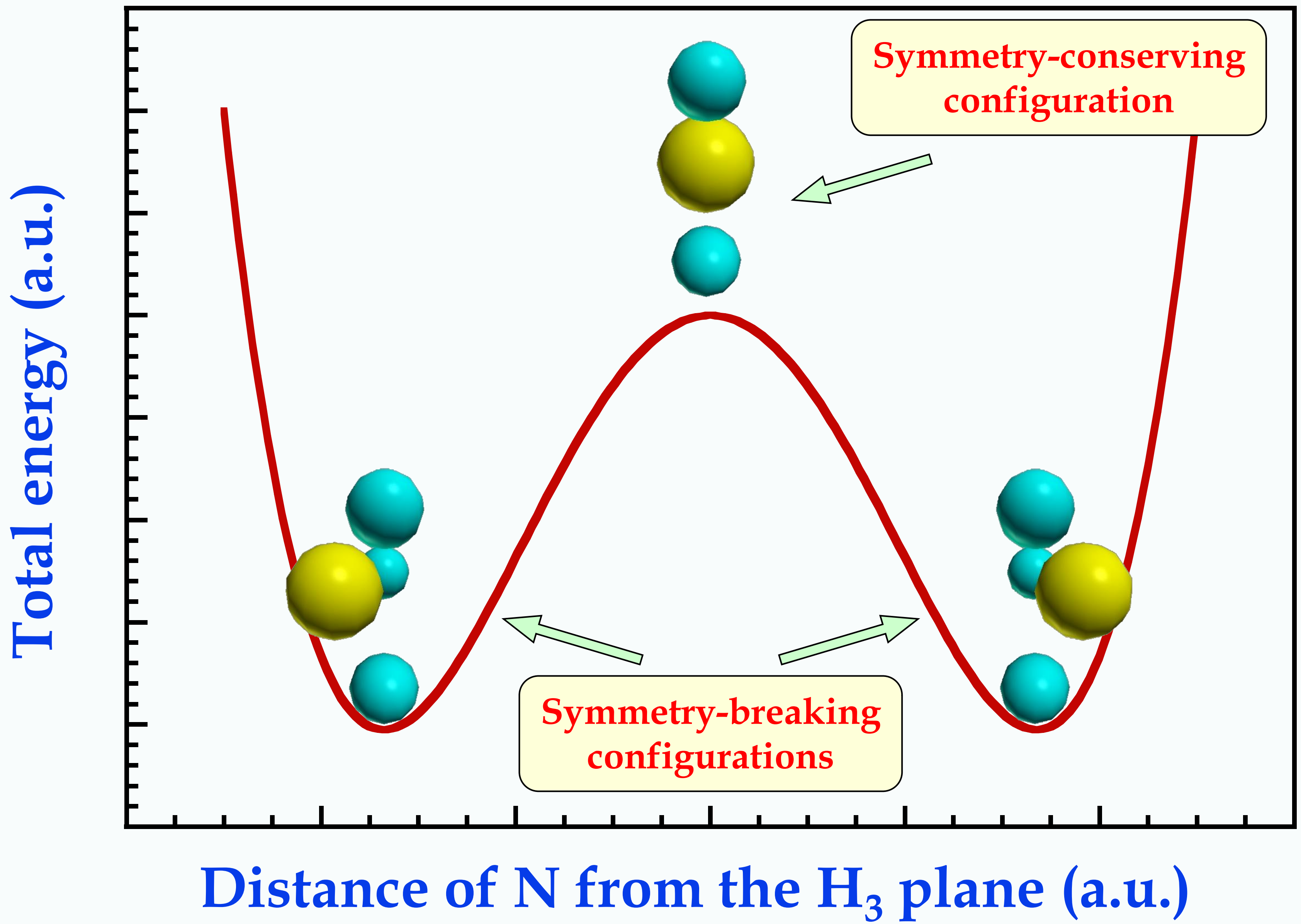


$$|L\rangle =$$ 

 $$= |R\rangle$$

left state **right state**

Ammonia molecule NH_3



Let P be the plane-reflection operator with respect to the H_3 plane, then

$$\begin{aligned} P|R\rangle &= |L\rangle \\ P|L\rangle &= |R\rangle \end{aligned}$$

Let us denote overlaps and matrix elements by

$$\begin{aligned} 1 &= \langle L|L\rangle = \langle R|R\rangle \\ \epsilon &= \langle L|R\rangle \\ E_0 &= \langle L|H|L\rangle = \langle R|H|R\rangle \\ \Delta &= \langle L|H|R\rangle \end{aligned}$$

In the non-orthogonal basis of $|L\rangle$, $|R\rangle$ the Hamiltonian matrix reads

$$H = \begin{pmatrix} E_0 & \Delta \\ \Delta & E_0 \end{pmatrix}$$

The eigenstates must correspond to the restored-symmetry states

$$|\pm\rangle = \frac{1}{\sqrt{2 \pm 2\epsilon}} \left(|L\rangle \pm R\rangle\right)$$

i.e.,

$$P|\pm\rangle = \pm|\pm\rangle$$

The eigenenergies read

$$E_\pm = \langle\pm|H|\pm\rangle = \frac{E_0 \pm \Delta}{1 \pm \epsilon}$$

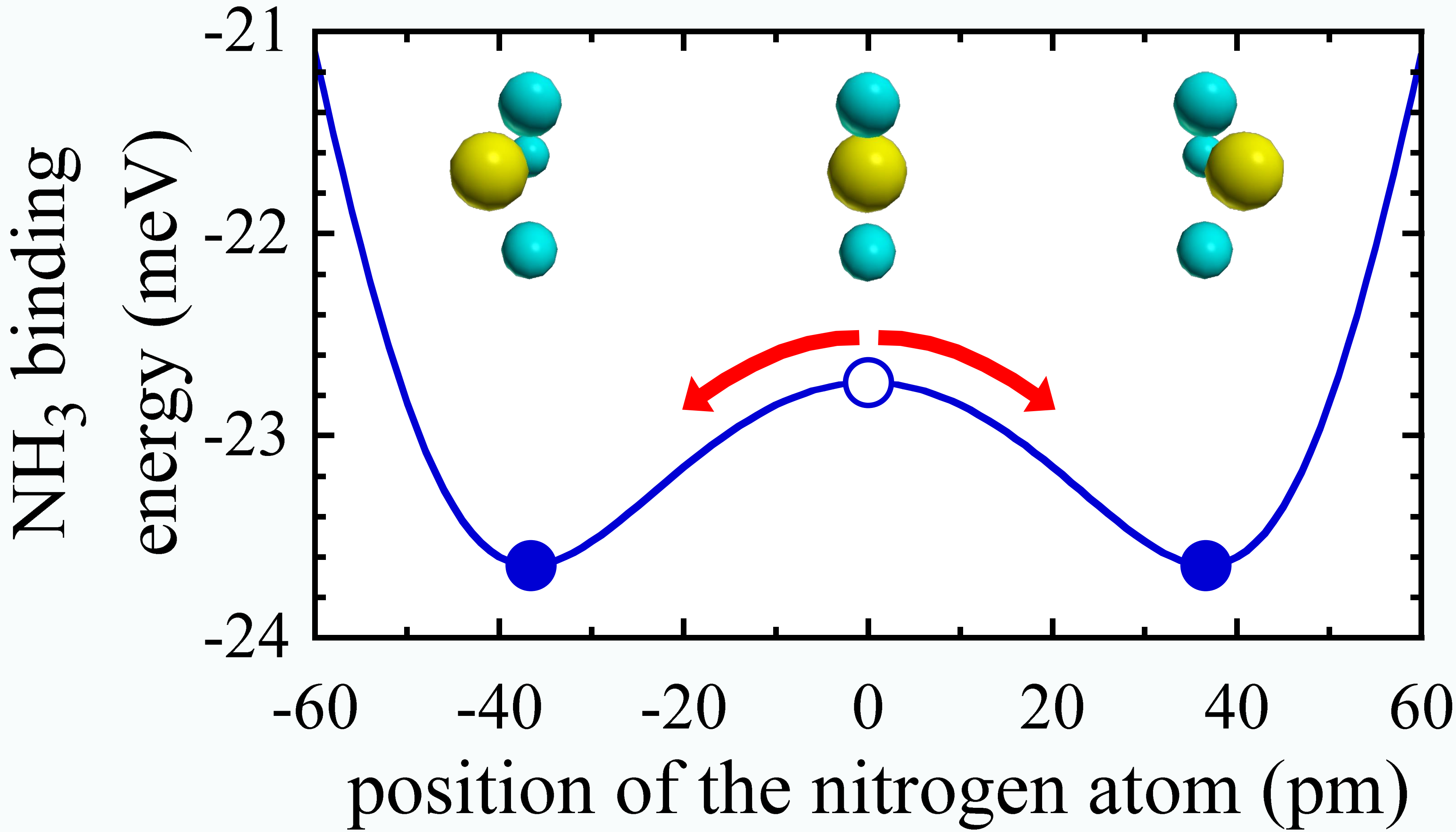
States $|L\rangle$ and $R\rangle$ are wave packets, e.g.,

$$|L\rangle = \frac{1}{2}\left(\sqrt{2+2\epsilon}|+\rangle + \sqrt{2-2\epsilon}|-\rangle\right)$$

which evolve in time ($\epsilon << \Delta/E_0$ assumed) as:,

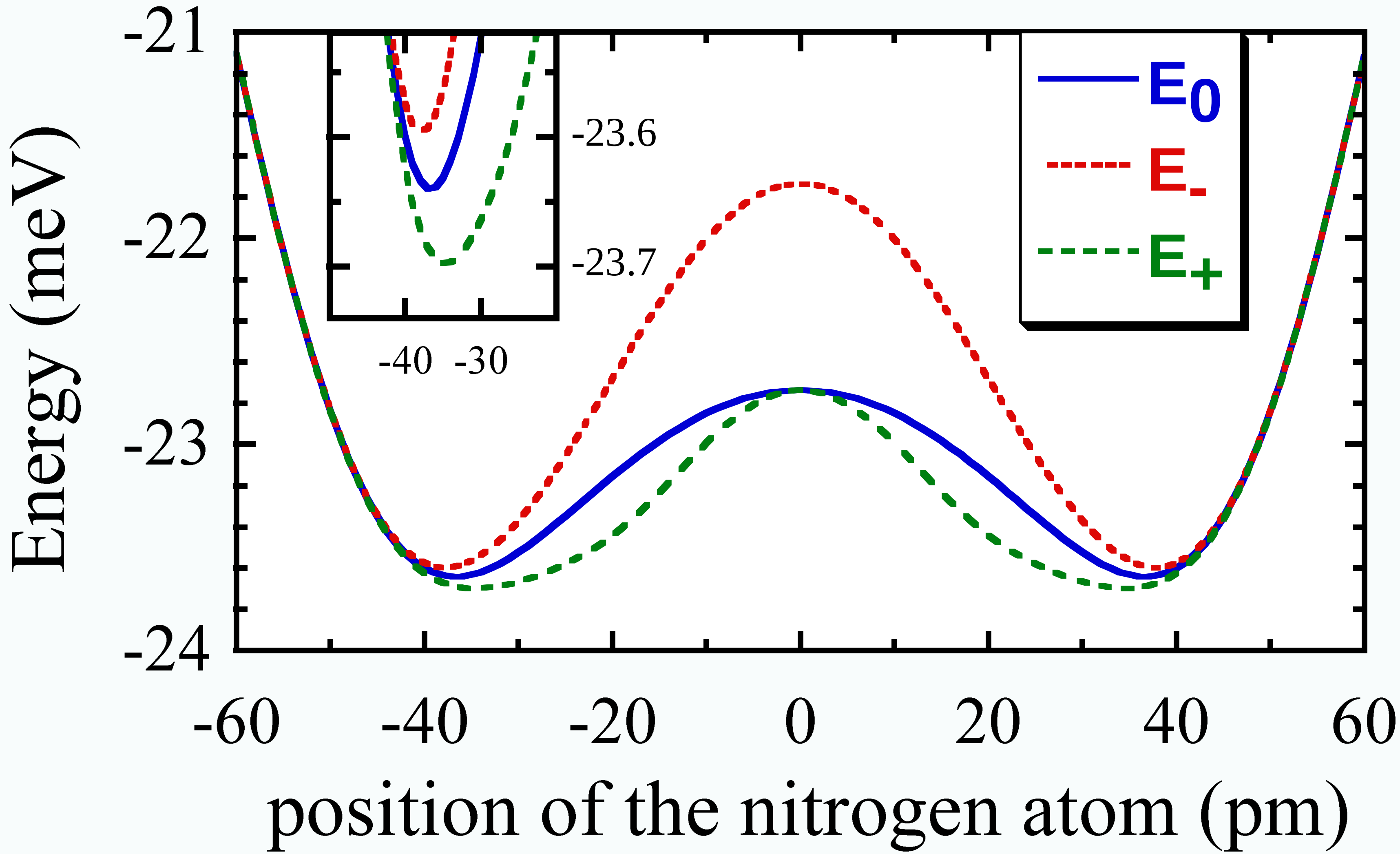
$$\boxed{|L,t\rangle = e^{iE_0t/\hbar}\left(\cos(\Delta t/\hbar)|L,0\rangle + i\sin(\Delta t/\hbar)|R,0\rangle\right)}$$

Ammonia molecule NH_3 - symmetry breaking



$$E_{\mathrm{NH}_3}(r_{\mathrm{NH}}, r_{\mathrm{HH}}) = 3\epsilon_{\mathrm{NH}}\left[\left(\frac{d_{\mathrm{NH}}}{r_{\mathrm{NH}}}\right)^{12} - 2\left(\frac{d_{\mathrm{NH}}}{r_{\mathrm{NH}}}\right)^{6}\right] + 3\epsilon_{\mathrm{HH}}\left[\left(\frac{d_{\mathrm{HH}}}{r_{\mathrm{HH}}}\right)^{12} - 2\left(\frac{d_{\mathrm{HH}}}{r_{\mathrm{HH}}}\right)^{6}\right]$$

Ammonia molecule NH_3 - symmetry restoration



Spontaneous symmetry breaking

Lessons learnt

Spontaneous symmetry breaking gives a description of the system in terms of wave packets instead of eigenstates. The wave packets corresponds to given configurations of constituents. If the configuration interaction energy is very small, the wave packets live a very long time and behave like classical objects. If configurations are orthogonal and degenerate, the wave packets are also eigenstates **(infinite systems)**.

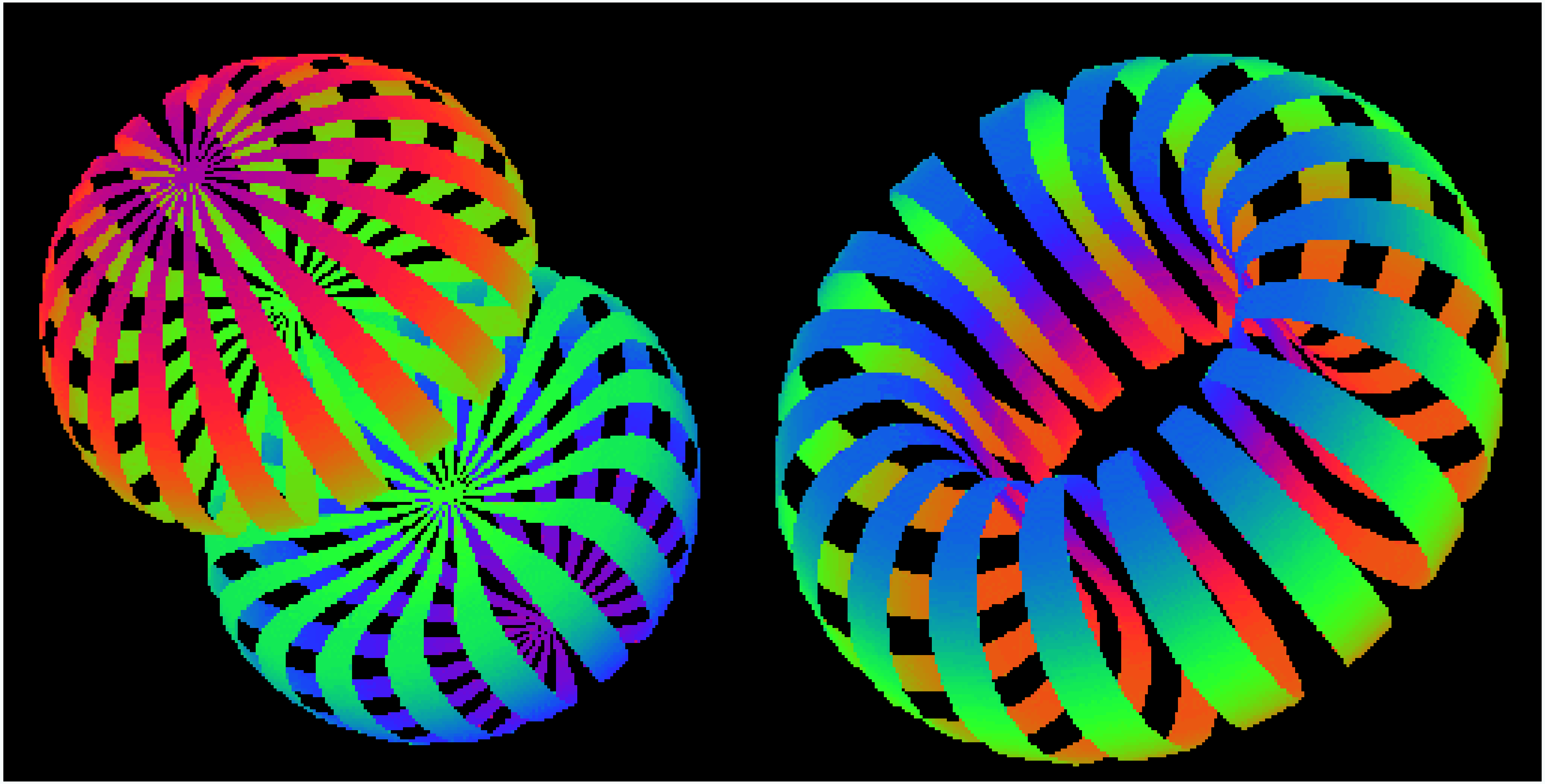
Symmetry restoration amounts to projecting states with good quantum numbers from the symmetry-breaking states. If the configuration interaction energy is very small, the energies of projected states are very close to those of the symmetry-breaking states. Symmetry restoration can be essential for calculating average values of symmetry-conserving observables other than the Hamiltonian. **After the symmetry restoration, the symmetry-breaking solutions do not break symmetry!**

Don’t let yourself confuse by the confusing traditional terminology

When you hear about: → **Think about:**

When you hear about:	Think about:
State in the <u>intrinsic</u> reference frame	State <u>before</u> the symmetry restoration
State in the <u>laboratory</u> reference frame	State <u>after</u> the symmetry restoration

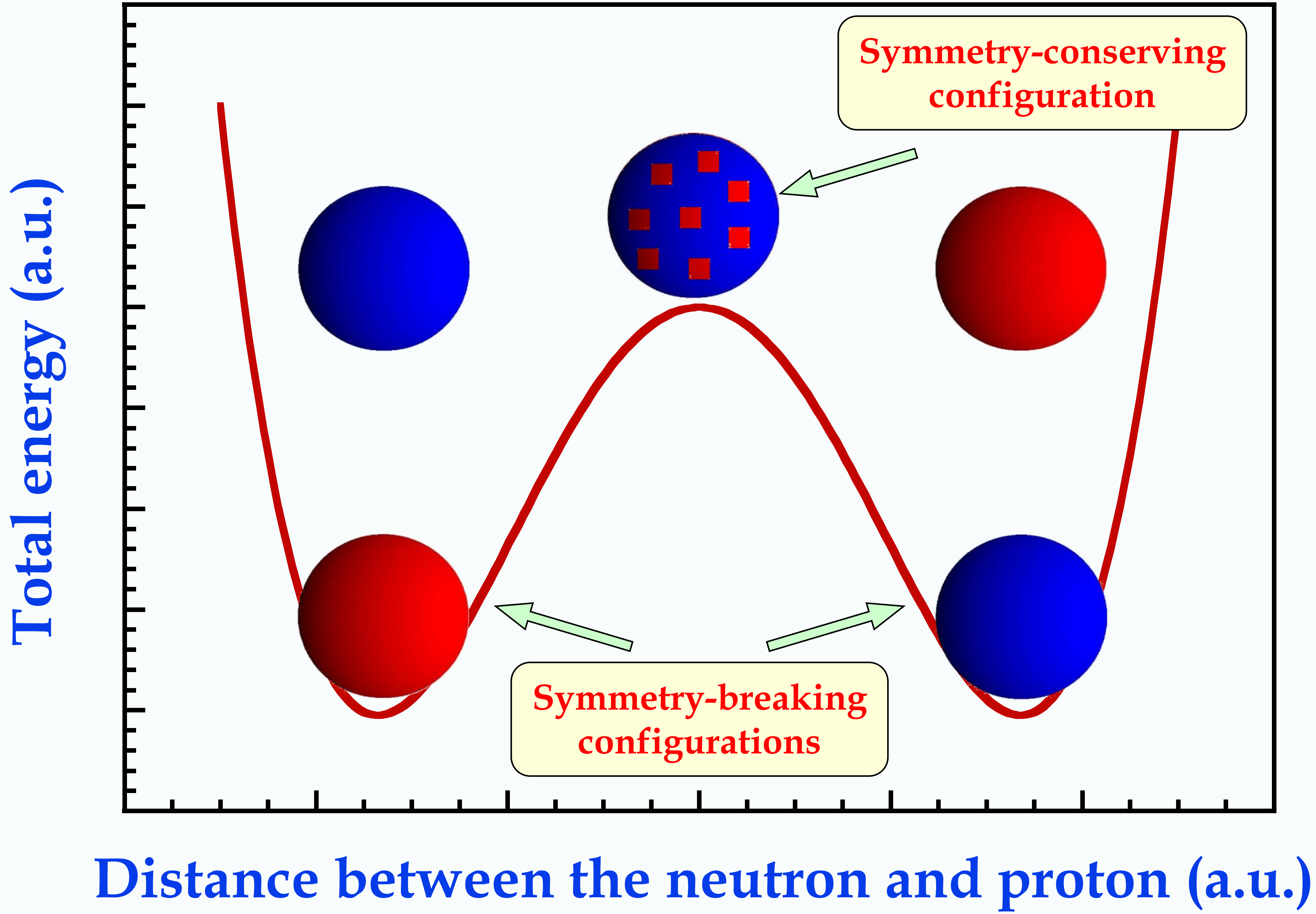
Structure of the deuteron



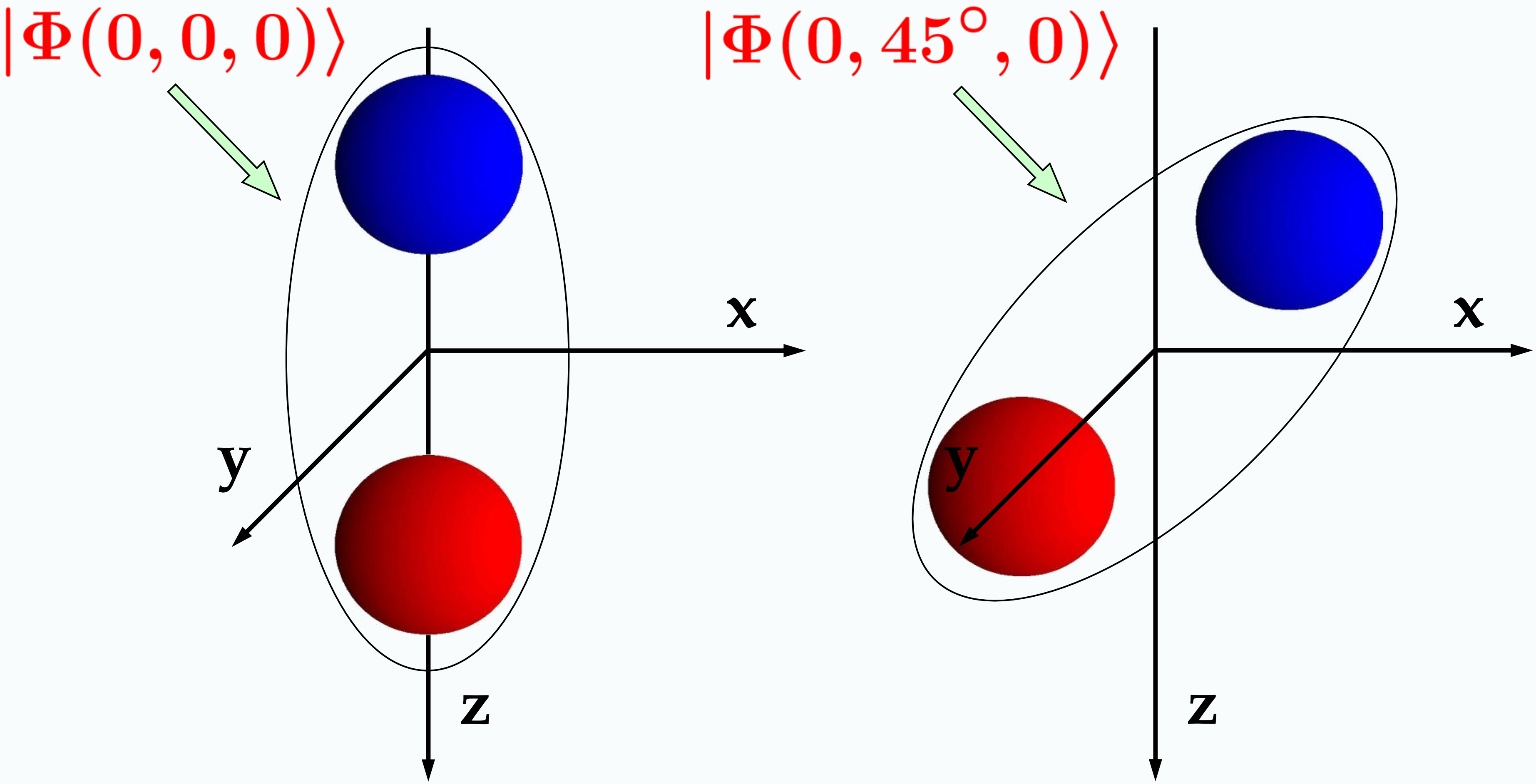
M=1 J=1 M=0

http://www.phy.anl.gov/theory/movie-run.html

Deuteron breaks the spherical symmetry



Restoration of the spherical symmetry

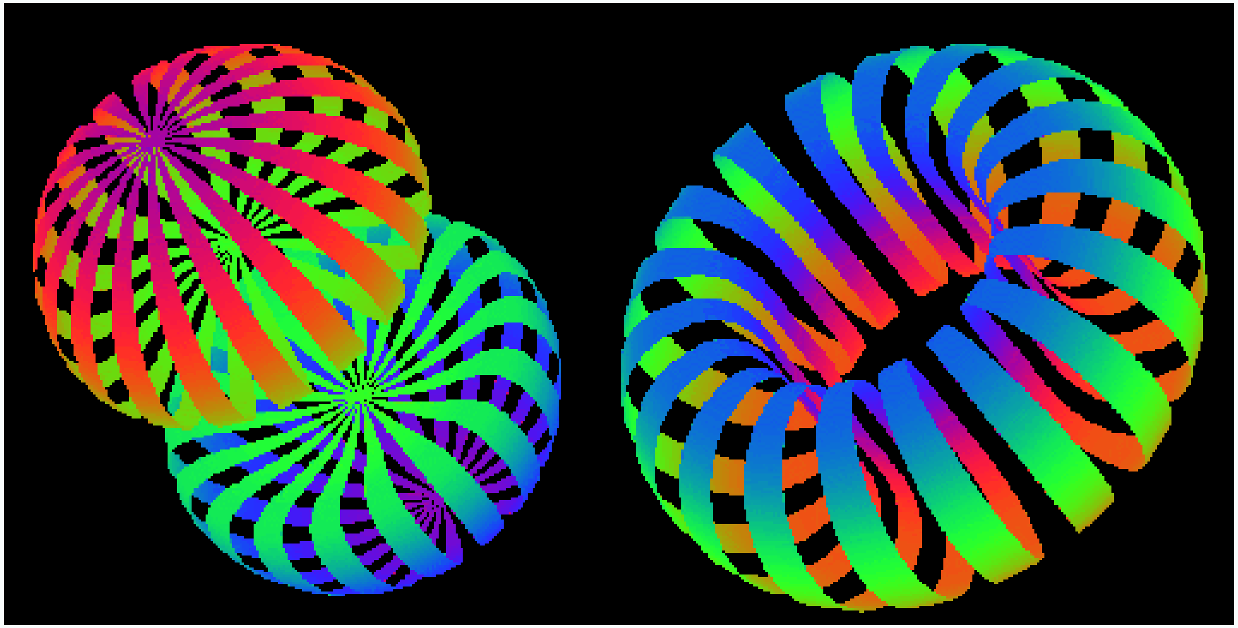


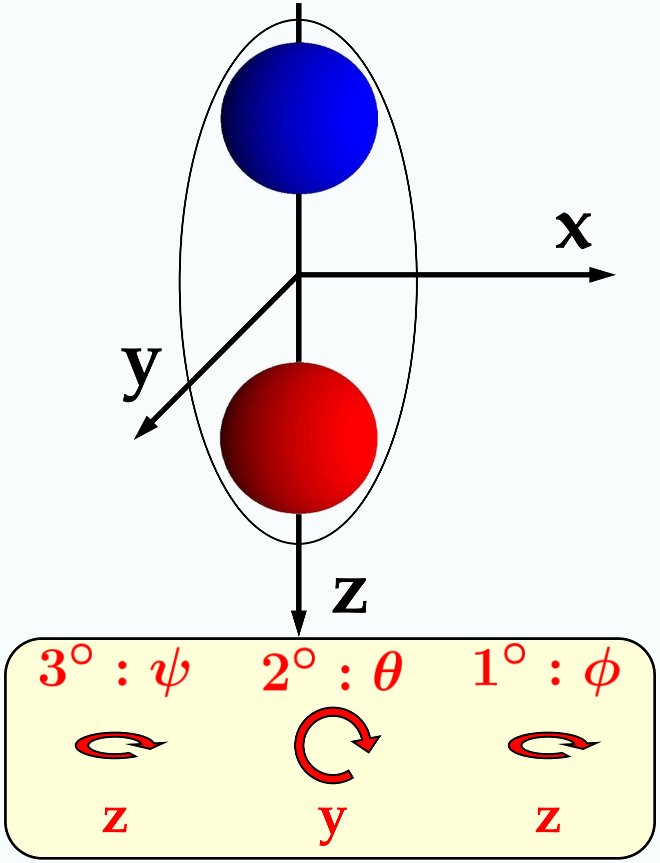
$$|\Psi^J_M\rangle = \int \mathrm{d}\psi\mathrm{d}\theta\mathrm{d}\phi\, D^{J*}_{MK}(\psi,\theta,\phi)\,|\Phi(\psi,\theta,\phi)\rangle$$

for the deuteron: $J=1, K=1$

Shapes of the deuteron

"Laboratory" frame **"Intrinsic" frame**





M=1 $\boxed{\mathbf{J=1}}$ **M=0**

$$|\Psi^1_1\rangle = \int \mathrm{d}\psi\mathrm{d}\theta\mathrm{d}\phi\, D^{1*}_{11}(\psi,\theta,\phi)|\Phi(\psi,\theta,\phi)\rangle \quad \text{for} \quad D^{1*}_{11} = e^{i\psi}\left(\frac{1+\cos\theta}{2}\right)e^{i\phi}$$

$$|\Psi^1_0\rangle = \int \mathrm{d}\psi\mathrm{d}\theta\mathrm{d}\phi\, D^{1*}_{01}(\psi,\theta,\phi)|\Phi(\psi,\theta,\phi)\rangle \quad \text{for} \quad D^{1*}_{01} = \left(\frac{\sin\theta}{\sqrt{2}}\right)e^{i\phi}$$

Shapes of the deuteron

Lessons learnt

Before the symmetry restoration the deuteron wave function is built of the proton wave function localized at a given point in space and a neutron wave function localized 2.3 fm away north, minus the same piece with the proton and neutron wave functions exchanged. This wave function does not have good angular momentum but represents a **wave packet with good orientation** angle towards one spontaneously chosen spatial direction.

After the symmetry restoration the deuteron M=1 wave function looks like a dumbbell and the M=0 wave function like a torus. These wave functions have good angular momentum J=1 but **undetermined orientation** angle in space. The symmetry axis of the wave function is just a quantization axis, which can be arbitrarily chosen in space.

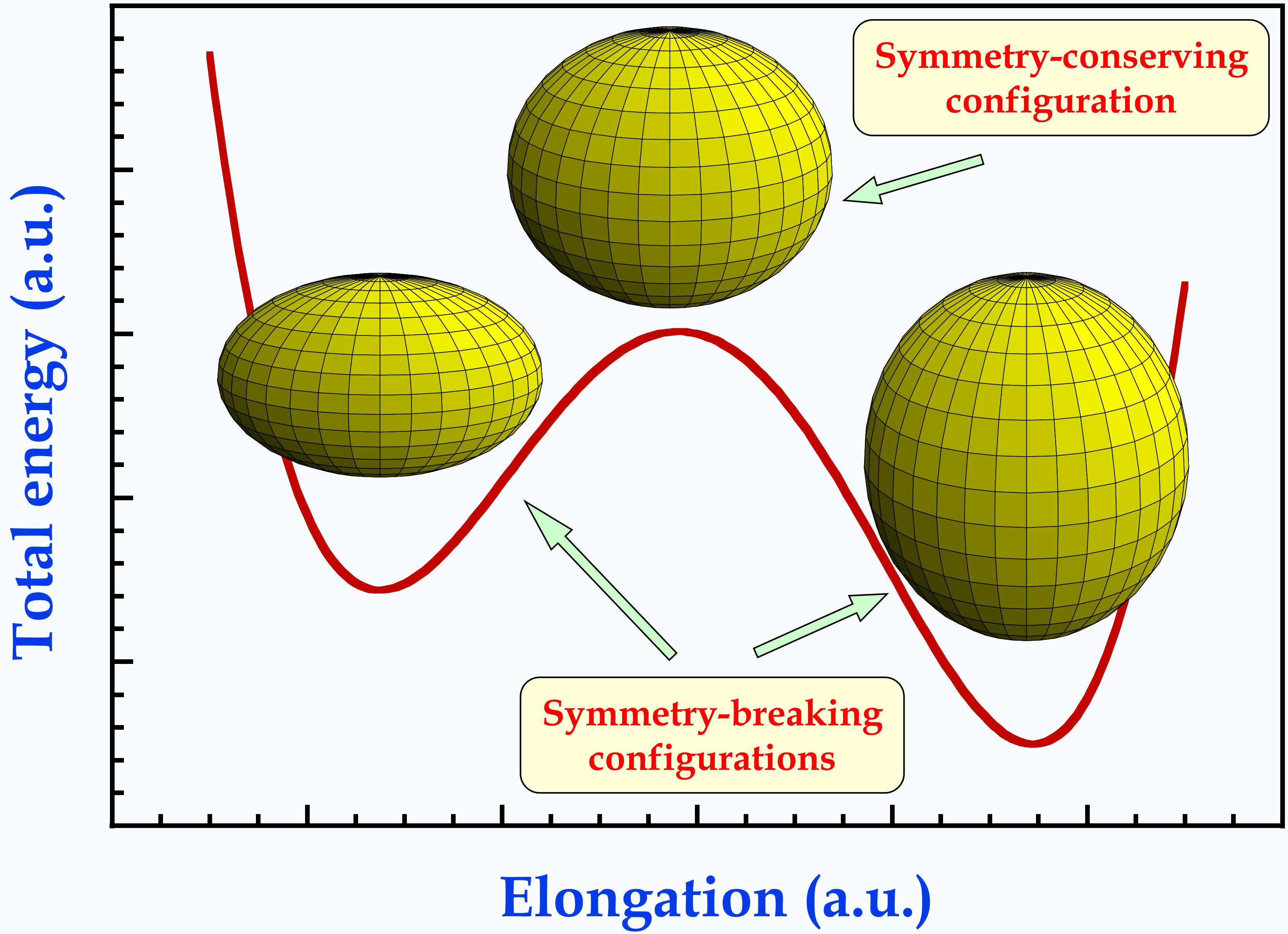
Don't let yourself confuse by the confusing traditional terminology

When you hear about: → **Think about:**

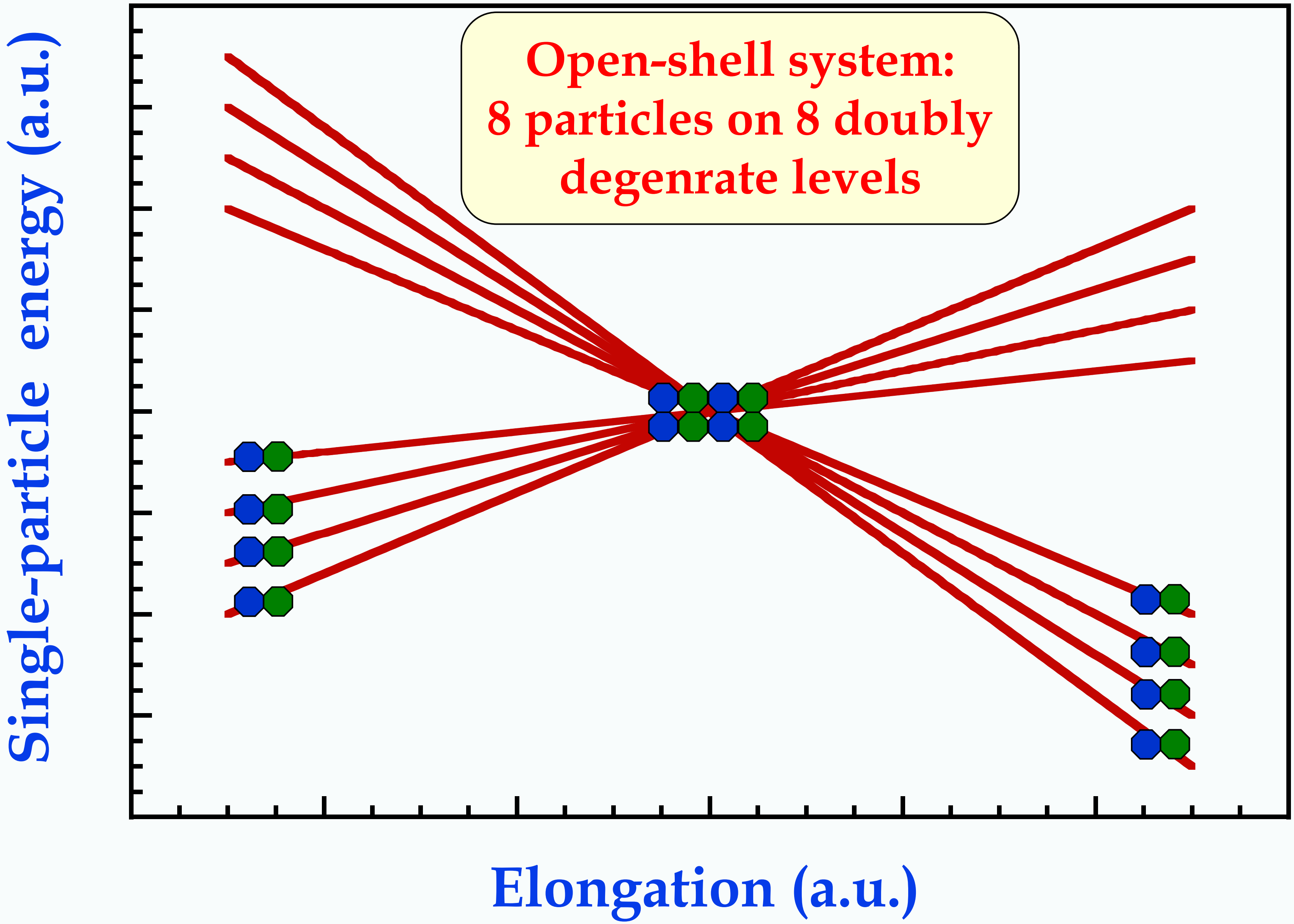
State in the intrinsic reference frame → State before the symmetry restoration

State in the laboratory reference frame → State after the symmetry restoration

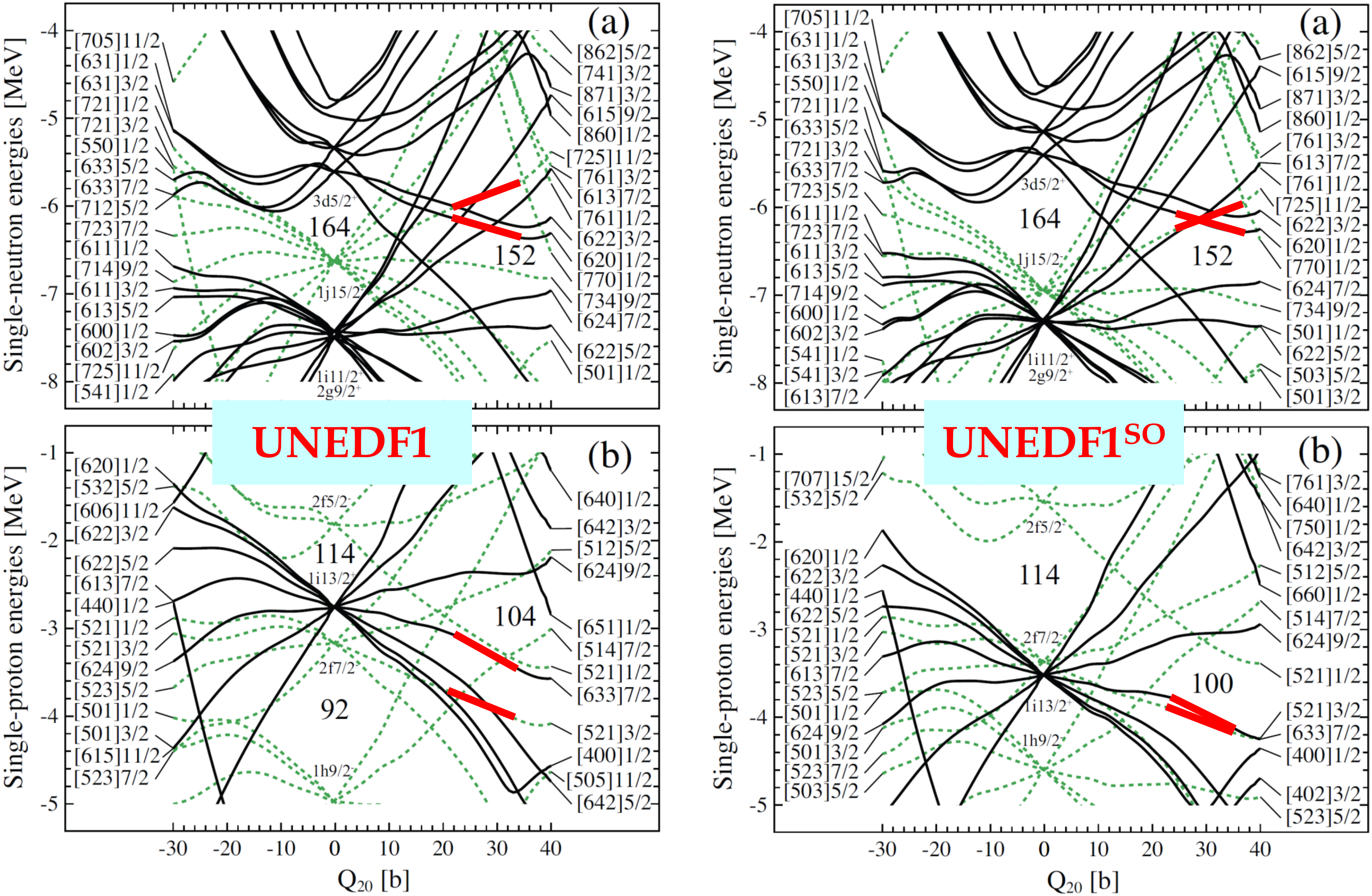
Nuclear deformation



Origins of nuclear deformation

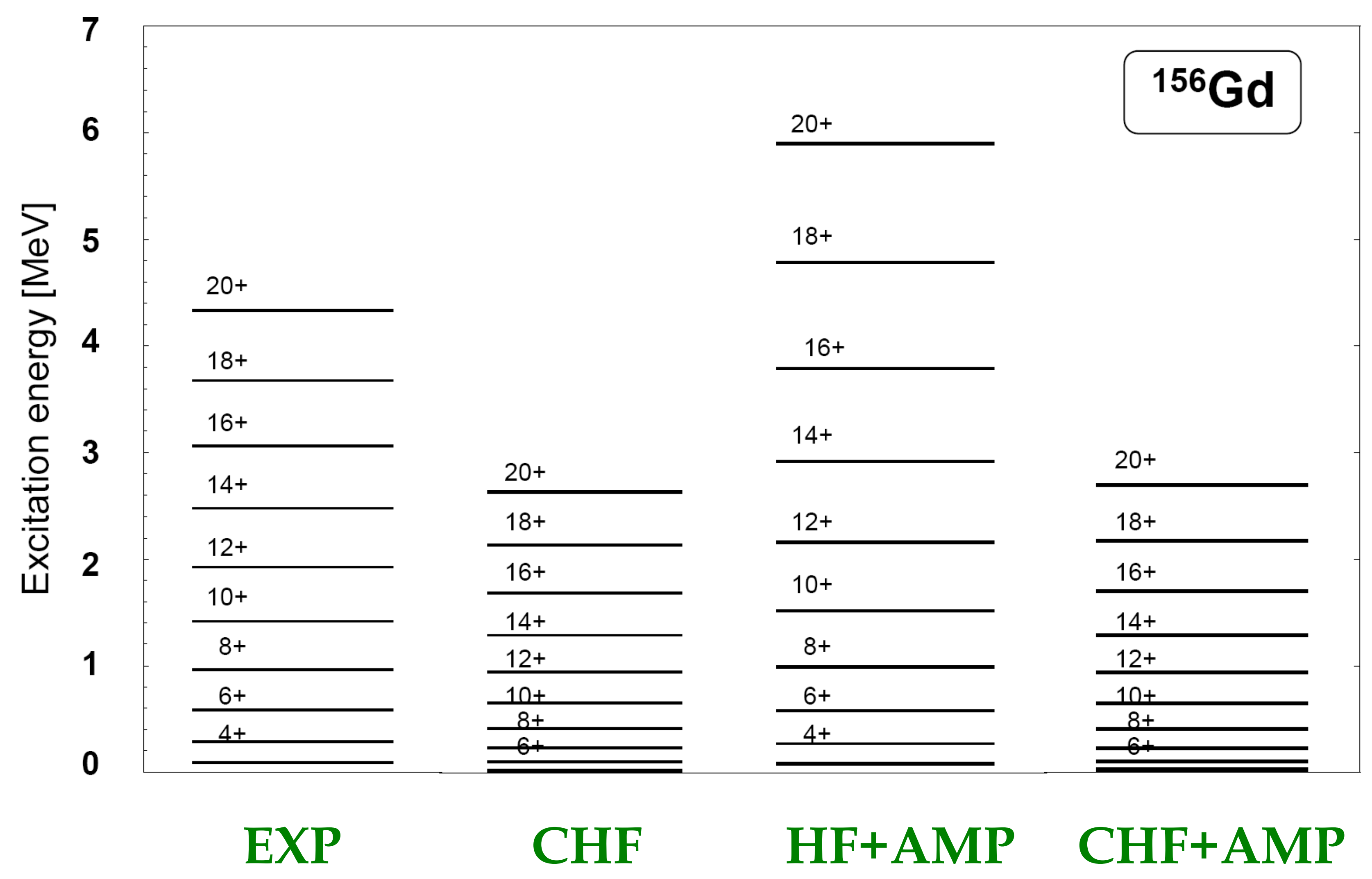


Nilsson diagrams in ^{254}No



Yue Shi, *et al.*, Phys. Rev. C89, 034309 (2014)

Angular-momentum projection



CHF = Cranked Hartree-Fock
AMP = Angular Momentum Projection

H. Zduńczuk, *et al.*, Int. Jour. Mod. Phys. E16, 377 (2007)

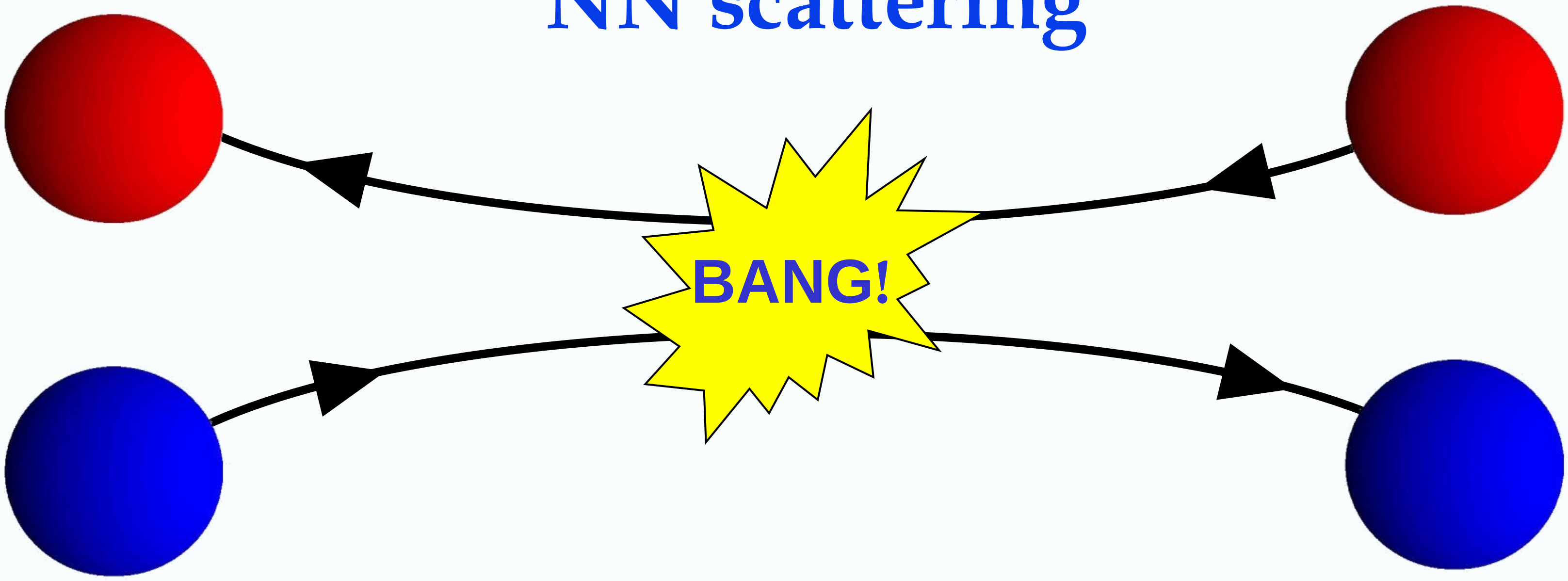
Nuclear deformation

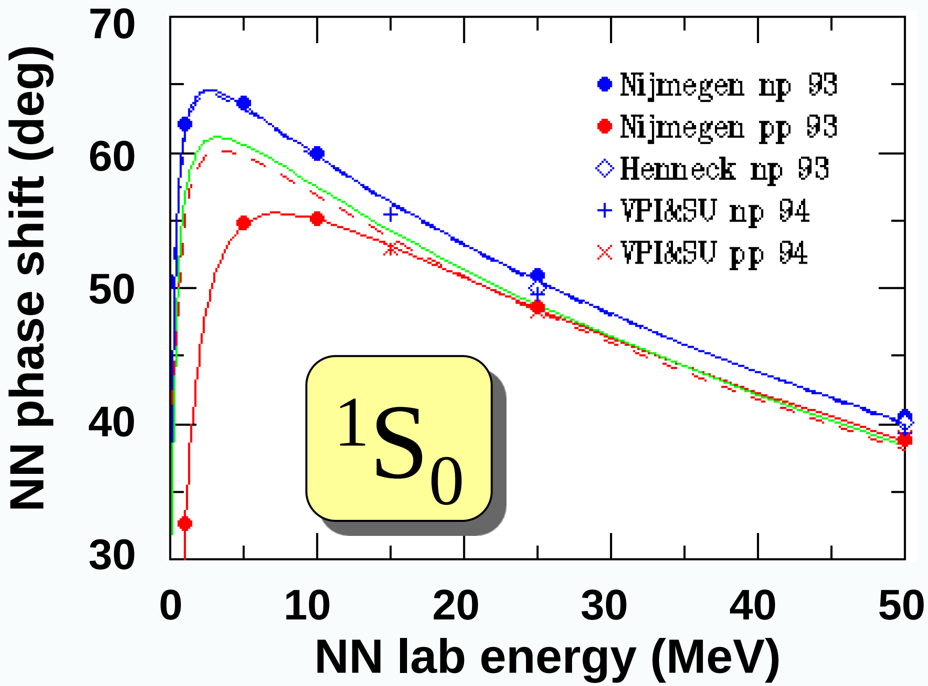
Lessons learnt

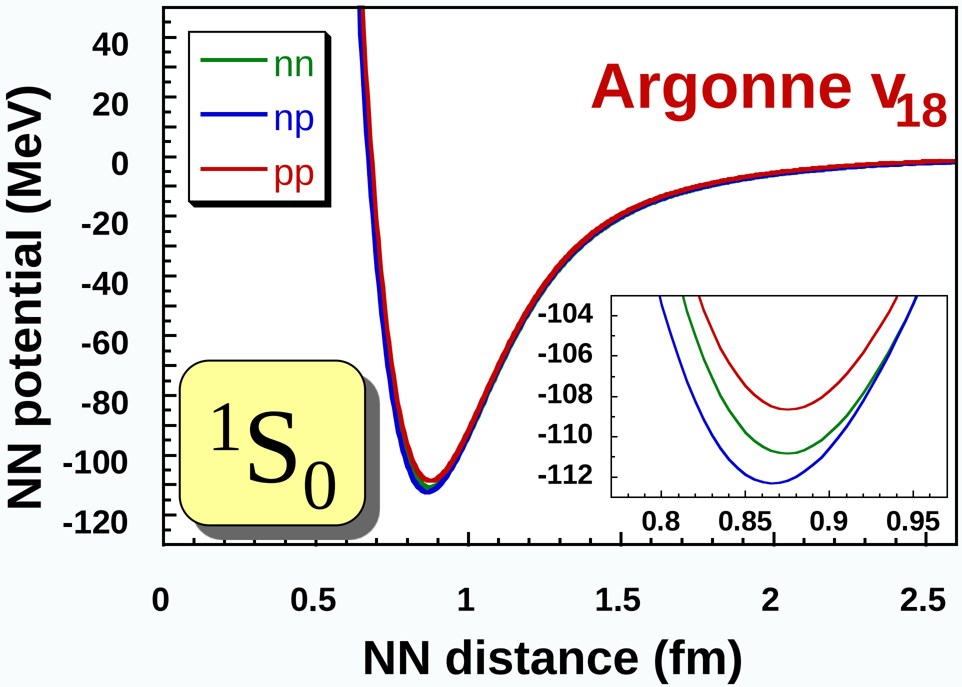
Nuclear deformation results form residual two-body interactions between valence nucleons that favor configurations in which nucleons occupy single-particle orbitals in a deformed mean field. Prolate deformations are preferred at the beginning of large shells (almost empty shells) and oblate deformations at the and of large shells (almost full shells), although for detailed and realistic situations the prolate ones appear more often in Nature. Deformed wave functions do not have good angular momenta and represent wave packets of very large widths (very many angular-momentum components). The average angular momentum squared is always large, while the average angular momentum can be zero (non-rotating wave packets) or non-zero (rotating wave packets). In the cranking approximation a rotating wave packet is stationary in the rotating reference frame.

Nucleon-nucleon interactions

NN scattering







Argonne v18 interaction

$$V(NN) = V^{\mathrm{EM}}(NN) + V^{\pi}(NN) + V^{\mathrm{R}}(NN)$$

- R.B. Wiringa, V.G.J. Stoks, and R. Schiavilla, Phys. Rev. C51, 38 (1995)
- Electromagnetic part: EM
- One-pion-exchange part: π
- Intermediate and short-range phenomenological part: R

$$V^{\pi}(pp) = f_{pp}^{2}\frac{1}{3}m_{\pi}\left[Y(r)\sigma_i\cdot\sigma_j + T(r)S_{ij}\right]$$

$$Y(r) = \frac{e^{-m_\pi r}}{r}\left(1 - e^{-cr^2}\right)$$

$$T(r) = \left(1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2}\right)\frac{e^{-m_\pi r}}{r}\left(1 - e^{-cr^2}\right)^2$$

$$V^{\mathrm{R}} = V^{\mathrm{c}} + V^{\mathrm{l2}}L^2 + V^{\mathrm{t}}S_{12} + V^{\mathrm{ls}}L\cdot S + V^{\mathrm{ls2}}(L\cdot S)^2$$

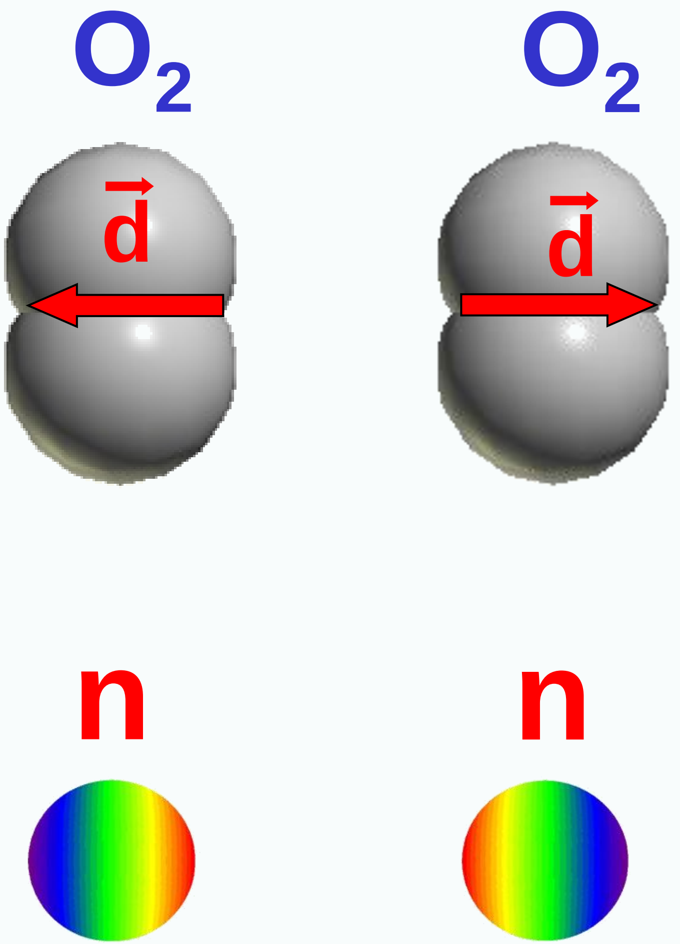
$$V^{\mathrm{i}}(r) = I^{\mathrm{i}}T^{2}(r) + \left[P^{\mathrm{i}} + m_\pi r Q^{\mathrm{i}} + (m_\pi r)^2 R^{\mathrm{i}}\right]\left(1 + e^{(r-r_0)/a}\right)^{-1}$$

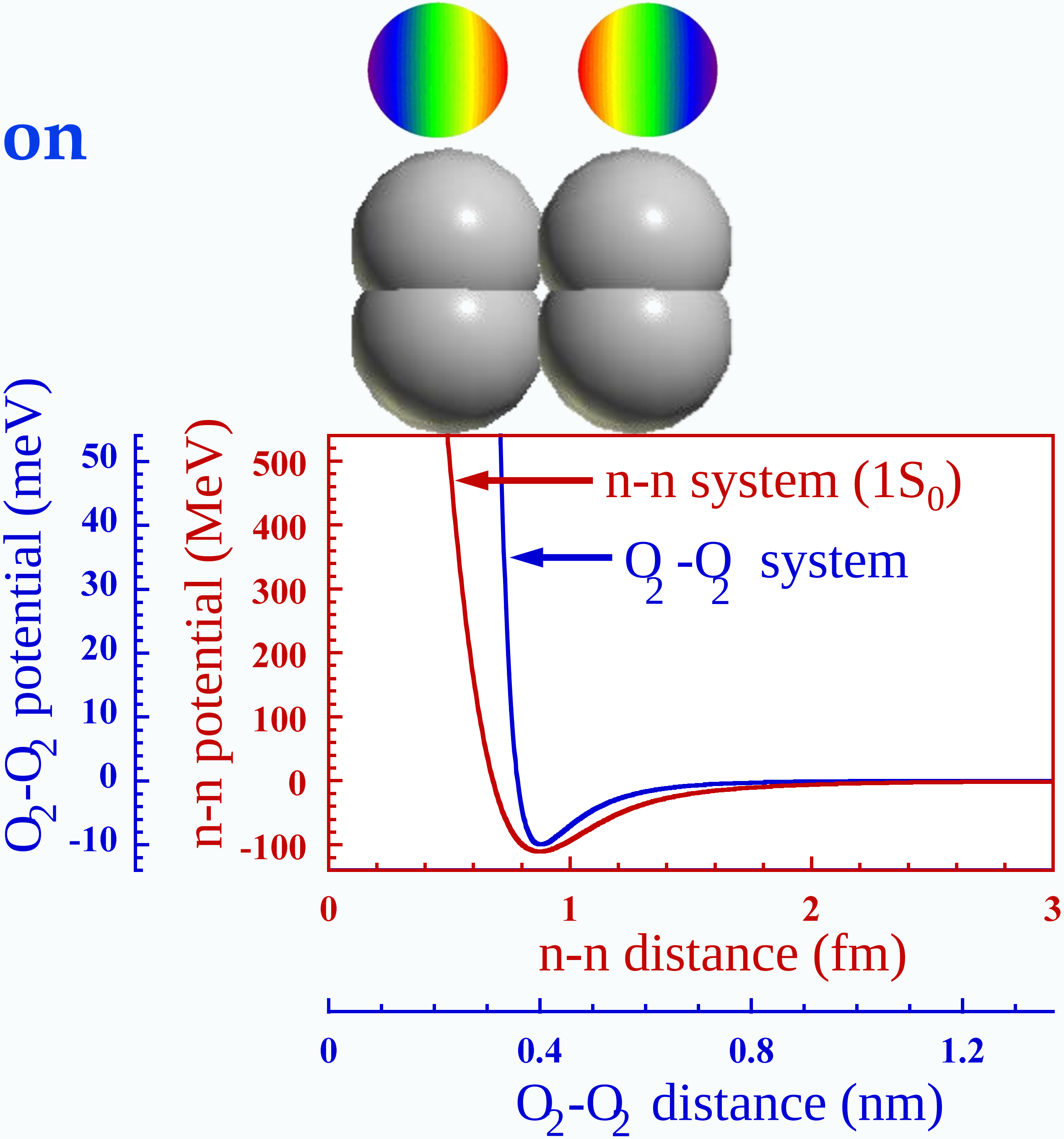
NN scattering

Lessons learnt

At low energies below 300 MeV, the quark structures of nucleons are not resolved and interactions between the nucleons can be approximated by potentials that have long attractive tails and strong repulsive cores. These potentials are best adjusted to the NN scattering data (phase shifts) in specific channels defined by L, S, and J, described by the spectroscopic notation of $^{2S+1}L_J$. The Pauli principle requires that the isospin of each channel be confined to T=0 for odd J and T=1 for even J. Interactions are very weakly depending on the isospin – mostly, but not uniquely, through the Coulomb force. Channels of J>0 mix two values of L=J±1, and therefore potentials in these channels are 2×2 matrices. Therefore, e.g., the deuteron wave function is a mixture of the L=0 (S) and L=2 (D) components.

n-n versus O_2-O_2 interaction





n-n versus O_2-O_2 interaction

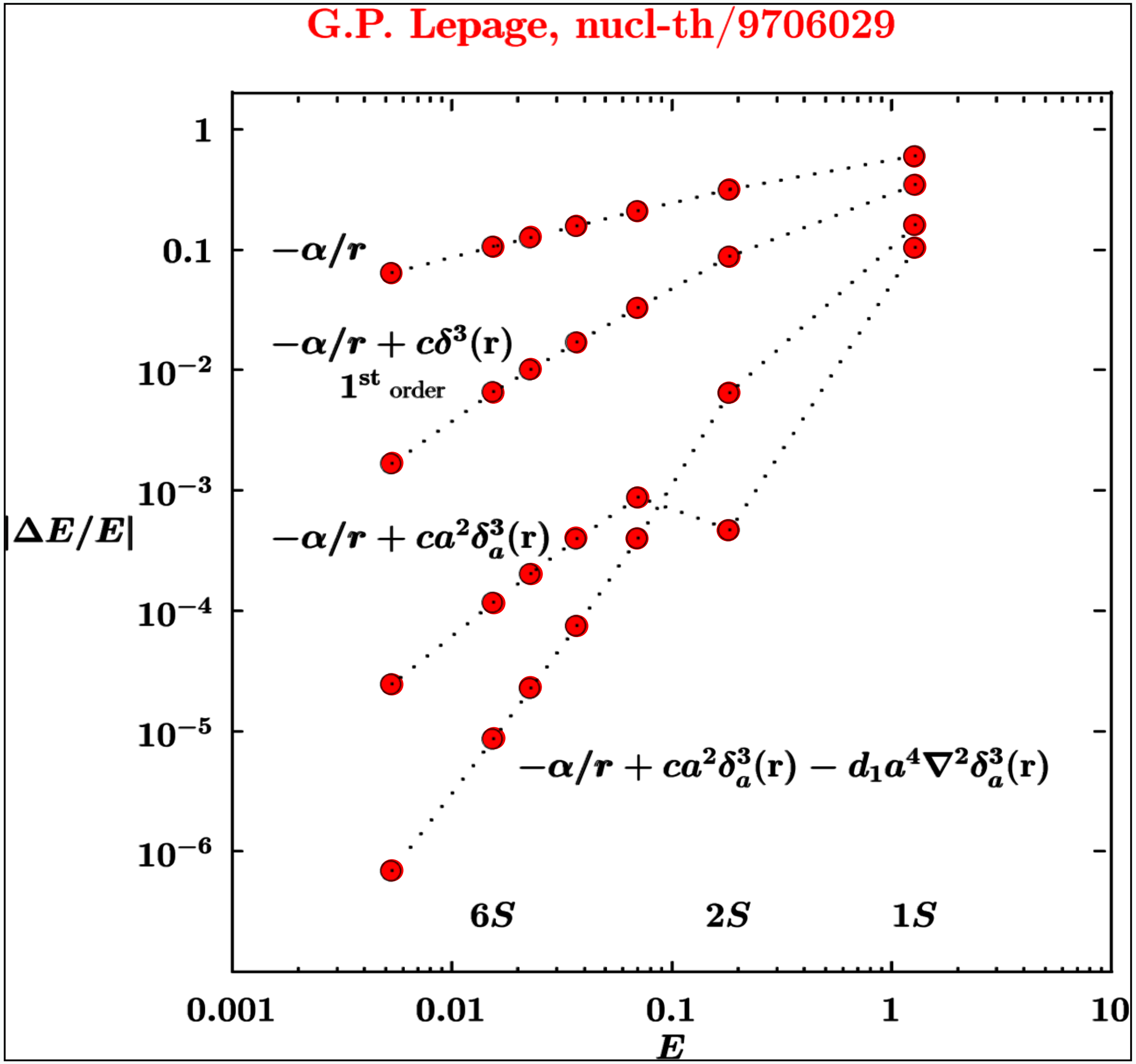
Lessons learnt

Neutrons are white (color singlets). When two are brought together they **attract** each other through the Yukawa force ($\sim e^{-r}/r$), (one-pion exchange) and higher forces (two-pion exchanges). At smaller distances, heavier mesons are exchanged and the Pauli blocking sets in, which can be modeled by adding a phenomenological **repulsive** hard core – the Argonne v_{18} **potential**. At low energies, the interaction can be approximated by a potential without any reference to the meson exchanges. Can this potential be expressed as resulting from polarized **color/flavor distributions** (nuclear Van der Waals force)?

O_2 molecules are neutral (have zero net charge) and non-polar (have zero dipole moment). When two are brought together they polarize each other and **attract** through a dipole-dipole interaction $E \cdot d$ where E ($\sim 1/r^3$) induces d ($\sim 1/r^3$), and hence the resulting Van der Waals force decreases as ($\sim 1/r^6$). At smaller distances, higher multipoles and Pauli blocking set in, which can be modeled by an ad hoc **repulsive** term ($\sim 1/r^{12}$) - the Lennard-Jones **potential**. EM interactions result from exchanging photons, but at low energies they can be approximated by the Coulomb force acting between the **charge distributions**.

Effective theories

Hydrogen atom perturbed near the center



Relative errors in the S-wave binding energies are plotted versus:

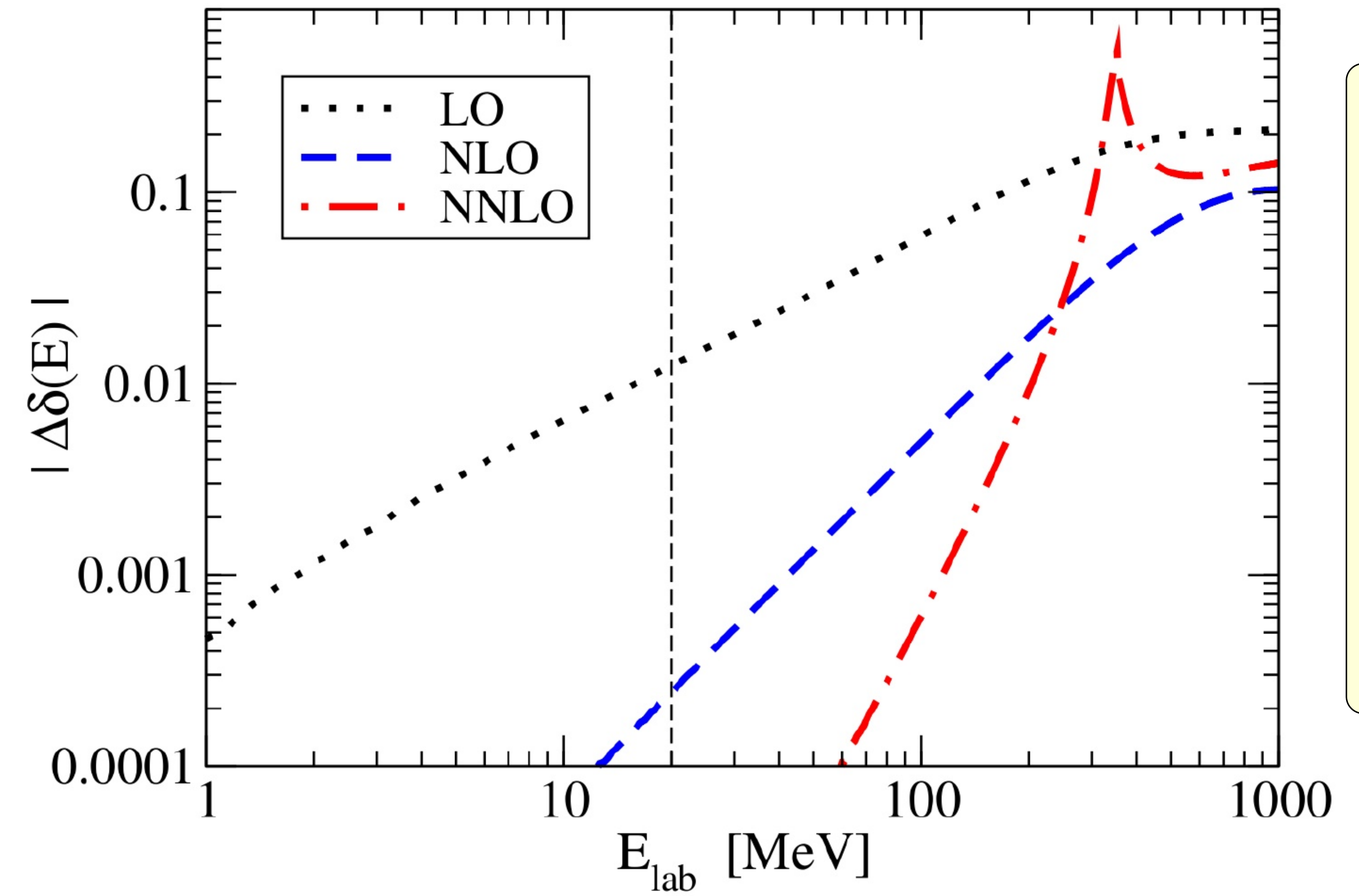
(i) the binding energy for the Coulomb theory

(ii) the Coulomb theory augmented with a delta function in first-order perturbation theory

(iii) the non-perturbative effective theory through a^2, and

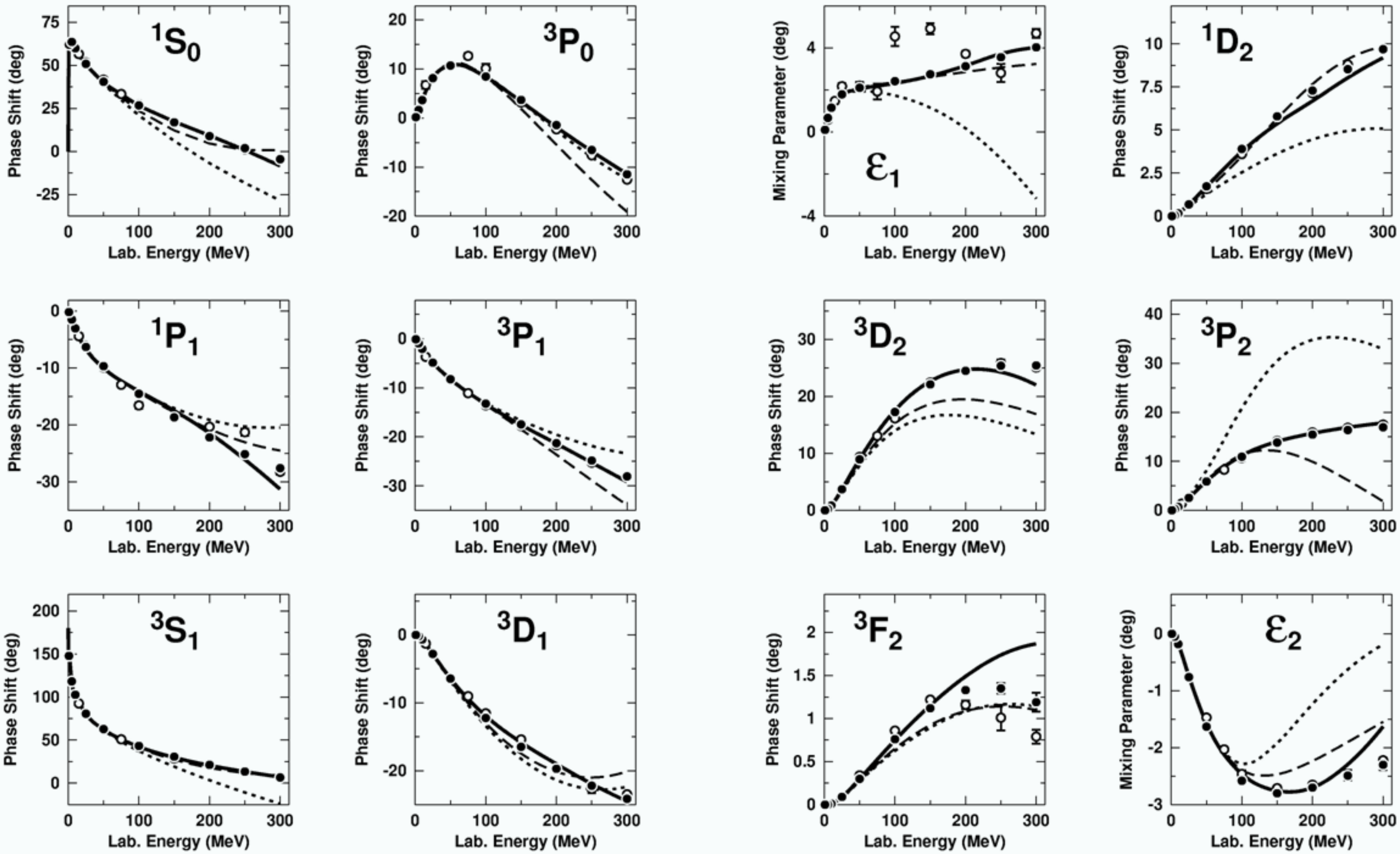
(iv) the effective theory through a^4.

Phase shifts in the low-momentum expansion



Evgeny Epelbaum, arXiv:1001.3229

EFT phase-shift analysis



np phase parameters below 300 MeV lab. energy for partial waves with J=0,1,2. The solid line is the result at N^{3}LO. The dotted and dashed lines are the phase shifts at NLO and NNLO, respectively, as obtained by Epelbaum *et al*. The solid dots show the Nijmegen multi-energy np phase shift analysis and the open circles are the VPI single-energy np analysis SM99.

D.R. Entem and R. Machleidt, Phys. Rev. C68 (2003) 041001

EFT phase-shift analysis

χ^2/datum for the reproduction of the 1999 *np* database [1] below 290 MeV by various *np* potentials.

Bin (MeV)	# of data	N^3LO^a	$NNLO^b$	NLO^b	$AV18^c$
0–100	1058	1.06	1.71	5.20	0.95
100–190	501	1.08	12.9	49.3	1.10
190–290	843	1.15	19.2	68.3	1.11
0–290	2402	1.10	10.1	36.2	1.04

^{a}This work. ^{b}Ref. [2]. ^{c}Ref. [3].

χ^2/datum for the reproduction of the 1999 *pp* database [1] below 290 MeV by various *pp* potentials.

Bin (MeV)	# of data	N^3LO^a	$NNLO^b$	NLO^b	$AV18^c$
0–100	795	1.05	6.66	57.8	0.96
100–190	411	1.50	28.3	62.0	1.31
190–290	851	1.93	66.8	111.6	1.82
0–290	2057	1.50	35.4	80.1	1.38

^{a}This work. ^{b}See footnote [4]. ^{c}Ref. [3].

[1] The 1999 NN data base is defined in R. Machleidt, Phys. Rev. C 63, 024001 (2001).

[2] E. Epelbaum *et al.*, Eur. Phys. J. A15, 543 (2002).

[3] R. B. Wiringa *et al.*, Phys. Rev. C 51, 38 (1995).

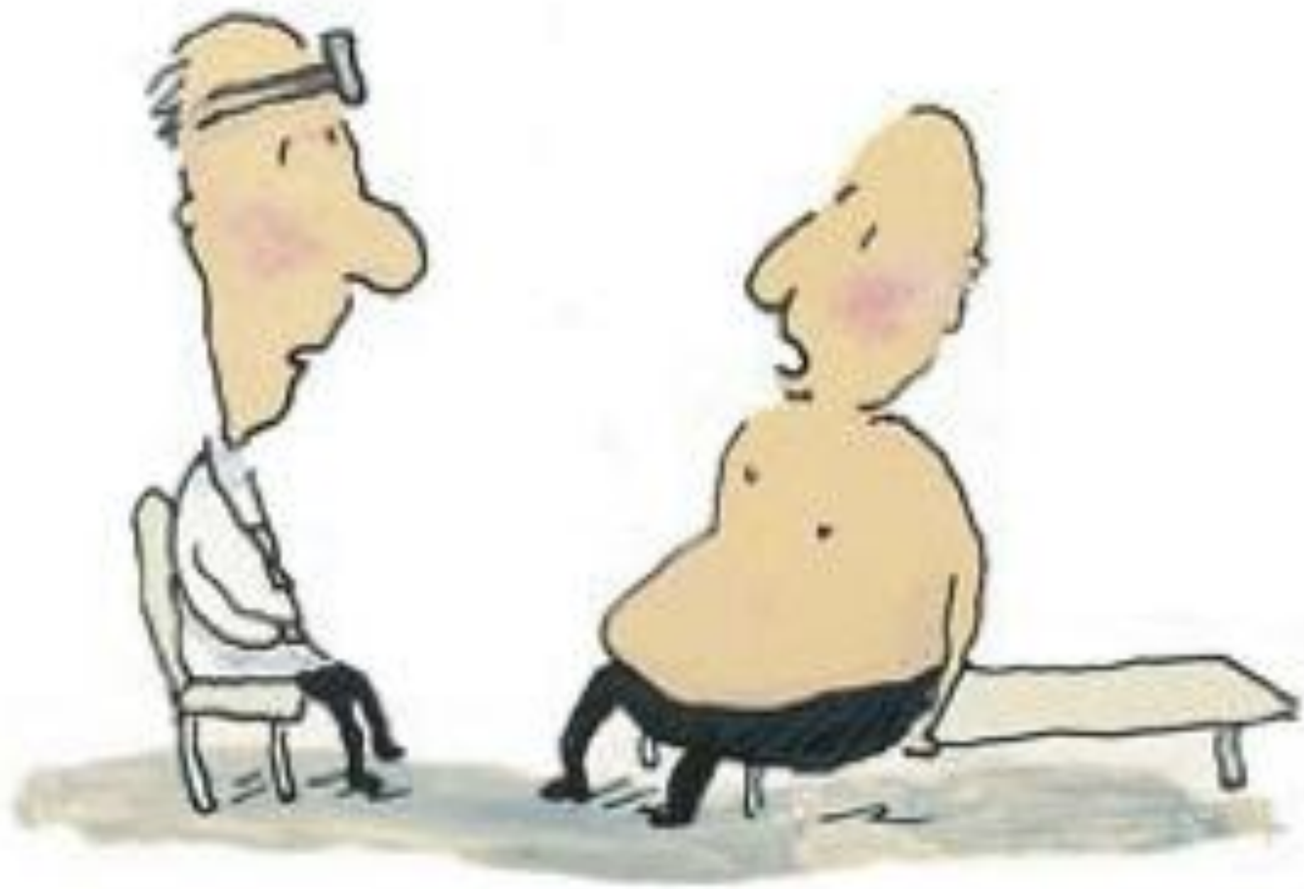
[4] Since Ref. [2] provides only the *np* versions of the NLO and NNLO potentials, we have constructed the *pp* versions by incorporating charge-dependence and minimizing the *pp* χ^2.

D.R. Entem and R. Machleidt, Phys. Rev. C68 (2003) 041001

Nuclear energy density functionals

Weinberg's Laws of Progress in Theoretical Physics

'e, MIT Press, 1983)



Patient: Doctor, doctor, it hurts when I do this!

Doctor: Then don't do that.

' *will get*

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ng ones, you'll be

Henry Youngman
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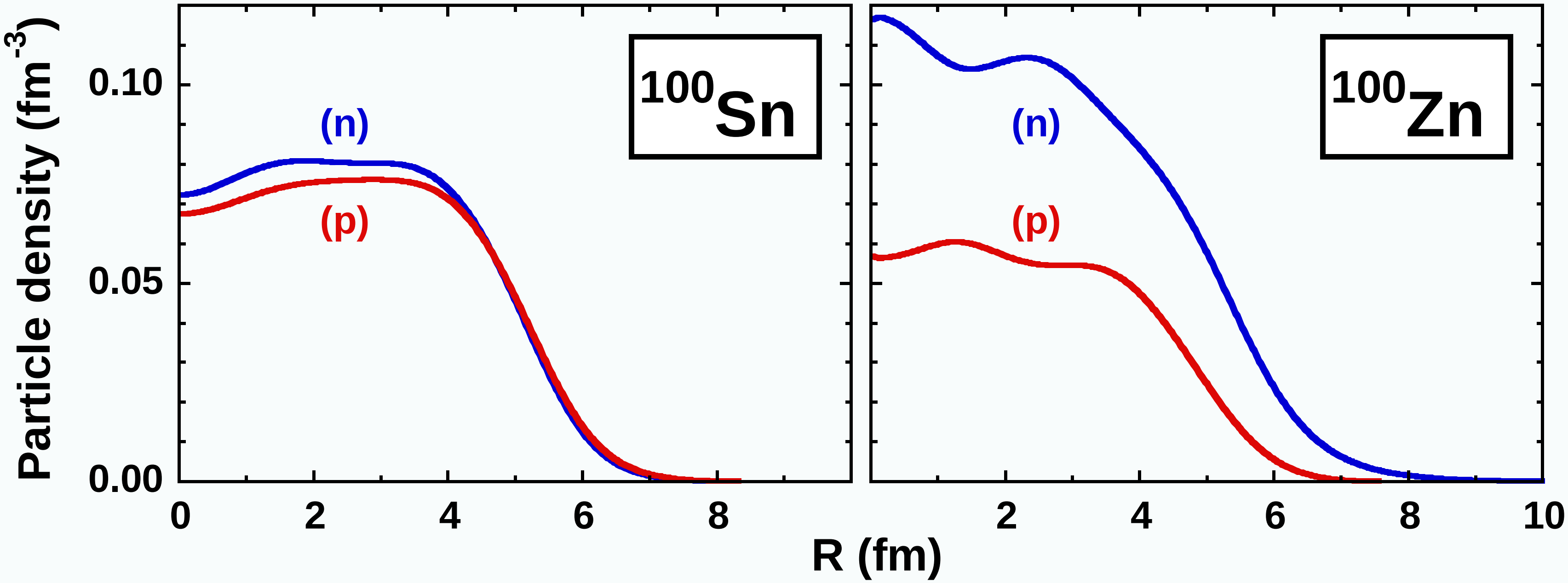
Slide by Witek Nazarewicz

Nuclear densities as composite fields



Modern Mean-Field Theory ≡ Energy Density Functional

ρ, τ, $\overleftrightarrow{J}$, $\vec{j}$, $\vec{T}$, $\vec{s}$, $\vec{F}$,

- Hohenberg-Kohn
- Kohn-Sham
- Negele-Vautherin
- Landau-Migdal
- Nilsson-Strutinsky

mean field ⇒ one-body densities

zero range ⇒ local densities

finite range ⇒ non-local densities

Rayleigh-Ritz Variational Principle

$$
\begin{aligned}
\hat{H}|\Psi_i\rangle &= E_i|\Psi_i\rangle \\
&\Downarrow \\
|\Psi\rangle &= a_0|\Psi_0\rangle + a_1|\Psi_1\rangle + a_2|\Psi_2\rangle + \ldots \\
\langle\Psi|\hat{H}|\Psi\rangle &= E_0|a_0|^2 + E_1|a_1|^2 + E_2|a_2|^2 + \ldots \\
&\Downarrow \\
\min_{a_0,a_1,a_2\ldots}\langle\Psi|\hat{H}|\Psi\rangle &= E_0 \quad \Longleftarrow \quad \text{Rayleigh-Ritz variational principle} \\
&\Downarrow \\
\min_{\alpha}\langle\Phi(\alpha)|\hat{H}|\Phi(\alpha)\rangle &= E_0^{\text{var}} \geq E_0 \quad \Longleftarrow \quad \text{variational approximation}
\end{aligned}
$$

What is DFT?

Density Functional Theory:

A variational method that uses observables as variational parameters.

$$\delta\langle\hat{H} - \lambda\hat{Q}\rangle = 0$$
$$\Downarrow$$
$$E = E(Q)$$

for $E(\lambda) \equiv \langle\hat{H}\rangle$ and $Q(\lambda) \equiv \langle\hat{Q}\rangle$

J. D., J. Phys.: Conf. Ser. 312, 092002 (2011)

What is the DFT good for?

$$\delta\langle\hat{H} - \lambda\hat{Q}\rangle = 0$$

$$\Downarrow$$

$$E = E(Q)$$

Energy E is a function(al) of Q

1) **Exact:** Minimization of E(Q) gives the exact E and exact Q
2) **Impractical:** Derivation of E(Q) requires the full variation δ (bigger effort than to find the exact ground state)
3) **Inspirational:** Can we build useful models E′(Q) of the exact E(Q)?
4) **Experiment-driven:** E′(Q) works better or worse depending on the physical input used to build it.

Which DFT?

$$\delta\langle \hat{H} - \lambda \hat{Q}\rangle = 0 \implies E = E(Q)$$

$$\delta\langle \hat{H} - \textstyle\sum_k \lambda_k \hat{Q}_k\rangle = 0 \implies E = E(Q_k)$$

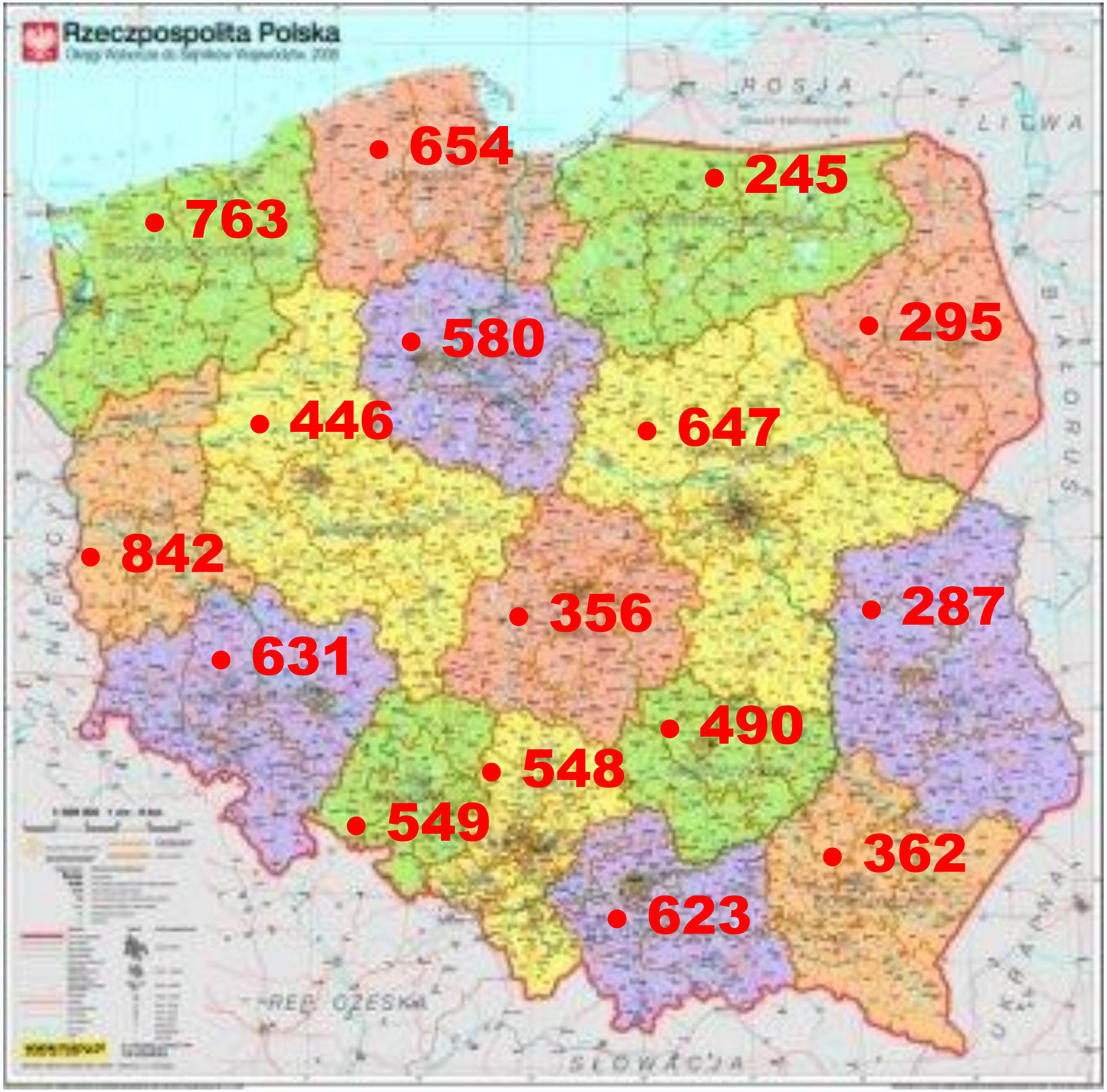
$$\delta\langle \hat{H} - \int \mathrm{d}q\, \lambda(q)\hat{Q}(q)\rangle = 0 \implies E = E[Q(q)]$$

$$\delta\langle \hat{H} - \int \mathrm{d}\vec{r}\, \lambda(\vec{r})\hat{\rho}(\vec{r})\rangle = 0 \implies E = E[\rho(\vec{r})]$$

$$\text{for} \quad \hat{\rho}(\vec{r}) \;=\; \textstyle\sum_{i=1}^{A} \delta(\vec{r} - \vec{r}_i)$$

$$\delta\langle \hat{H} - \int\!\!\int \mathrm{d}\vec{r}\mathrm{d}\vec{r}\,'\, \lambda(\vec{r},\vec{r}\,')\hat{\rho}(\vec{r},\vec{r}\,')\rangle = 0 \implies E = E[\rho(\vec{r},\vec{r}\,')]$$

J. D., J. Phys.: Conf. Ser. 312, 092002 (2011)



Price of land in Poland per district

Price district functional

Energy density functional

Price of land in Poland per county

Price county functional

Energy density functional

Price of land in Eurpe per country

Price country functional

Energy density functional

How the nuclear EDF is built?

$$E[\rho(\vec{r}_1,\vec{r}_2)] \;=\; \int\!\!\int \mathrm{d}\vec{r}_1 \mathrm{d}\vec{r}_2\, \mathcal{H}(\rho(\vec{r}_1,\vec{r}_2))$$

Energy Density Functional (EDF)

Energy Density

$$\boxed{\mathcal{H}(\rho(\vec{r}_1,\vec{r}_2)) \;=\; V(\vec{r}_1-\vec{r}_2)\Big[\rho(\vec{r}_1)\rho(\vec{r}_2) - \rho(\vec{r}_1,\vec{r}_2)\rho(\vec{r}_2,\vec{r}_1)\Big]}$$

Direct

Exchange

How the nuclear EDF is built?

$$E'[\rho(\vec{r})] = \int \mathrm{d}\vec{r}\, \mathcal{H}(\rho(\vec{r}))$$

LDA

Local energy density is a function of local density

$$E'[\rho(\vec{r},\vec{r}\,')] = \int\!\!\int \mathrm{d}\vec{r}\,\mathrm{d}\vec{r}\,'\, \mathcal{H}(\rho(\vec{r},\vec{r}\,'))$$

Gogny, M3Y,...

Non-local energy density is a function of non-local density

$$\mathcal{H}(\rho(\vec{r},\vec{r}\,')) = V(\vec{r}-\vec{r}\,')\Big[\rho(\vec{r})\rho(\vec{r}\,') - \rho(\vec{r},\vec{r}\,')\rho(\vec{r}\,',\vec{r})\Big]$$

How the nuclear EDF is built?

$$E' = \int d\vec{r}\, \mathcal{H}(\rho(\vec{r}), \tau(\vec{r}), \Delta\rho(\vec{r}), \ldots)$$

Quasi-local energy density is a function of local densities and gradients

Skyrme, BCP, point-coupling,...

$$E' = \int\int d\vec{r} d\vec{r}'\, \mathcal{H}(\rho(\vec{r}), \rho(\vec{r}'))$$

Non-local energy density is a function of local densities

RMF (Hartree)

Nuclear densities as composite fields

Density matrix:

$$\rho(\vec{r}\sigma,\vec{r}'\sigma') = \langle\Phi|a^+(\vec{r}'\sigma')a(\vec{r}\sigma)|\Phi\rangle$$

Scalar and vector part:

$$\begin{aligned}\rho(\vec{r},\vec{r}') &= \textstyle\sum_\sigma \rho(\vec{r}\sigma,\vec{r}'\sigma)\\ \vec{s}(\vec{r},\vec{r}') &= \textstyle\sum_{\sigma\sigma'} \rho(\vec{r}\sigma,\vec{r}'\sigma')\langle\sigma'|\vec{\sigma}|\sigma\rangle\end{aligned}$$

Symmetries:

$$\begin{aligned}\rho^T(\vec{r},\vec{r}') &= \rho^*(\vec{r},\vec{r}') &&= \rho(\vec{r}',\vec{r})\\ \vec{s}^T(\vec{r},\vec{r}') &= -\vec{s}^*(\vec{r},\vec{r}') &&= -\vec{s}(\vec{r}',\vec{r})\end{aligned}$$

Local densities:

Matter:	$\rho(\vec{r}) = \rho(\vec{r},\vec{r})$
Momentum:	$\vec{j}(\vec{r}) = (1/2i)[(\vec{\nabla}-\vec{\nabla}')\rho(\vec{r},\vec{r}')]_{r=r'}$
Kinetic:	$\tau(\vec{r}) = [\vec{\nabla}\cdot\vec{\nabla}'\rho(\vec{r},\vec{r}')]_{r=r'}$
Spin:	$\vec{s}(\vec{r}) = \vec{s}(\vec{r},\vec{r})$
Spin momentum:	$J_{\mu\nu}(\vec{r}) = (1/2i)[(\nabla_\mu-\nabla'_\mu)s_\nu(\vec{r},\vec{r}')]_{r=r'}$
Spin kinetic:	$\vec{T}(\vec{r}) = [\vec{\nabla}\cdot\vec{\nabla}'\vec{s}(\vec{r},\vec{r}')]_{r=r'}$
Tensor kinetic:	$\vec{F}(\vec{r}) = \frac{1}{2}[(\vec{\nabla}\otimes\vec{\nabla}'+\vec{\nabla}'\otimes\vec{\nabla})\cdot\vec{s}(\vec{r},\vec{r}')]_{r=r'}$

Local energy density: (no isospin, no pairing)

Density	Derivative	Symmetry T	Symmetry P	Symmetry space	Energy density
$\rho(\vec{r})$		$+$	$+$	scalar	ρ^2
	$\vec{\nabla}\rho(\vec{r})$	$+$	$-$	vector	$\vec{\nabla}\rho\cdot\vec{J}$
	$\Delta\rho(\vec{r})$	$+$	$+$	scalar	$\rho\Delta\rho$
$\tau(\vec{r})$		$+$	$+$	scalar	$\rho\tau$
$J^{(0)}(\vec{r})$		$+$	$-$	scalar	$J^{(0)}J^{(0)}$
	$\vec{\nabla}J^{(0)}(\vec{r})$	$+$	$+$	vector	
$\vec{J}(\vec{r})$		$+$	$-$	vector	$\vec{J}^2$
	$\vec{\nabla}\cdot\vec{J}(\vec{r})$	$+$	$+$	scalar	$\rho\vec{\nabla}\cdot\vec{J}$
	$\vec{\nabla}\times\vec{J}(\vec{r})$	$+$	$+$	vector	
$J^{(2)}_{\mu\nu}(\vec{r})$		$+$	$-$	tensor	$\sum_{\mu\nu}J^{(2)}_{\mu\nu}J^{(2)}_{\mu\nu}$
$\vec{s}(\vec{r})$		$-$	$+$	vector	$\vec{s}^2$
	$\vec{\nabla}\cdot\vec{s}(\vec{r})$	$-$	$-$	scalar	$(\vec{\nabla}\cdot\vec{s})^2$
	$\vec{\nabla}\times\vec{s}(\vec{r})$	$-$	$-$	vector	$\vec{j}\cdot\vec{\nabla}\times\vec{s}$
	$\Delta\vec{s}(\vec{r})$	$-$	$+$	vector	$\vec{s}\cdot\Delta\vec{s}$
$\vec{j}(\vec{r})$		$-$	$-$	vector	$\vec{j}^2$
	$\vec{\nabla}\cdot\vec{j}(\vec{r})$	$-$	$+$	scalar	
	$\vec{\nabla}\times\vec{j}(\vec{r})$	$-$	$+$	vector	$\vec{s}\cdot\vec{\nabla}\times\vec{j}$
$\vec{T}(\vec{r})$		$-$	$+$	vector	$\vec{s}\cdot\vec{T}$
$\vec{F}(\vec{r})$		$-$	$+$	vector	$\vec{s}\cdot\vec{F}$

Complete local energy density

The energy density can be written in the following form:

$$\mathcal{H}(\vec{r}) = \frac{\hbar^2}{2m}\tau_0(\vec{r}) + \sum\nolimits_{t=0,1} \left(\chi_t(\vec{r}) + \breve{\chi}_t(\vec{r})\right),$$

The p-h and p-p interaction energy densities, $\chi_t(\vec{r})$ and $\breve{\chi}_t$, for t=0 depend quadratically on the isoscalar densities, and for t=1 – on the isovector ones. Based on general rules of constructing the energy density, one obtains

Mean field

$$\begin{aligned}
\chi_0(\vec{r}) &= C_0^{\rho}\rho_0^2 + C_0^{\Delta\rho}\rho_0\Delta\rho_0 + C_0^{\tau}\rho_0\tau_0 \\
&+ C_0^{J0}J_0^2 + C_0^{J1}\vec{J}_0^2 + C_0^{J2}\underline{\mathsf{J}}_0^2 + C_0^{\nabla J}\rho_0\vec{\nabla}\cdot\vec{J}_0 \\
&+ C_0^{s}\vec{s}_0^2 + C_0^{\Delta s}\vec{s}_0\cdot\Delta\vec{s}_0 + C_0^{T}\vec{s}_0\cdot\vec{T}_0 \\
&+ C_0^{j}\vec{j}_0^2 + C_0^{\nabla j}\vec{s}_0\cdot(\vec{\nabla}\times\vec{j}_0) \\
&+ C_0^{\nabla s}(\vec{\nabla}\cdot\vec{s}_0)^2 + C_0^{F}\vec{s}_0\cdot\vec{F}_0, \\
\chi_1(\vec{r}) &= C_1^{\rho}\vec{\rho}^2 + C_1^{\Delta\rho}\vec{\rho}\circ\Delta\vec{\rho} + C_1^{\tau}\vec{\rho}\circ\vec{\tau} \\
&+ C_1^{J0}\vec{J}^2 + C_1^{J1}\vec{\vec{J}}^2 + C_1^{J2}\vec{\underline{\mathsf{J}}}^2 + C_1^{\nabla J}\vec{\rho}\circ\vec{\nabla}\cdot\vec{\vec{J}} \\
&+ C_1^{s}\vec{\vec{s}}^2 + C_1^{\Delta s}\vec{\vec{s}}\cdot\circ\Delta\vec{\vec{s}} + C_1^{T}\vec{\vec{s}}\cdot\circ\vec{\vec{T}} \\
&+ C_1^{j}\vec{\vec{j}}^2 + C_1^{\nabla j}\vec{\vec{s}}\cdot\circ(\vec{\nabla}\times\vec{\vec{j}}) \\
&+ C_1^{\nabla s}(\vec{\nabla}\cdot\vec{\vec{s}})^2 + C_1^{F}\vec{\vec{s}}\cdot\circ\vec{\vec{F}},
\end{aligned}$$

where $\times$ stands for the vector product

Pairing

$$\begin{aligned}
\breve{\chi}_0(\vec{r}) &= \breve{C}_0^{s}|\breve{\vec{s}}_0|^2 + \breve{C}_0^{\Delta s}\Re(\breve{\vec{s}}_0^{\,*}\cdot\Delta\breve{\vec{s}}_0) \\
&+ \breve{C}_0^{T}\Re(\breve{\vec{s}}_0^{\,*}\cdot\breve{\vec{T}}_0) + \breve{C}_0^{j}|\breve{\vec{j}}_0|^2 \\
&+ \breve{C}_0^{\nabla j}\Re(\breve{\vec{s}}_0^{\,*}\cdot(\vec{\nabla}\times\breve{\vec{j}}_0)) \\
&+ \breve{C}_0^{\nabla s}|\vec{\nabla}\cdot\breve{\vec{s}}_0|^2 \\
&+ \breve{C}_0^{F}\Re(\breve{\vec{s}}_0^{\,*}\cdot\breve{\vec{F}}_0), \\
\breve{\chi}_1(\vec{r}) &= \breve{C}_1^{\rho}|\breve{\vec{\rho}}|^2 + \breve{C}_1^{\Delta\rho}\Re(\breve{\vec{\rho}}^{\,*}\circ\Delta\breve{\vec{\rho}}) \\
&+ \breve{C}_1^{\tau}\Re(\breve{\vec{\rho}}^{\,*}\circ\breve{\vec{\tau}}) \\
&+ \breve{C}_1^{J0}|\breve{\vec{J}}|^2 + \breve{C}_1^{J1}|\vec{\breve{\vec{J}}}|^2 \\
&+ \breve{C}_1^{J2}|\breve{\vec{\underline{\mathsf{J}}}}|^2 \\
&+ \breve{C}_1^{\nabla J}\Re(\breve{\vec{\rho}}^{\,*}\circ\vec{\nabla}\cdot\vec{\breve{\vec{J}}}).
\end{aligned}$$

Mean-field equations

Mean-field potentials:

$$
\begin{aligned}
\Gamma_t^{\mathrm{even}} &= -\vec{\nabla}\cdot M_t(\vec{r})\vec{\nabla} + U_t(\vec{r}) + \tfrac{1}{2i}\left(\overleftrightarrow{\nabla}\sigma\cdot\overleftrightarrow{B}_t(\vec{r}) + \overleftrightarrow{B}_t(\vec{r})\cdot\overleftrightarrow{\nabla}\sigma\right) \\
\Gamma_t^{\mathrm{odd}} &= -\vec{\nabla}\cdot(\vec{\sigma}\cdot\vec{C}_t(\vec{r}))\vec{\nabla} + \vec{\sigma}\cdot\vec{\Sigma}_t(\vec{r}) + \tfrac{1}{2i}(\vec{\nabla}\cdot\vec{I}_t(\vec{r}) + \vec{I}_t(\vec{r})\cdot\vec{\nabla}) - \vec{\nabla}\cdot\vec{D}_t(\vec{r})\vec{\sigma}\cdot\vec{\nabla}
\end{aligned}
$$

where

$$
\begin{aligned}
U_t &= 2C_t^{\rho}\rho_t + 2C_t^{\Delta\rho}\Delta\rho_t + C_t^{\tau}\tau_t + C_t^{\nabla J}\vec{\nabla}\cdot\vec{J}_t, \\
\vec{\Sigma}_t &= 2C_t^{s}\vec{s}_t + 2C_t^{\Delta s}\Delta\vec{s}_t + C_t^{T}\vec{T}_t + C_t^{\nabla j}\vec{\nabla}\times\vec{j}_t, -2C_t^{\nabla s}\Delta\vec{s}_t + C_t^{F}\vec{F}_t - 2C_t^{\nabla s}\vec{\nabla}\times(\vec{\nabla}\times\vec{s}_t) \\
M_t &= C_t^{\tau}\rho_t, \\
\vec{C}_t &= C_t^{T}\vec{s}_t, \\
\overleftrightarrow{B}_t &= 2C_t^{J}\overleftrightarrow{J}_t - C_t^{\nabla J}\overleftrightarrow{\nabla}\rho_t, \\
\vec{I}_t &= 2C_t^{j}\vec{j}_t + C_t^{\nabla j}\vec{\nabla}\times\vec{s}_t, \\
\vec{D}_t &= C_t^{F}\vec{s}_t,
\end{aligned}
$$

Neutron and proton mean-field Hamiltonians:

$$
\begin{aligned}
h_n &= -\frac{\hbar^2}{2m}\Delta + \Gamma_0^{\mathrm{even}} + \Gamma_0^{\mathrm{odd}} + \Gamma_1^{\mathrm{even}} + \Gamma_1^{\mathrm{odd}}, \\
h_p &= -\frac{\hbar^2}{2m}\Delta + \Gamma_0^{\mathrm{even}} + \Gamma_0^{\mathrm{odd}} - \Gamma_1^{\mathrm{even}} - \Gamma_1^{\mathrm{odd}}.
\end{aligned}
$$

HF equation for single-particle wave functions:

$$
h_\alpha\psi_{i,\alpha}(\vec{r}\sigma) = \epsilon_{i,\alpha}\psi_{i,\alpha}(\vec{r}\sigma),
$$

where i numbers the neutron ($\alpha{=}n$) and proton ($\alpha{=}p$) eigenstates.

Nuclear densities as composite fields

Lessons learnt

Density distributions of **matter, spin, and current** in a nucleus can be used as fields defining new degrees of freedom that describe nucleus as a composite particle. In terms of these fields, the most general **energy density functional** can be constructed by using symmetry arguments. The functional depends on a number of **coupling constants**, which have to be either adjusted to the experimental data or determined from a higher-level theory. In the spirit of the **Effective Field Theory**, the energy density can be supplemented by higher-order contact terms, which amount to a density dependence of the coupling constants. These terms take into account **high-energy phenomena** that are not resolved when one looks at nuclear phenomena within the scale of the nucleon binding (~10 MeV). On the other hand, all effects within the **low-energy scale** of collective excitations (~1 MeV) have to be treated explicitly (deformations, zero-point motion, pairing corrections, symmetry restoration, etc.).

Phenomenological effective interactions

- Gogny force.*

$$\tilde{G}_{xyx'y'} = \delta(\vec{x}-\vec{x}')\delta(\vec{y}-\vec{y}')G(x,y),$$

where the tilde denotes a non-antisymmetrized matrix element ($G_{xyx'y'} = \tilde{G}_{xyx'y'} - \tilde{G}_{xyy'x'}$), and $G(x,y)$ is a sum of two Gaussians, plus a zero-range, density dependent part,

$$G(x,y) = \sum_{i=1,2} e^{-(\vec{x}-\vec{y})^2/\mu_i^2} \times (W_i + B_i P_\sigma - H_i P_\tau - M_i P_\sigma P_\tau)$$
$$+ t_3(1+P_\sigma)\delta(\vec{x}-\vec{y})\rho^{1/3}\left[\tfrac{1}{2}(\vec{x}+\vec{y})\right].$$

In this Equation, $P_\sigma = \frac{1}{2}(1+\vec{\sigma}_1\cdot\vec{\sigma}_2)$ and $P_\tau = \frac{1}{2}(1+\vec{\tau}_1\cdot\vec{\tau}_2)$ are, respectively, the spin and isospin exchange operators of particles 1 and 2, $\rho(\vec{r})$ is the total density of the system at point $\vec{r}$, and μ_i=0.7 and 1.2 fm, W_i, B_i, H_i, M_i, and t_3 are parameters.

- Skyrme force.*

$$\tilde{G}_{xyx'y'} = \Big\{ t_0(1+x_0P^\sigma) + \tfrac{1}{6}t_3(1+x_3P^\sigma)\rho^\alpha\left(\tfrac{1}{2}(\vec{x}+\vec{y})\right)$$
$$+ \tfrac{1}{2}t_1(1+x_1P^\sigma)[\vec{k}^2+\vec{k}'^2] + t_2(1+x_2P^\sigma)\vec{k}^*\cdot\vec{k}' \Big\}\delta(\vec{x}-\vec{x}')\delta(\vec{y}-\vec{y}')\delta(\vec{x}-\vec{y}),$$

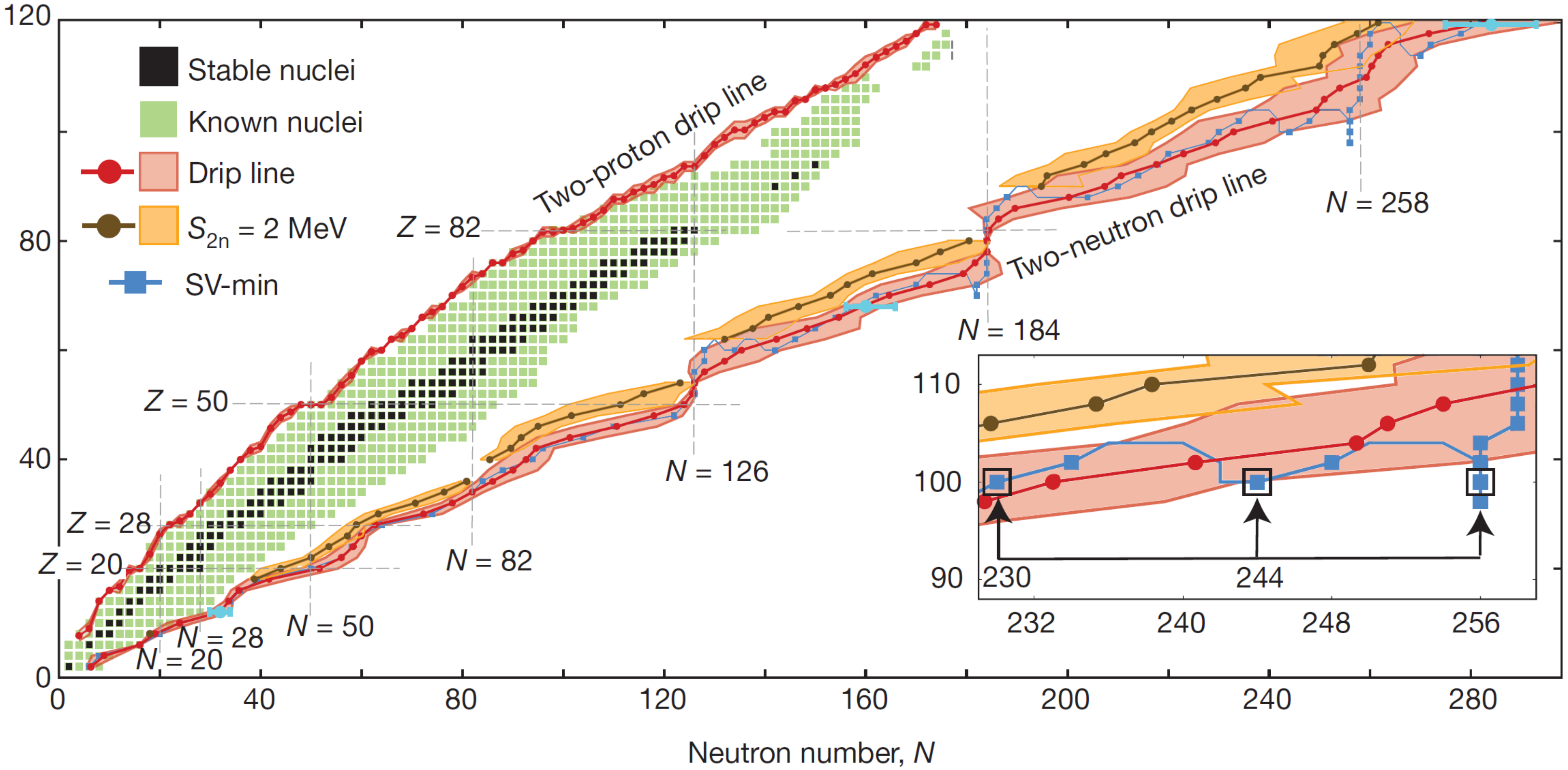
where the relative momentum operators read

$$\hat{\vec{k}} = \frac{1}{2i}\left(\vec{\nabla}_x - \vec{\nabla}_y\right), \qquad \hat{\vec{k}}' = \frac{1}{2i}\left(\vec{\nabla}'_x - \vec{\nabla}'_y\right).$$

*We omit the spin-orbit and tensor terms for simplicity.

What the energy density functionals can give us?

6900±500 bound nuclei

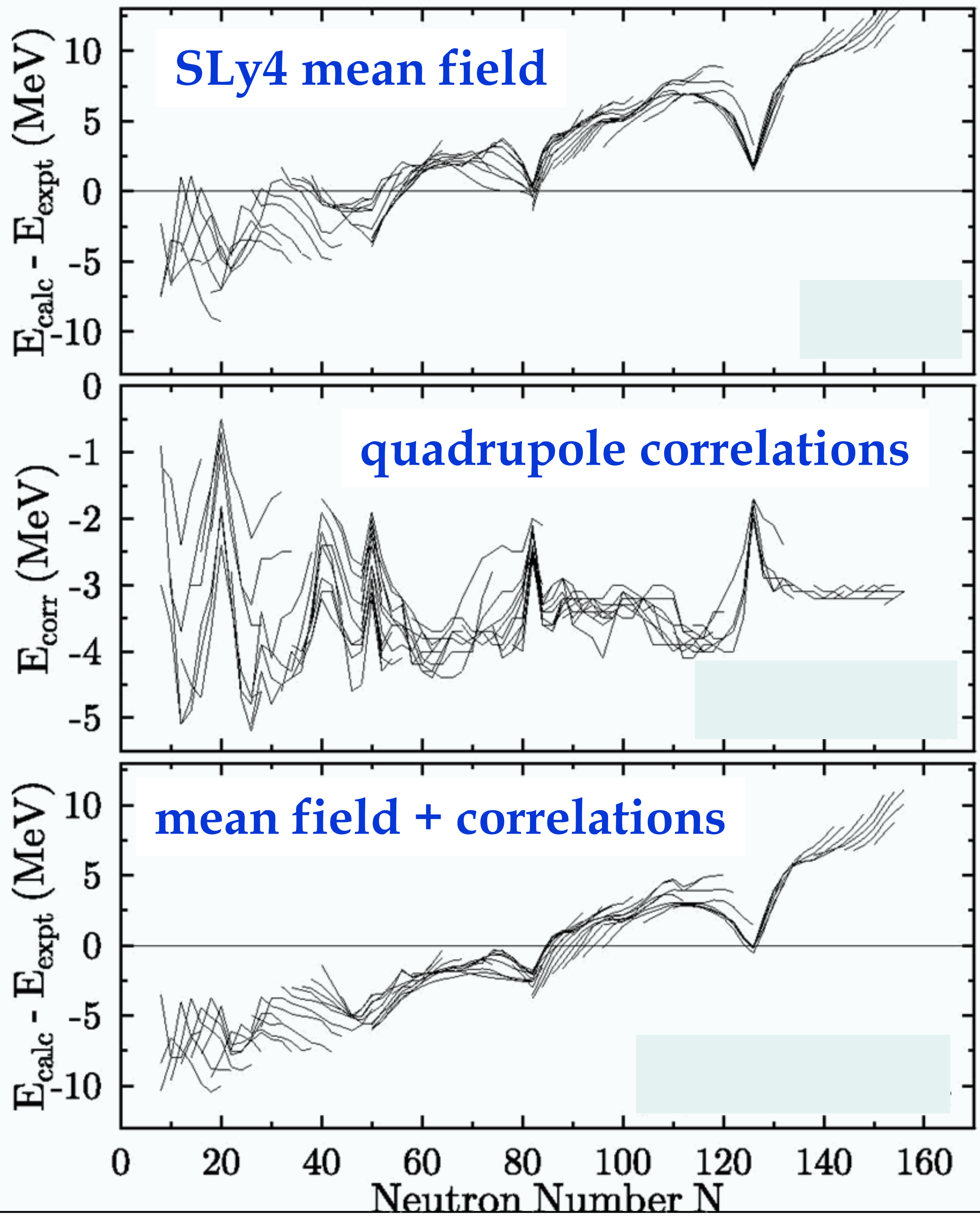


The limits of the nuclear landscape

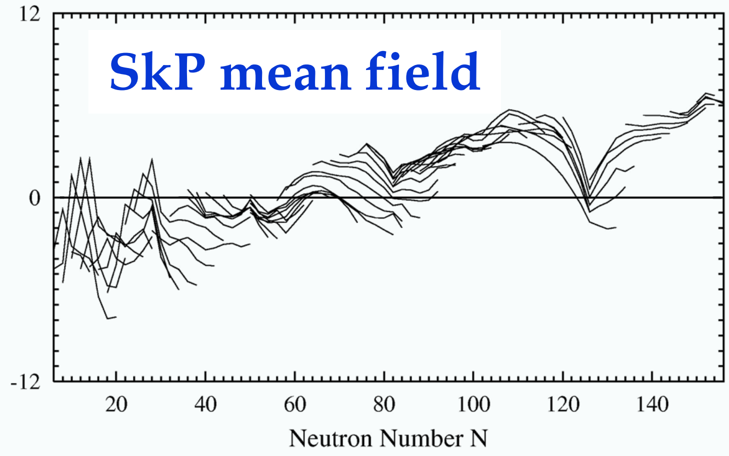
J. Erler, N. Birge, M. Kortelainen, W. Nazarewicz, E. Olsen, A.M. Perhac, M. Stoitsov,

Nature 486, 509 (2012)

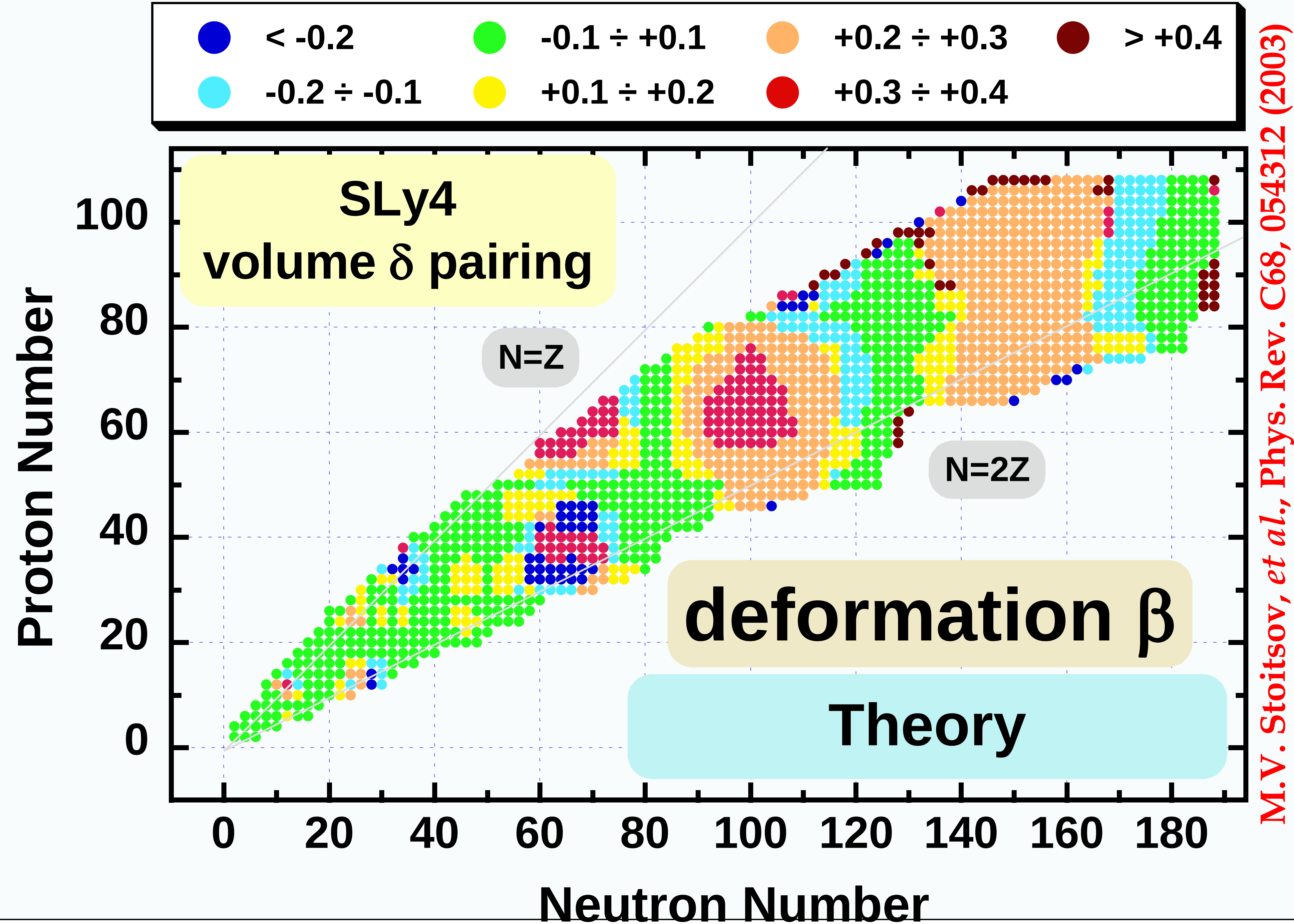
M. Bender, *et al.*, nucl-th/0410023

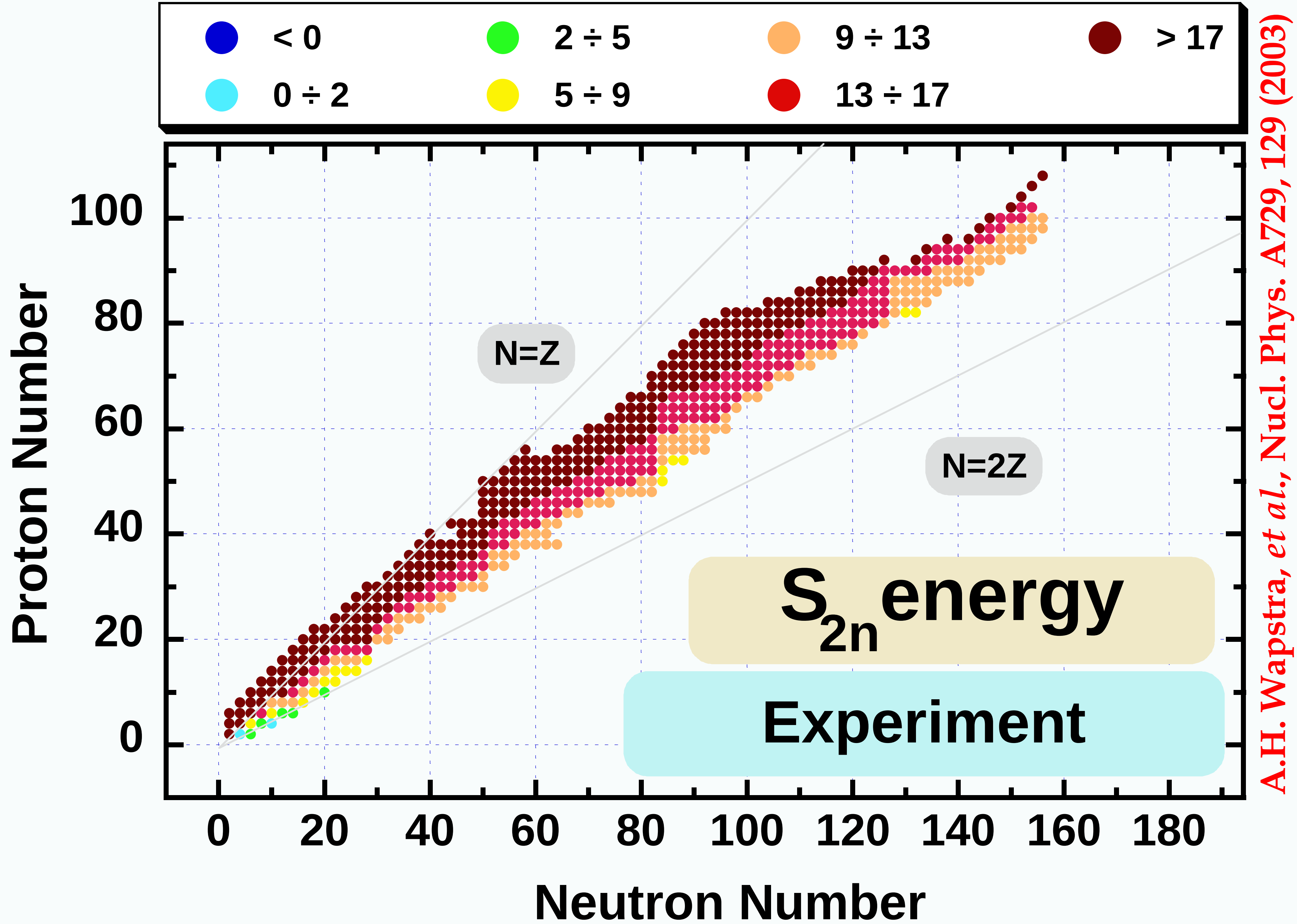


J. D., *et al.*, nucl-th/0404077



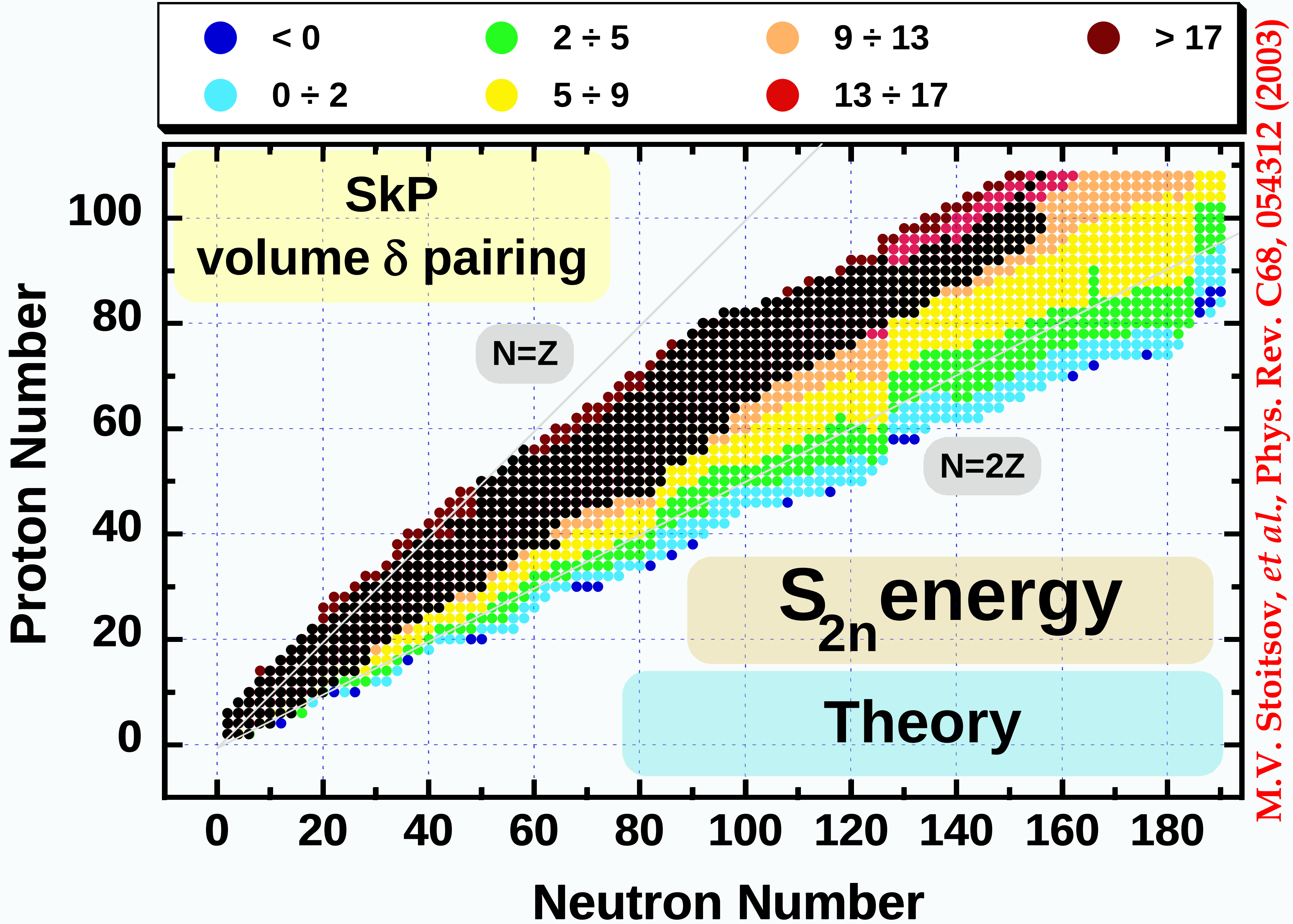
- First systematic **microscopic** calculations of the quadrupole collective correlation energies become available.
- Magnitude of shell effects depends on the mean field **and** on the collective correlations.
- More data far from stability needed to pin down the isotopic dependence

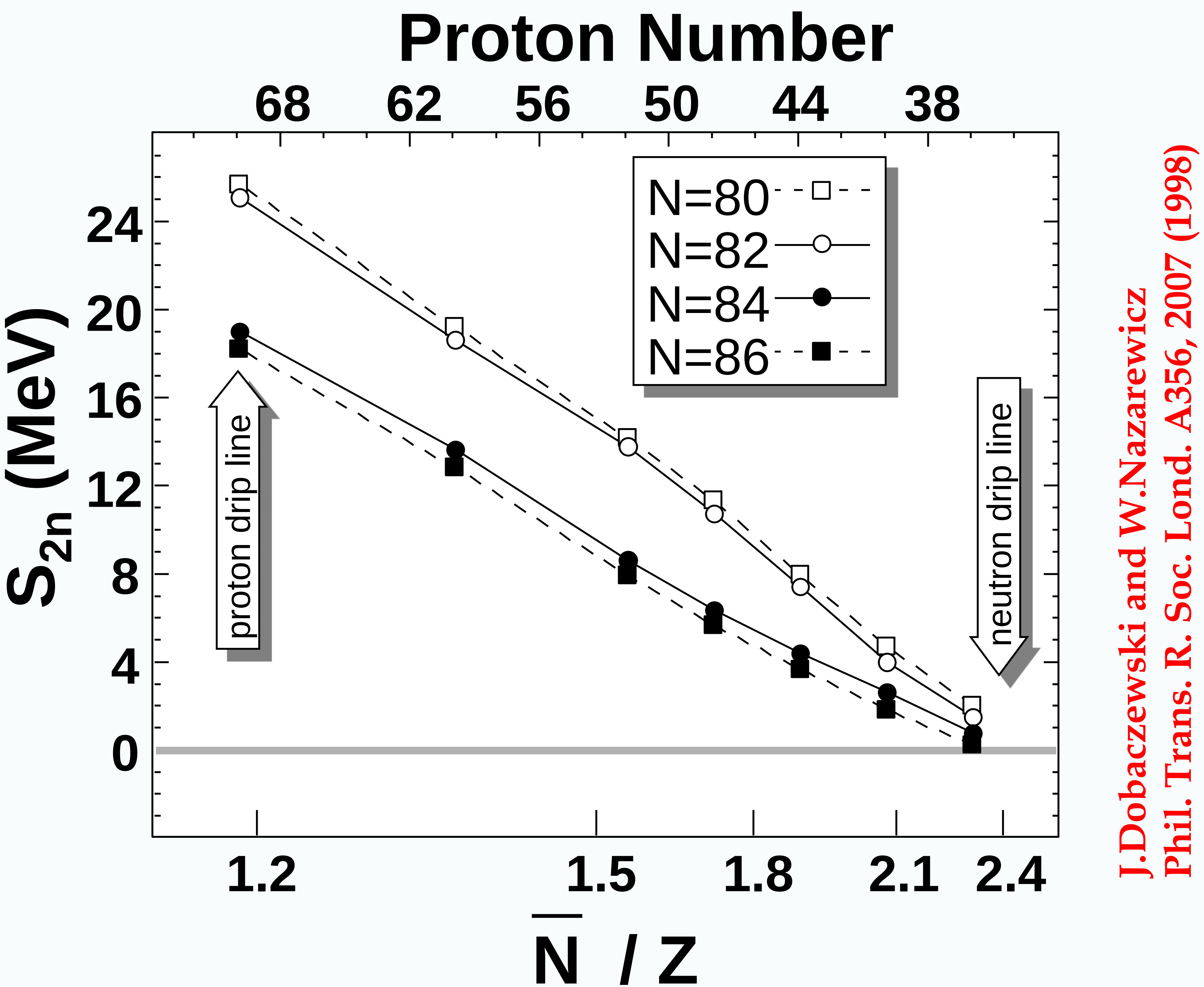

< -0.2
-0.2 ÷ -0.1
-0.1 ÷ +0.1
+0.1 ÷ +0.2
+0.2 ÷ +0.3
+0.3 ÷ +0.4
> +0.4
SLy4
volume δ pairing
N=Z
N=2Z
deformation β
Theory
Proton Number
Neutron Number
0
20
40
60
80
100
120
140
160
180
M.V. Stoitsov, et al., Phys. Rev. C68, 054312 (2003)



S_{2n} energy

Experiment


< 0
0 ÷ 2
2 ÷ 5
5 ÷ 9
9 ÷ 13
13 ÷ 17
> 17
SkP
volume δ pairing
N=Z
N=2Z
S_{2n} energy
Theory
Proton Number
Neutron Number
0
20
40
60
80
100
120
140
160
180
M.V. Stoitsov, et al., Phys. Rev. C68, 054312 (2003)


Proton Number
68
62
56
50
44
38
N=80
N=82
N=84
N=86
proton drip line
neutron drip line
S2n (MeV)
24
20
16
12
8
4
0
1.2
1.5
1.8
2.1
2.4
N / Z
J.Dobaczewski and W.Nazarewicz
Phil. Trans. R. Soc. Lond. A356, 2007 (1998)

Two-neutron separation energy S_{2n}

$$S_{2n}(N,Z) = E(N,Z) - E(N-2,Z)$$

where E(N,Z) is the ground-state energy (negative) of the nucleus with N neutrons and Z protons.

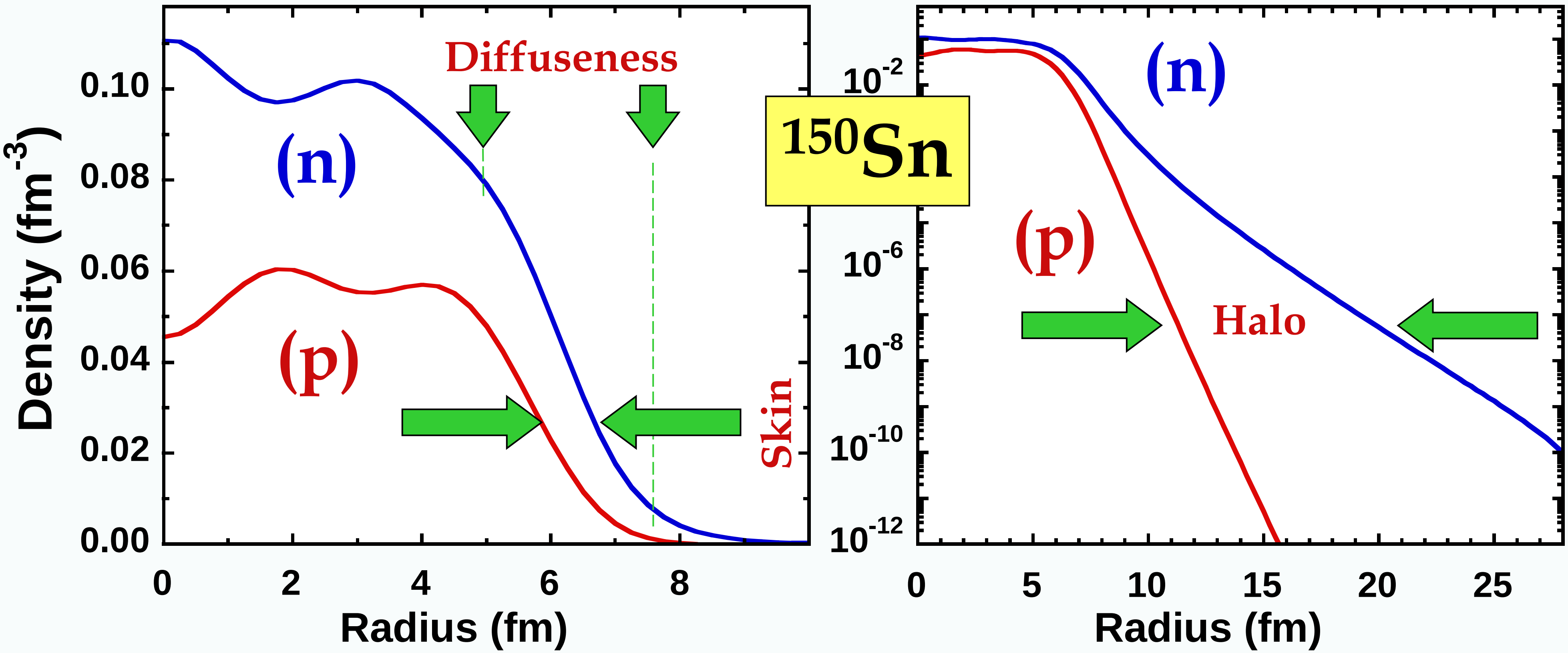
Two-neutron separation energies are insensitive to pairing correlations, because complete pairs are simultaneously removed.

Two-neutron separation energies exhibit jumps when crossing magic neutron numbers. The magnitude of the jump is a measure of the neutron magic shell gap for a given proton number.

Two-neutron unbound nuclei may exist beyond the neutron drip line due to the shape coexistence phenomenon.

Lessons learnt

Neutron & proton density distributions



The Helm model

Helm density distribution:

$$\rho^{(\mathrm{H})}(\vec{r})=\int d^3\vec{r}'\,\rho_0\Theta(R_0-|\vec{r}'|)\left(\frac{1}{(2\pi\sigma^2)^{-3/2}}e^{-\frac{|\vec{r}-\vec{r}'|^2}{2\sigma^2}}\right)$$

The Fourier transform of the Helm density is a product of the Fourier transforms of the Gaussian profile and step function, i.e.,

$$F^{(\mathrm{H})}(q)=\frac{3}{R_0 q}j_1(qR_0)e^{-\frac{\sigma^2q^2}{2}}.$$

Since the central density $\rho_0=\frac{3N}{4\pi R_0^3}$ is fixed by the particle-number condition, the Helm density has 2 parameters: R_0 and σ. These parameters can be determined from the first zero q_1 and the first maximum q_m of the Fourier transform $F^{(\mathrm{H})}(q)$:

$$R_0=4.49341/q_1,$$

$$\sigma^2=\frac{2}{q_m^2}\ln\frac{3R_0^2 j_1(q_m R_0)}{R_0 q_m F(q_m)}.$$

By comparing the root-mean-squared (rms) radius of the real density distribution:

$$R_{\mathrm{geom}}=\sqrt{\frac{5}{3}}R_{\mathrm{rms}}=\sqrt{\frac{5}{3}}\sqrt{\frac{\int d^3\vec{r}\,r^2\rho(\vec{r})}{\int d^3\vec{r}\,\rho(\vec{r})}}$$

with that of the Helm density distribution:

$$R_{\mathrm{Helm}}=\sqrt{\frac{5}{3}}R^{(\mathrm{H})}_{\mathrm{rms}}=\sqrt{(R_0^2+5\sigma^2)}$$

we obtain a simple quantitative measure of the halo size:

$$\delta R_{\mathrm{halo}}\equiv R_{\mathrm{geom}}-R_{\mathrm{Helm}}$$

By comparing the neutron and proton Helm radii, we obtain a simple quantitative measure of the skin:

$$\delta R_{\mathrm{skin}}\equiv R_{\mathrm{Helm}}(n)-R_{\mathrm{Helm}}(p)$$

S. Mizutori, et al., Phys. Rev. C61, 044326 (2000)

Neutron & proton density distributions

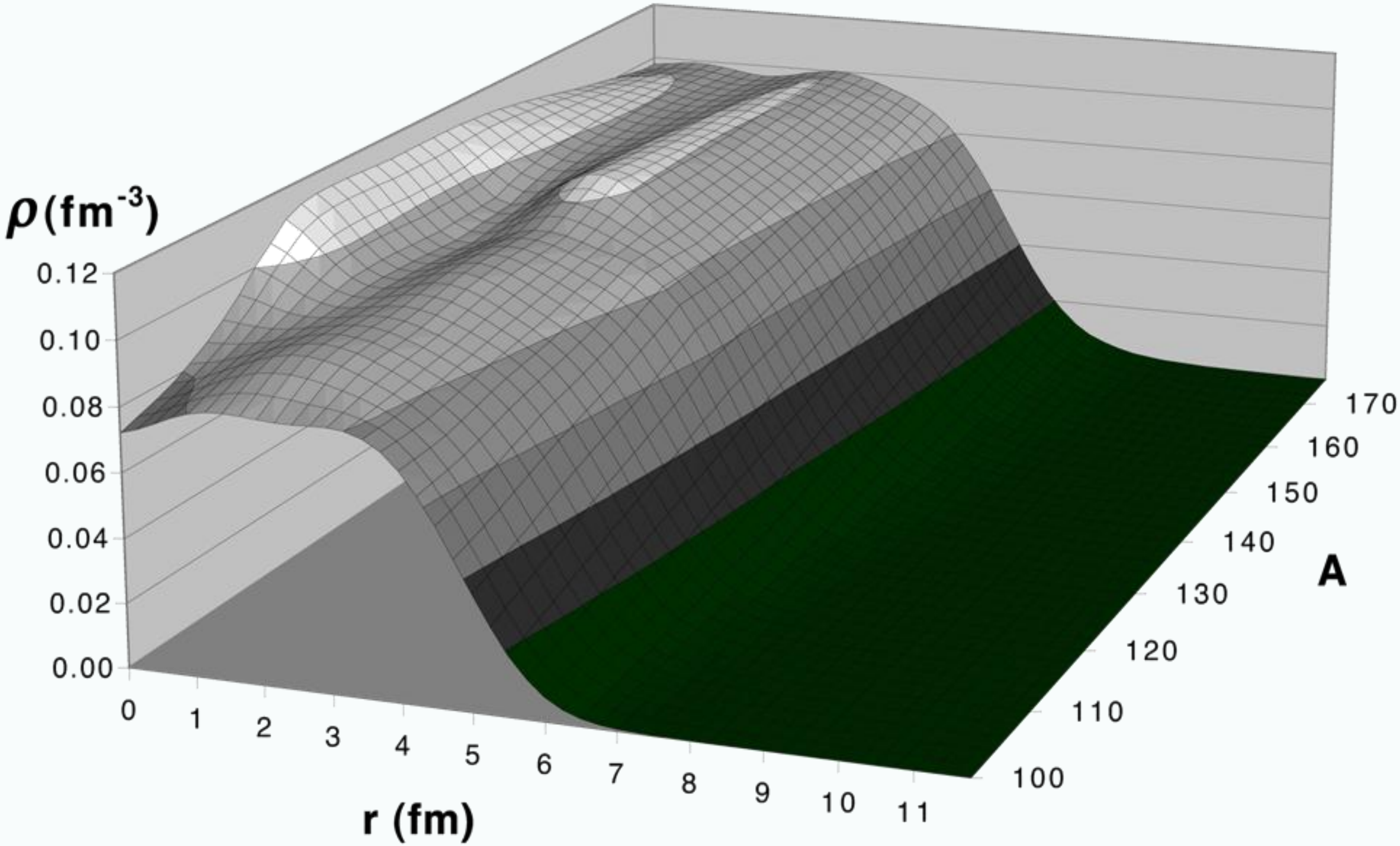
Lessons learnt

Diffuseness of the density distribution is equal to the difference of radii where the density has values of 10% and 90% of the average central density. Better quantitative measure of the diffuseness can be formulated within the Helm model (step-like distribution folded with a Gaussian).

Neutron skin size is equal to the difference of radii where the neutron and proton densities have values of 50% of their respective average central densities Better quantitative measure of the skin can be formulated within the Helm model as the difference of neutron and proton diffraction radii.

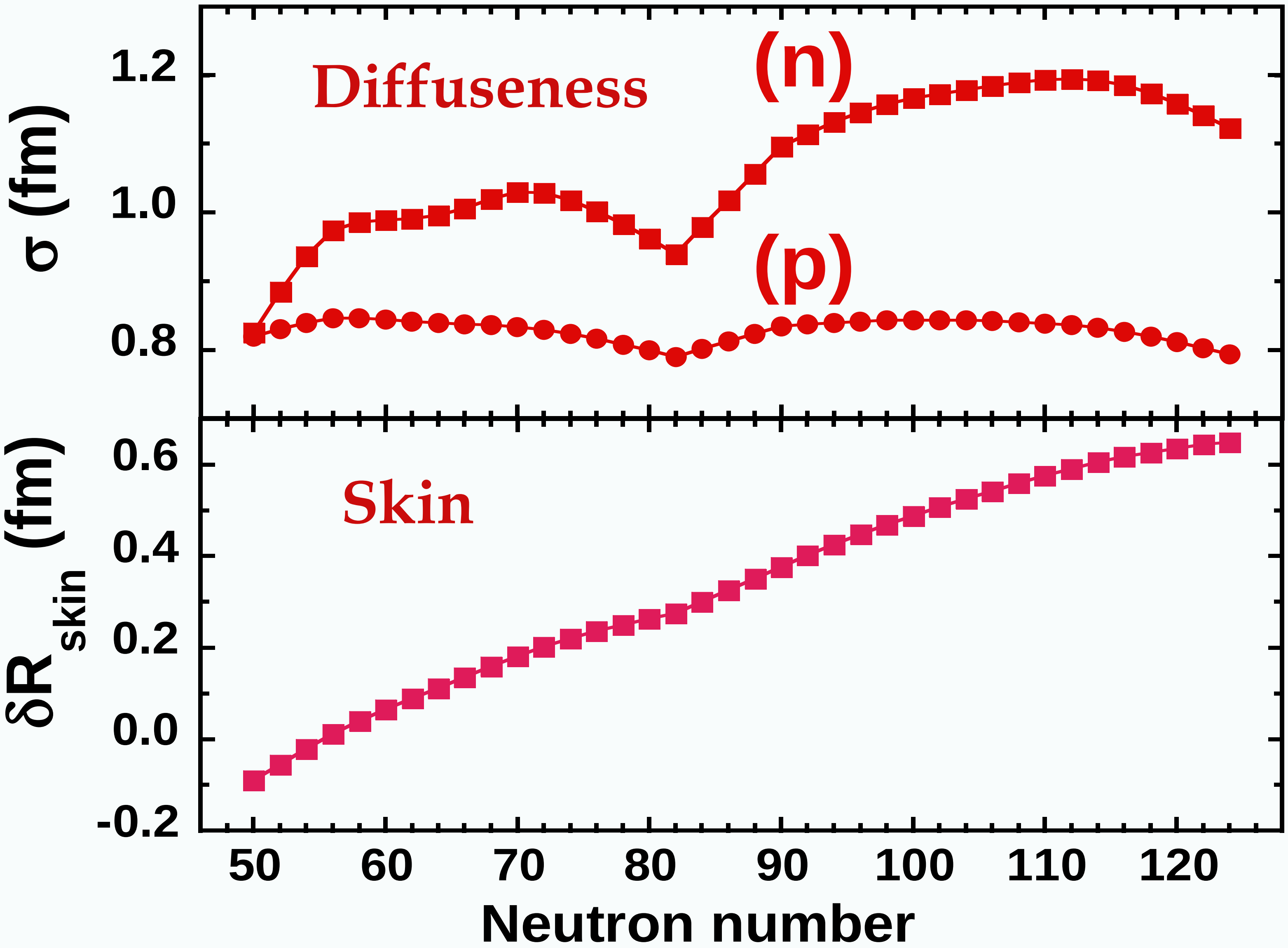
Neutron halo size is the difference between the neutron root-mean-squared and diffraction radii. Properties of the neutron halo are governed by the asymptotic features of tails of quantal wave functions.

Neutron densities in Sn



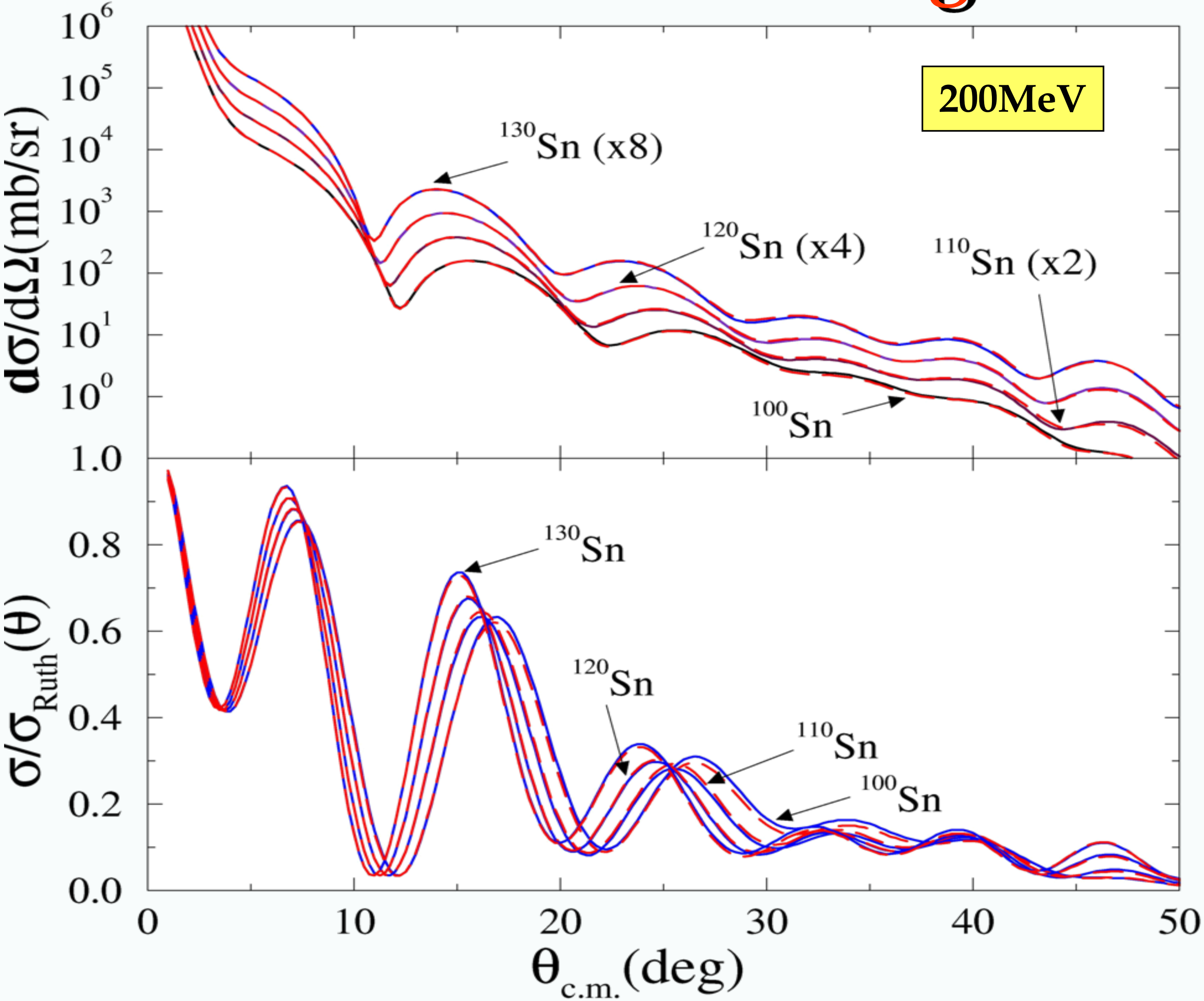
K. Amos, *et al.*, Phys. Rev. C70 (2004) 024607

Neutron densities in Sn



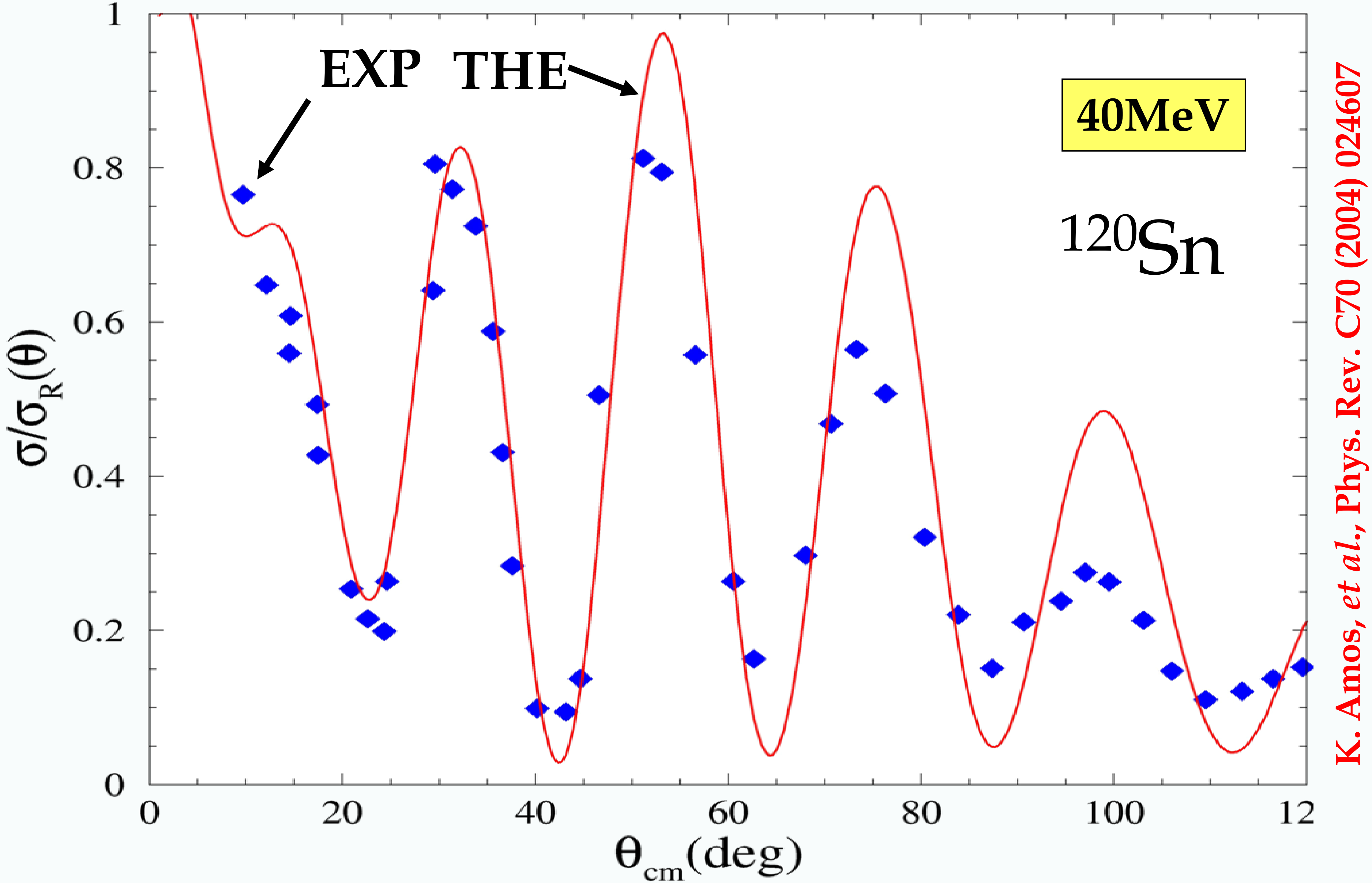
K. Amos, *et al.*, Phys. Rev. C70 (2004) 024607

Proton Scattering



K. Amos, *et al.*, Phys. Rev. C70 (2004) 024607

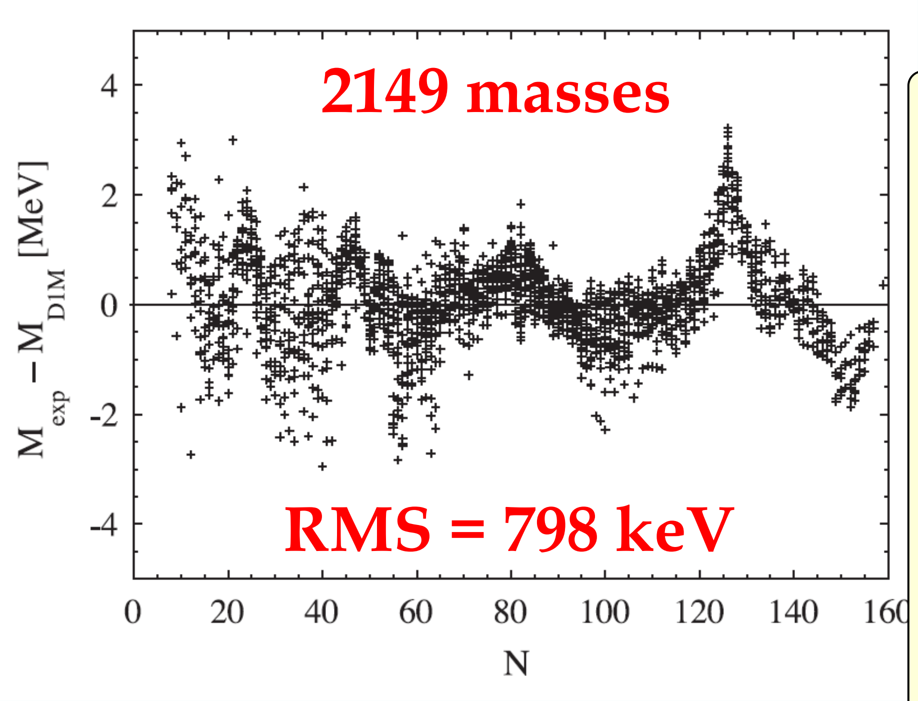
Proton Scattering



Precision frontier

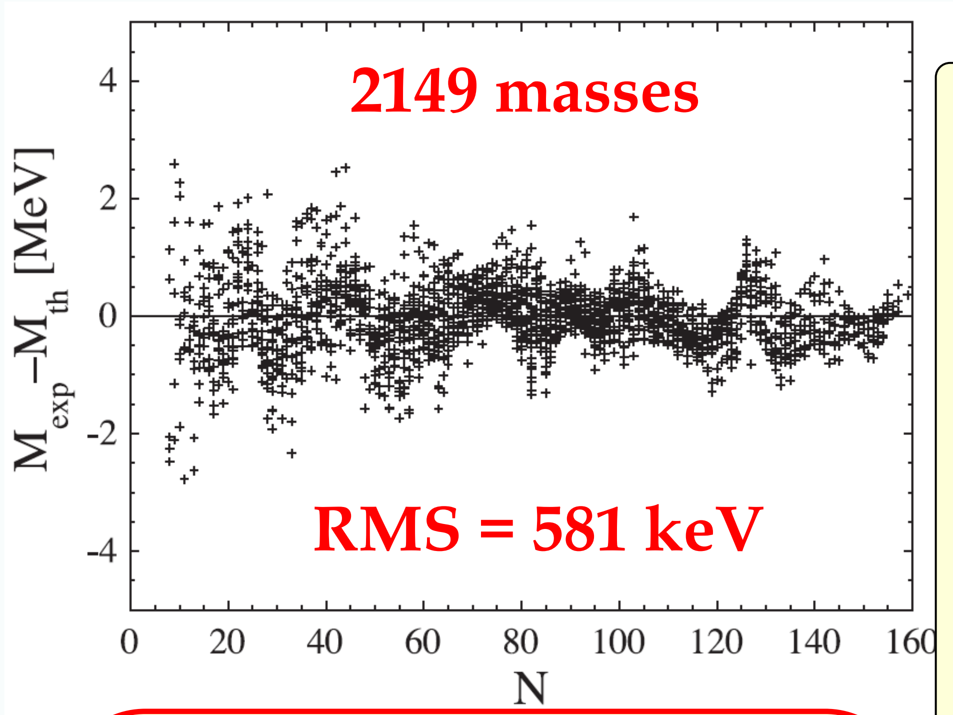
1) **“Remember that all models are wrong; the practical question is how wrong do they have to be to not be useful”**
G.E.P. Box and N.R. Draper
Empirical Model Building and Response Surfaces
(John Wiley & Sons, New York, 1987)

Nuclear binding energies (masses)



S. Goriely *et al.*, Phys. Rev. Lett. 102, 242501 (2009)

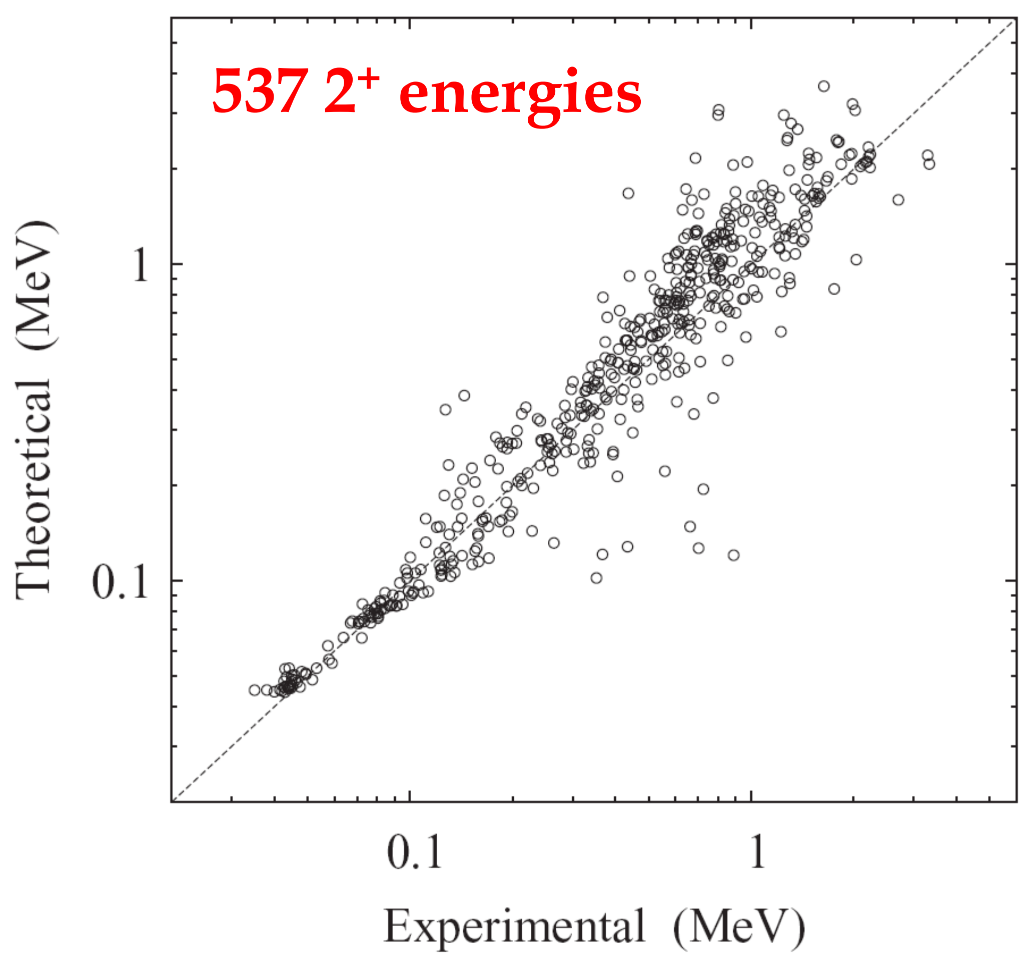
The first Gogny HFB mass model. An explicit and self-consistent account of all the quadrupole correlation energies are included within the 5D collective Hamiltonian approach.



S. Goriely *et al.*, Phys. Rev. Lett. 102, 152503 (2009)

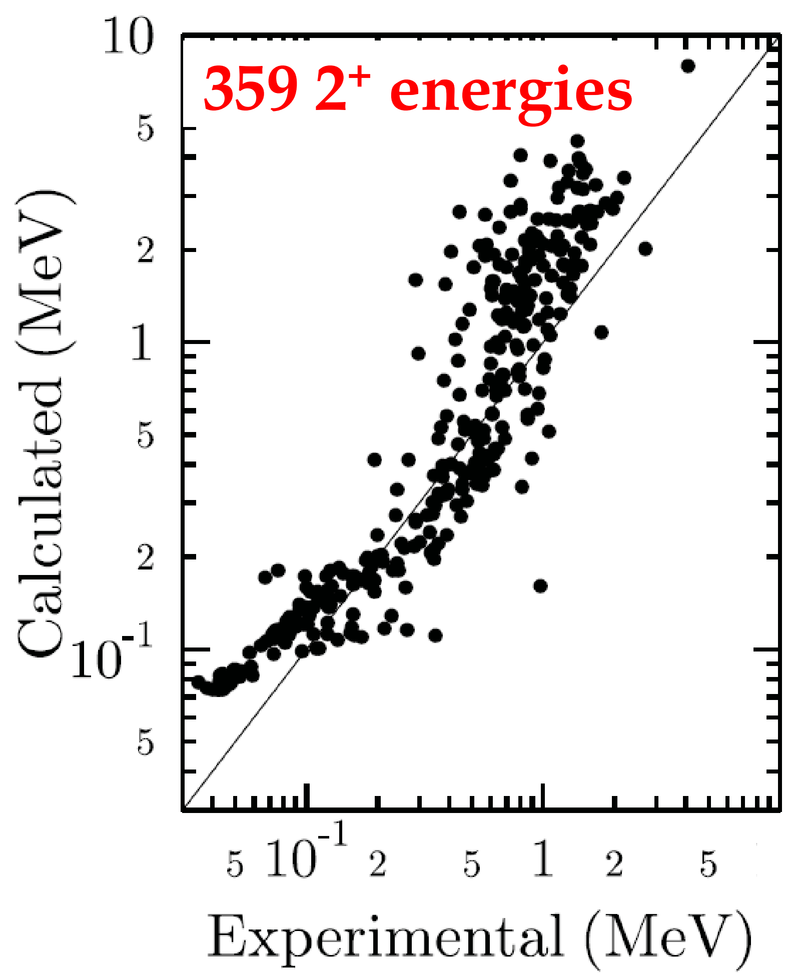
The new Skyrme HFB nuclear-mass model, in which the contact-pairing force is constructed from microscopic pairing gaps of symmetric nuclear matter and neutron matter.

First 2^+ excitations of even-even nuclei



J.-P. Delaroche *et al.*, Phys. Rev. C81, 014303 (2010)

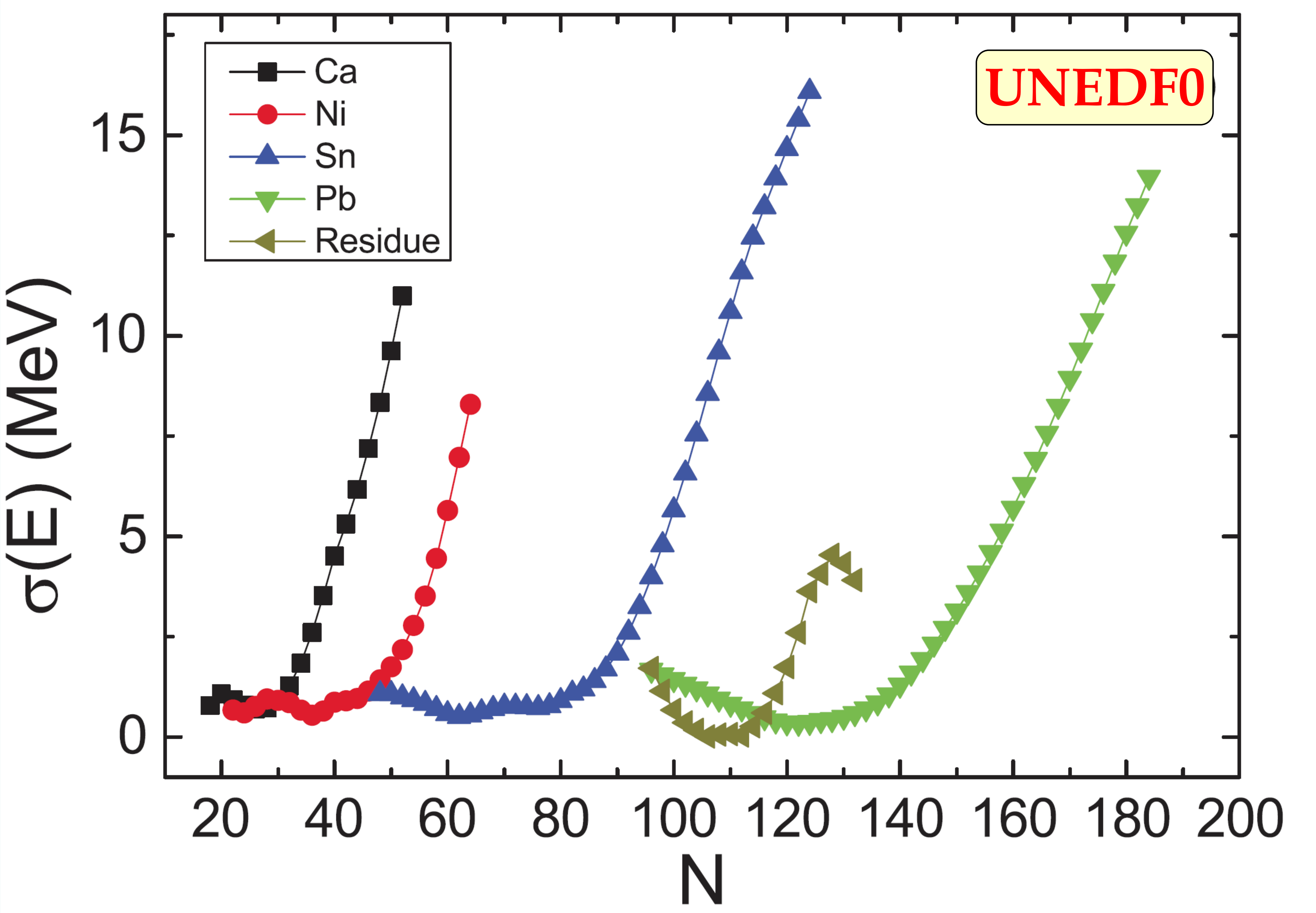
Gogny HFB calculations plus the 5D collective Hamiltonian approach.



B. Sabbey *et al.*, Phys. Rev. C75, 044305 (2007)

Skyrme HF+BCS calculations plus the particle-number and angular-momentum projection and shape mixing.

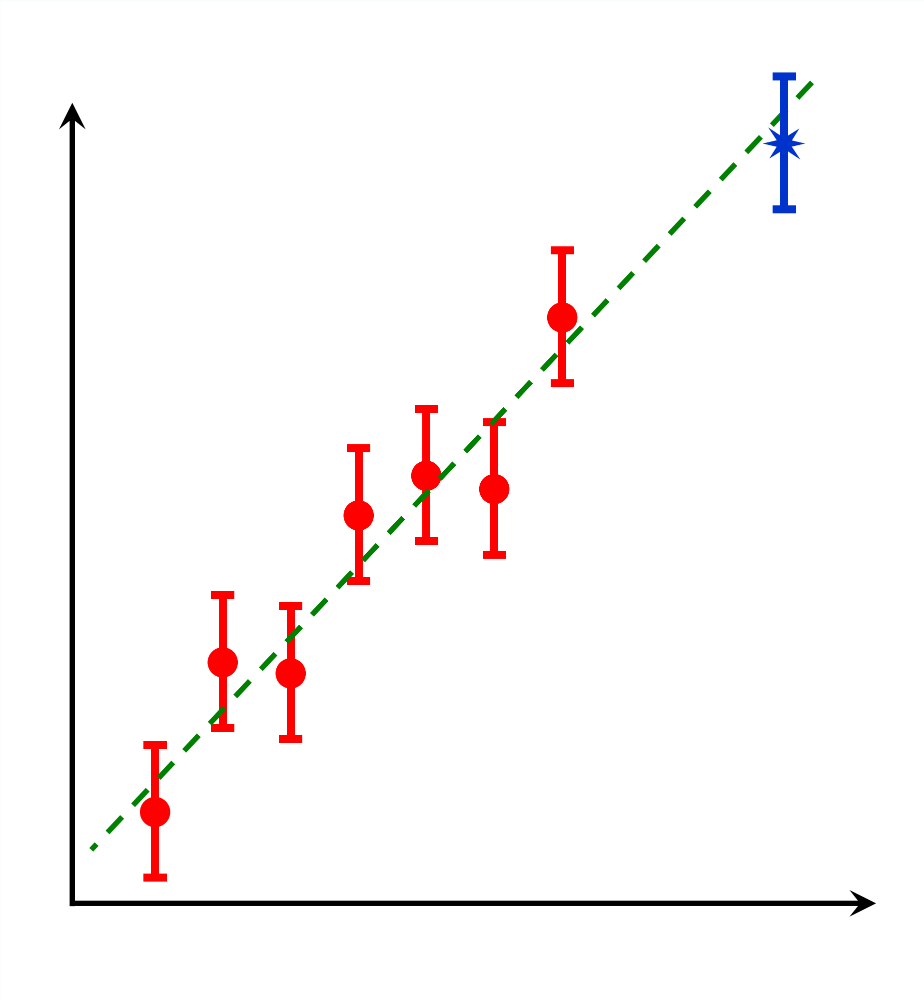
Propagation of uncertainties

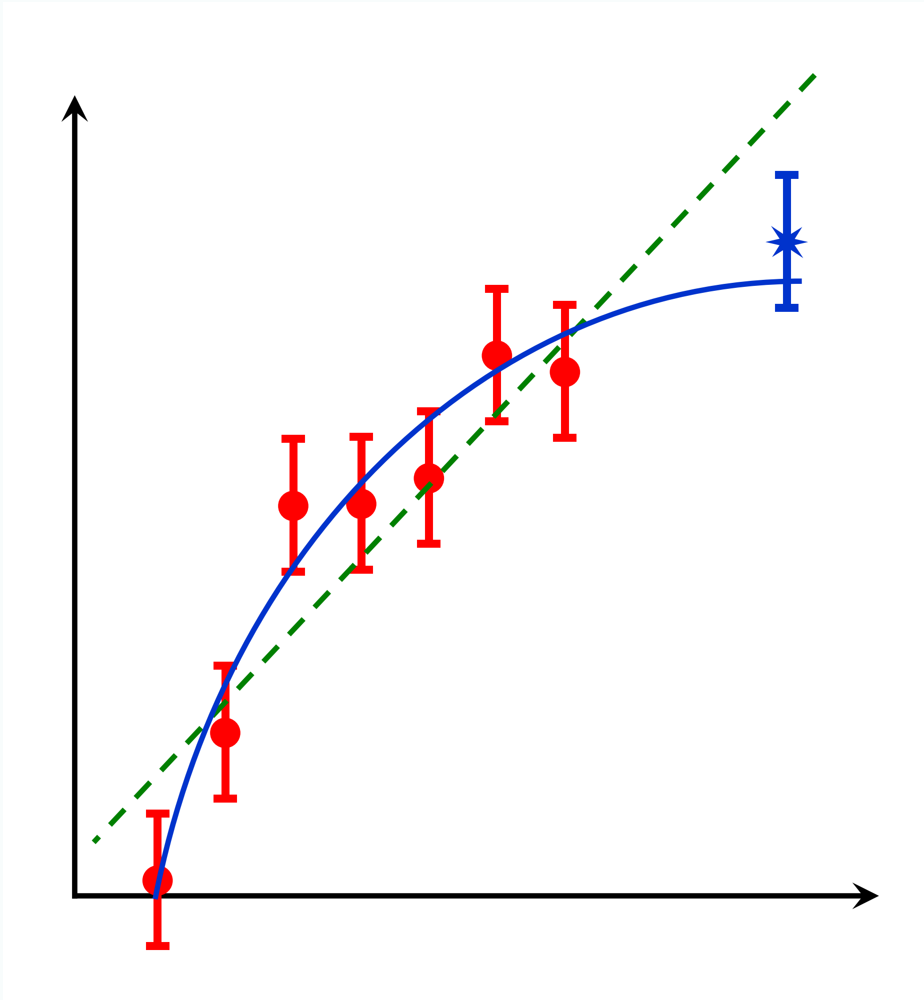


Y. Gao, *et al.*, Phys. Rev C 87, 034324 (2013)

Exact model

Inaccurate model





Exact model

J. Tovanen, *et al.*, Phys. Rev. C 78, 034306 (2008)

Collectivity

Collectivity

beyond mean field, ground-state correlations, shape coexistence, symmetry restoration, projection on good quantum numbers, configuration interaction, generator coordinate method, multi-reference DFT, etc….

$$E = \langle\Psi|\hat{H}|\Psi\rangle \simeq \iint \mathrm{d}\vec{r}\mathrm{d}\vec{r}\,'\,\mathcal{H}(\rho(\vec{r},\vec{r}\,'))$$

True for interaction

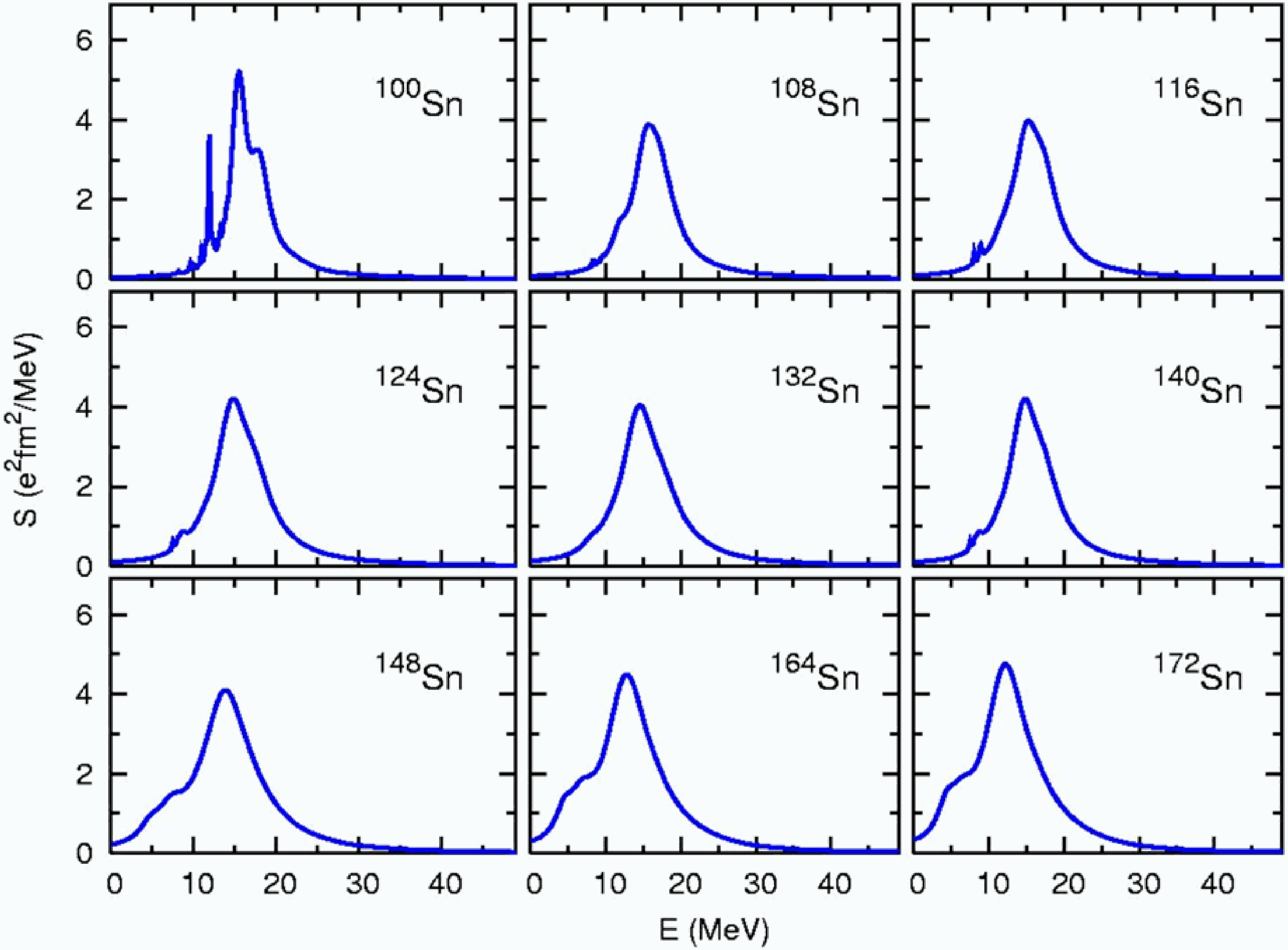
$$\Downarrow$$

$$\text{for} \quad \rho(\vec{r},\vec{r}\,') = \frac{\langle\Psi|a^+(\vec{r}\,')a(\vec{r}\,')|\Psi\rangle}{\langle\Psi|\Psi\rangle}$$

$$\langle\Psi_1|\hat{H}|\Psi_2\rangle \simeq \iint \mathrm{d}\vec{r}\mathrm{d}\vec{r}\,'\,\mathcal{H}(\rho_{12}(\vec{r},\vec{r}\,'))$$

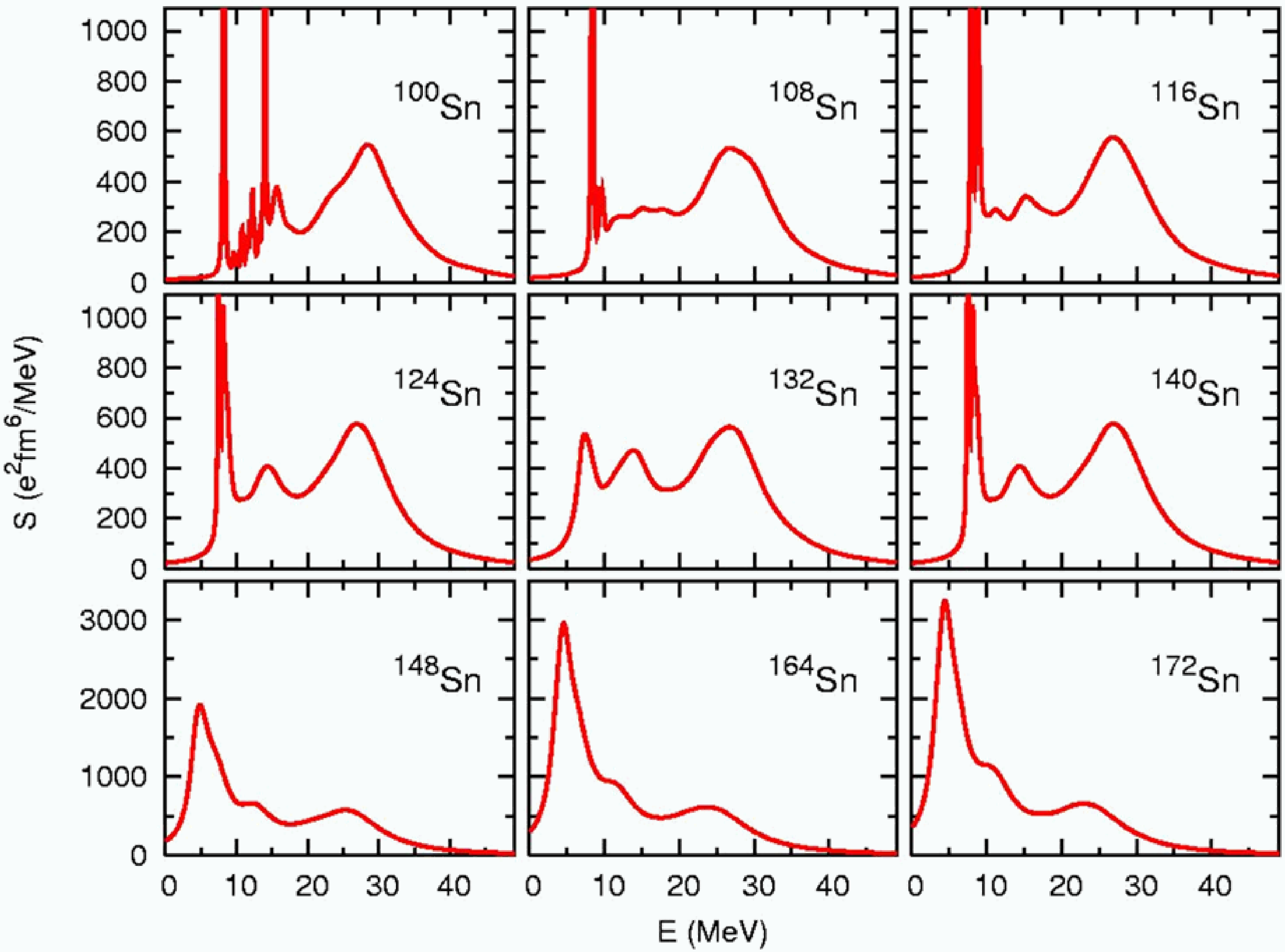
$$\text{for} \quad \rho_{12}(\vec{r},\vec{r}\,') = \frac{\langle\Psi_1|a^+(\vec{r}\,')a(\vec{r}\,')|\Psi_2\rangle}{\langle\Psi_1|\Psi_2\rangle}$$

Isovector 1^- modes, SkM* + volume-type pairing



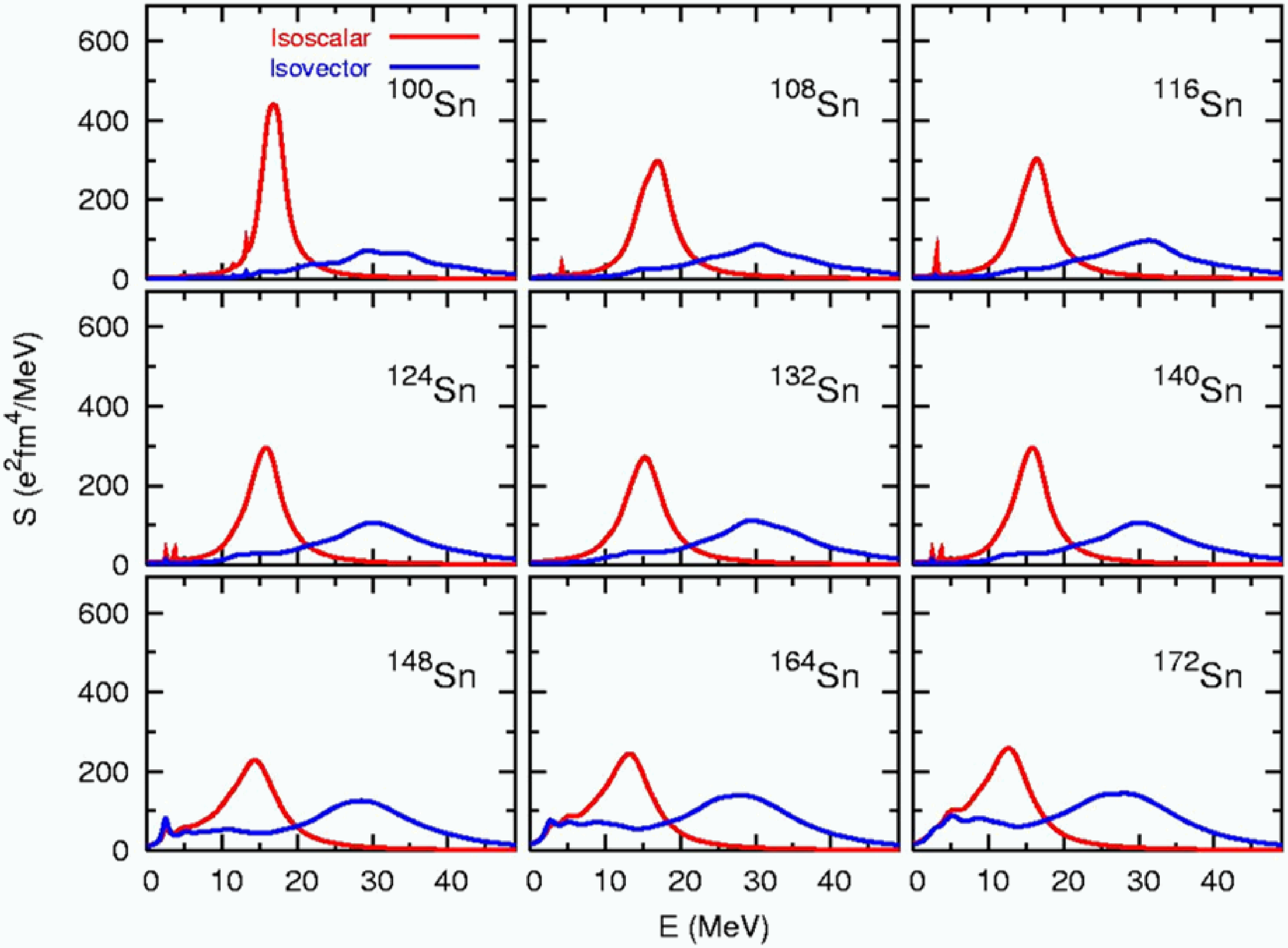
J. Terasaki, *et al.*, to be published & nucl-th/0407111

Isoscalar 1$^-$ modes, SkM* + volume-type pairing



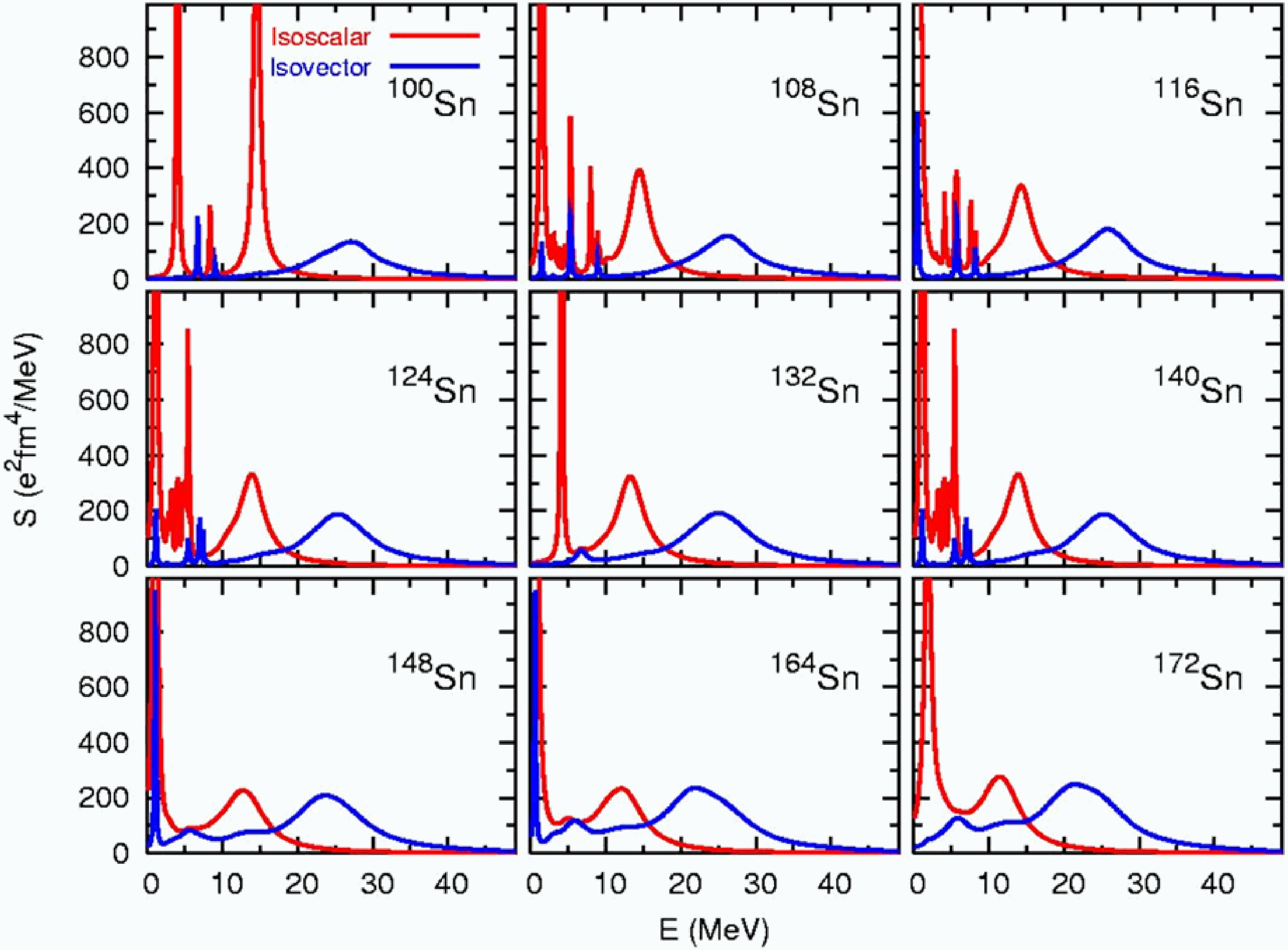
J. Terasaki, *et al.*, to be published & nucl-th/0407111

0^+ modes, SkM* + volume-type pairing



J. Terasaki, *et al.*, to be published & nucl-th/0407111

2^+ modes, SkM* + volume-type pairing



J. Terasaki, *et al.*, to be published & nucl-th/0407111

Lessons learnt

Conclusions

Lessons learnt

- Differences of **energy scales** between the QCD and nuclear structure allow for derivations of nuclear properties from fundamental principles.
- Properties of heavy exotic nuclei, including those near drip lines, can now be studied within **microscopic models** based on effective interactions and energy density functionals.
- Search for the best possible universal interactions and functionals should be continued taking into account experimental data on **exotic nuclei**.
- In weakly bound nuclei, the **positive-energy phase space** (the so-called continuum) significantly contributes to the ground- and excited-state wave functions, and has to be taken into account.
- **Collective correlations** are essential for a detailed description of ground state properties.
- New features of **collective excitations** are expected to appear in exotic nuclei.

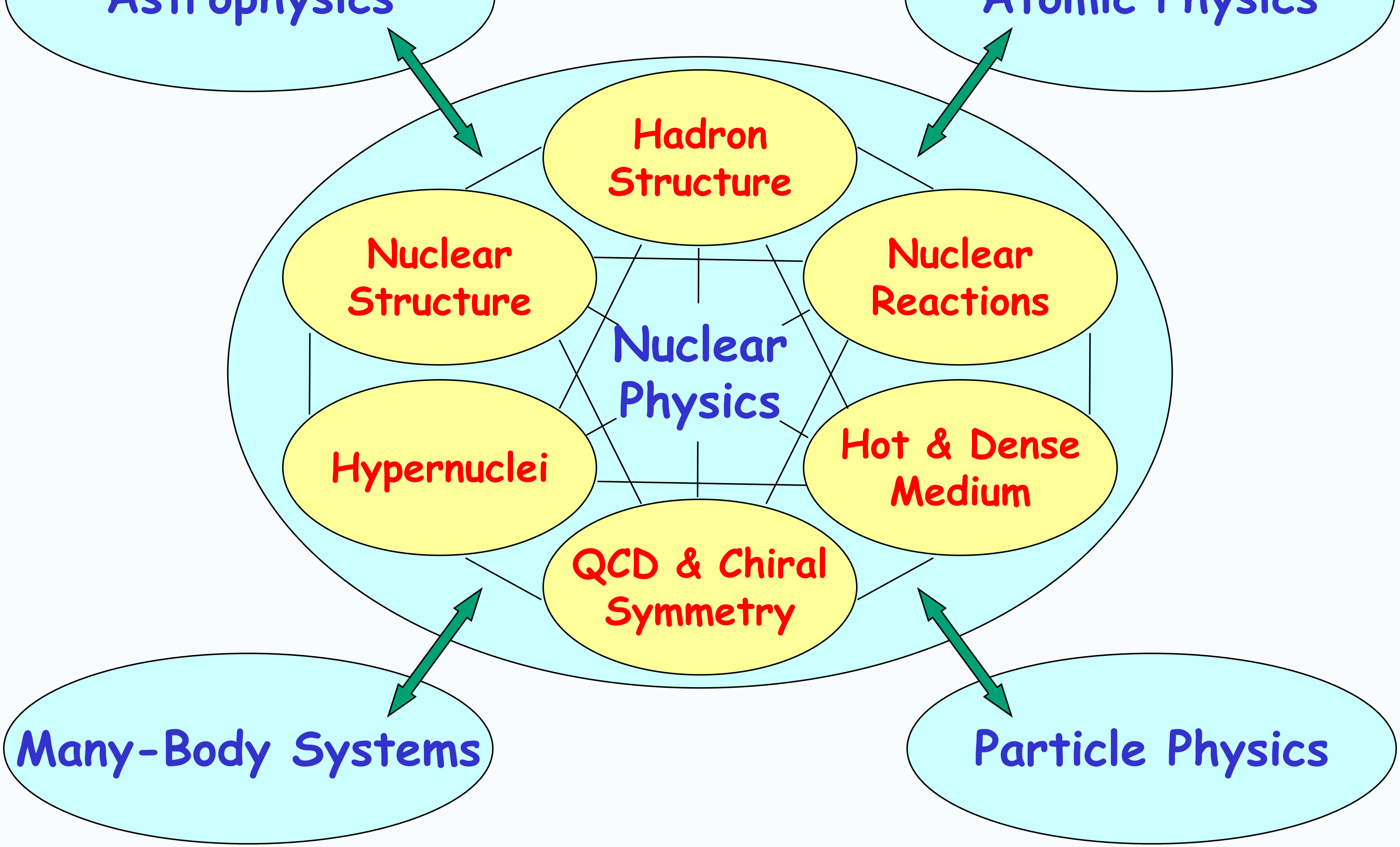
Lessons learnt

Lessons learnt

In collaboration with: K. Amos,, M. Bender, K. Bennaceur, J. Engel, S. Karataglidis W. Nazarewicz, M.V. Stoitsov, and J. Terasaki.

Thank you

Nuclear-Physics connections



Nuclear-Physics connections

Lessons learnt

The present-day nuclear physics contains not only the traditional subdomains, such as the nuclear structure, nuclear reactions, and physics of hypernuclei, but also those that not-so-long-ago where part and parcel of the particle physics, like the hadron structure, hot and dense nuclear matter, and low-energy aspects of the QCD.

Nuclear physics not only has very strong ties to astrophysics (stars are born and die via nuclear processes) and particle physics (weak interactions and neutrinos), but also to atomic physics (tests of fundamental interactions through the electron-nucleus interaction), and to the general physics of many-body systems (condensed matter and solid state physics).

Fundamental matter and interaction fields

FERMIONS
matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2		
Flavor	Mass GeV/c^2	Electric charge
ν_e electron neutrino	$<1\times10^{-8}$	0
e electron	0.000511	−1
ν_μ muon neutrino	<0.0002	0
μ muon	0.106	−1
ν_τ tau neutrino	<0.02	0
τ tau	1.7771	−1

Quarks spin = 1/2		
Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.003	2/3
d down	0.006	−1/3
c charm	1.3	2/3
s strange	0.1	−1/3
t top	175	2/3
b bottom	4.3	−1/3

BOSONS
force carriers
spin = 0, 1, 2, ...

Unified Electroweak spin = 1		
Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W$^-$	80.4	−1
W$^+$	80.4	+1
Z^0	91.187	0

Strong (color) spin = 1		
Name	Mass GeV/c^2	Electric charge
g gluon	0	0

Nuclear Physics studies composite objects that are built of light quarks **uds** and interact by exchanging gluons **g**. The complete theory is defined by the QCD Lagrangian:

$$\begin{aligned}\mathcal{L} = \; & - \; \tfrac{1}{4}F^{\alpha}_{\mu\nu}F^{\mu\nu}_{\alpha} \\ & - \; \Sigma_n\bar{\psi}_n\gamma^{\mu}[\partial_\mu - igA^{\alpha}_{\mu}t_\alpha]\psi_n \\ & - \; \Sigma_n m_n\bar{\psi}_n\psi_n\end{aligned}$$

where

$$F^{\alpha}_{\mu\nu} = \partial_\mu A^{\alpha}_{\nu} - \partial_\nu A^{\alpha}_{\mu} + C^{\alpha}_{\beta\gamma}A^{\beta}_{\mu}A^{\gamma}_{\nu}$$

Quarks and gluons also interact electro-weakly with electrons **e** and neutrinos, ν_e and ν_μ, by exchanging photons γ and bosons W^-, W^+, Z^0.

http://www.cpepweb.org/

Quantum field theory in four minutes

Minute 1

- Classical mechanics:
  - Lagrangian:

$$\mathcal{L} = \mathcal{L}(q_i, \dot{q}_i, t) = T - U$$

  - Euler-Lagrange equations:

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_i} - \frac{\partial \mathcal{L}}{\partial q_i} = 0$$

  - Hamiltonian:

$$\mathcal{H}(q_i, p_i, t) = \sum_i p_i \dot{q}_i - \mathcal{L} \quad , \quad \text{for} \quad p_i \equiv \frac{\partial \mathcal{L}}{\partial \dot{q}_i}$$

Quantum field theory in four minutes

$\boxed{\text{Minute 2}}$

- Quantum mechanics:
  - Quantization:

$$\hat{q}_i = q_i \quad , \quad \hat{p}_i = -i\hbar\frac{\partial}{\partial q_i}$$

$$\mathcal{H}(q_i, p_i, t) \longrightarrow \hat{\mathcal{H}}(\hat{q}_i, \hat{p}_i, t)$$

  - Schrödinger equation:

$$i\hbar\frac{\partial}{\partial t}\Psi(q_i, t) = \hat{\mathcal{H}}(\hat{q}_i, \hat{p}_i, t)\Psi(q_i, t)$$

Quantum field theory in four minutes

Minute 3

- Classical field theory:
  - Generalized variables:

$$i \longrightarrow \vec{x}$$

$$q_i \longrightarrow \psi_{\vec{x}} \equiv \psi(\vec{x})$$

  - Local Lagrangian density $\mathcal{L}[\psi(\vec{x}), \partial_\mu \psi(\vec{x})]$ defines the Lagrangian:

$$L = \int d^3\vec{x}\ \mathcal{L}[\psi(\vec{x}), \partial_\mu \psi(\vec{x})]$$

  - Action is relativisticaly invariant, $x \equiv (\vec{x}, t)$:

$$I = \int dt\ L[\psi(\vec{x}), \dot{\psi}(\vec{x})] = \int d^4x\ \mathcal{L}[\psi(x), \partial_\mu \psi(x)]$$

Quantum field theory in four minutes

$\boxed{\text{Minute 4}}$

- Quantum field theory:
  - Quantization:

$$\hat{\psi}(\vec{x}) = \psi(\vec{x}) \quad , \quad \widehat{\text{momentum}} = -i\hbar\frac{\delta}{\delta\psi(\vec{x})}$$

  - Some “details”:
    1. Spin
    2. Statistics
    3. Perturbation theory
    4. . . .

Chiral symmetry and isospin

$$\mathcal{L}_\chi = -\bar{q}\gamma^\mu D_\mu q = -\bar{u}\gamma^\mu D_\mu u - \bar{d}\gamma^\mu D_\mu d$$

- Covariant derivative: $D_\mu = \partial_\mu - igA^\alpha_\mu t_\alpha$
- Quark iso-spinor: $q = \begin{pmatrix} u \\ d \end{pmatrix}$
- Quark masses neglected for a moment.
- $\mathcal{L}_\chi$ is invariant with respect to the SU(2)×SU(2) group generated by the mixing of the u and d quarks.
- SU(2)×SU(2) generators:
  1° the isospin matrices: $\vec{t} = \frac{1}{2}\vec{\tau}$

  $\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

  2° the γ_5×isospin matrices: $\vec{x} = \gamma_5\vec{t}$ (remember that $(\gamma_5)^2 = 1$)
- SU(2)×SU(2) commutation relations:
  $\vec{t}_L = \frac{1}{2}(1+\gamma_5)\vec{t} = \frac{1}{2}(\vec{t}+\vec{x}) \quad \longleftarrow$ left-handed
  $\vec{t}_R = \frac{1}{2}(1-\gamma_5)\vec{t} = \frac{1}{2}(\vec{t}-\vec{x}) \quad \longleftarrow$ right-handed
  $[t_{Li}, t_{Lj}] = i\epsilon_{ijk}t_{Lk}, \qquad [t_{Ri}, t_{Rj}] = i\epsilon_{ijk}t_{Rk}, \qquad [t_{Li}, t_{Rj}] = 0$
- Group isomorphism: SU(2)×SU(2) $\equiv$ O(4)

Chiral symmetry breaking

$$\mathcal{L}_\chi = -\bar{u}\gamma^\mu D_\mu u - \bar{d}\gamma^\mu D_\mu d - m_u\bar{u}u - m_d\bar{d}d$$

- Quark masses are small as compared to the QCD scale:
  $m_u \approx 3\,\mathrm{MeV}$, $m_d \approx 6\,\mathrm{MeV}$, $\Lambda_{QCD} \approx 1000\,\mathrm{MeV}$
- Quark masses weakly break the chiral symmetry
- Chiral symmetry is strongly broken in the real world: pairs of particles having similar masses and opposite parities are not observed.
- Strong chiral symmetry breaking: SU(2)×SU(2) broken while the isospin SU(2) conserved.
- Effective theories are needed to describe complicated composite objects like mesons and nucleons.
- Fields of composite objects can be treated as elementary fields.
- Lagrangians of effective fields can be built based on the symmetry requirements.
- Pion mass is given by the quark masses:

$$m_\pi = -4(m_u + m_d)\langle\Phi_4^+\rangle_{\mathrm{vac}}/F_\pi^2$$

Linear σ model

Let ϕ_n, n=1,2,3,4, denote the pseudoscalar (real) fields described by the Lagrangian

$$\mathcal{L}_\sigma = -\tfrac{1}{2}\partial_\mu\phi_n\partial^\mu\phi_n - \tfrac{1}{2}\mathcal{M}^2\phi_n\phi_n - \tfrac{1}{4}g(\phi_n\phi_n)^2$$

$\mathcal{L}_\sigma$ is explicitly invariant with respect to rotations in 4 dimensions: O(4) $\equiv$ SU(2)×SU(2) symmetry. However, the potential energy depends only on the radial variable σ^2=$\phi_n\phi_n$

$$V(\phi) = V(\sigma) = \tfrac{1}{2}\mathcal{M}^2\sigma^2 + \tfrac{1}{4}g\sigma^4$$

For g>0 and $\mathcal{M}^2$<0 it has the minimum at

$$\sigma = |\mathcal{M}|/\sqrt{g}$$

and does not depend on the orientation of ϕ in the 4-dim space.

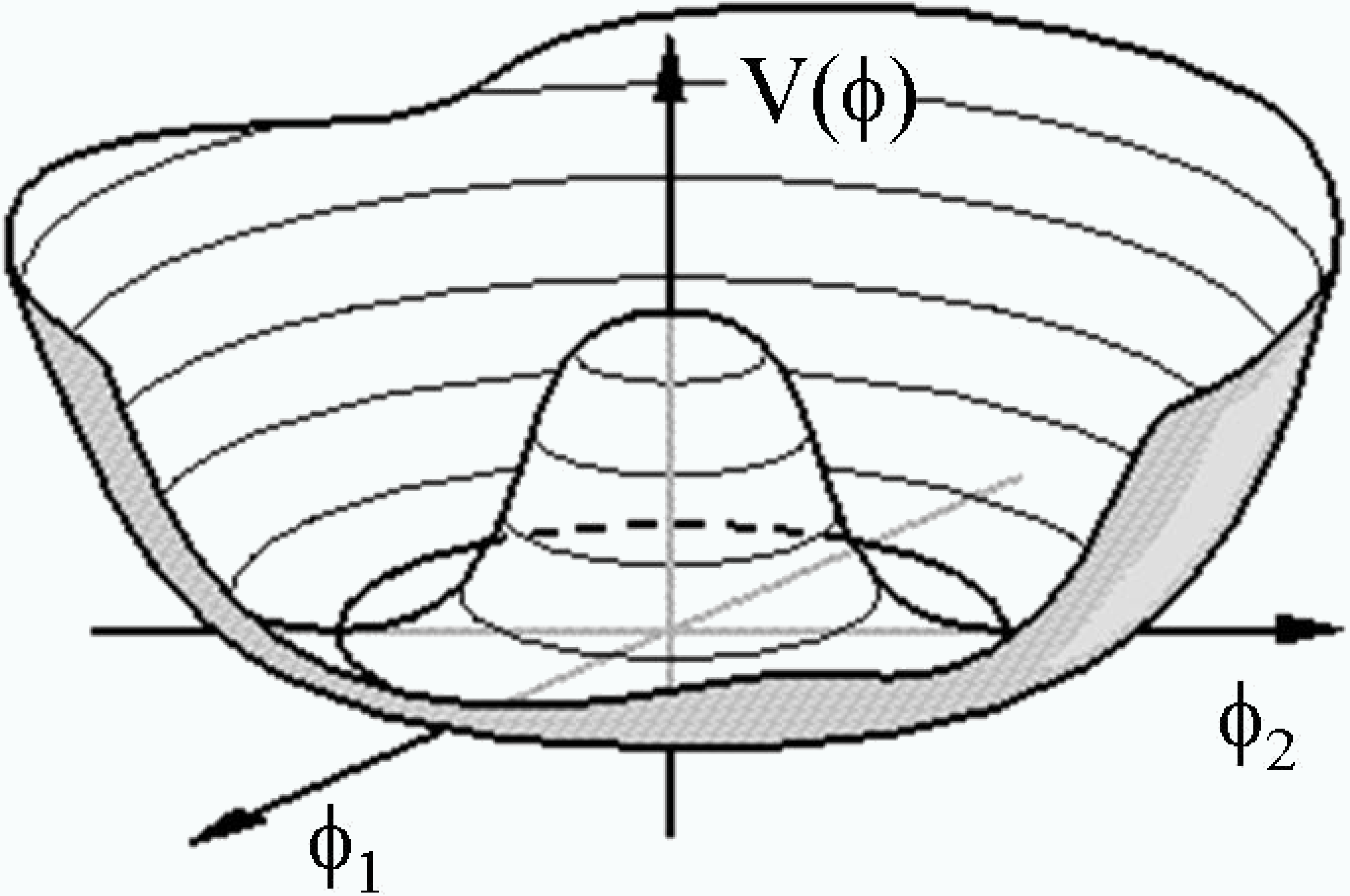
By picking one solution $\bar{\phi}$ out of the infinitely-many existing ones, we break the $O(4)$ symmetry.

The mass-matrix (the stiffness of the potential) calculated at $\bar{\phi}$ reads:

$$\mathcal{M}_{nm} = \frac{\partial^2 V}{\partial\phi_n\partial\phi_m} = 2g\bar{\phi}_n\bar{\phi}_m$$

It has one eigenvalue equal to $m^2 = 2g\sigma^2 = -2\mathcal{M}^2$ (eigenvector $\bar{\phi}$) and three eigenvalues m^2=0 (three vectors orthogonal to $\bar{\phi}$) — the Goldstone bosons — the three pions: $\vec{\pi} = (\pi_+, \pi_0, \pi_-)$.

Mexican-hat potential



Non-linear σ model

In order to separate out the radial variable σ in the 4-dimensional space we set:

$$\vec{\phi} = \frac{2\vec{z}}{1+\vec{z}^2}\sigma \qquad (n = 1, 2, 3)$$

$$\phi_4 = \frac{1-\vec{z}^2}{1+\vec{z}^2}\sigma$$

The Lagrangian expressed in the new fields $\vec{z}$ and σ reads

$$\mathcal{L}_\sigma = -2\sigma^2 \frac{\partial_\mu \vec{z} \partial^\mu \vec{z}}{(1+\vec{z}^2)^2} - \tfrac{1}{2}\partial_\mu \sigma \partial^\mu \sigma - \tfrac{1}{2}\mathcal{M}^2\sigma^2 - \tfrac{1}{4}g\sigma^4$$

- Pion field $\vec{\pi}$ is equal to $\vec{z}$ up to a normalization constant:

$$\vec{\pi} = F\vec{z}$$

- Details of the part depending on σ are irrelevant (high-energy part).
- Pions fields must couple to other fields (e.g. to nucleons) through the covariant derivatives $\vec{D}_\mu = \frac{\partial_\mu \vec{z}}{1+\vec{z}^2}$

Chiral symmetry breaking

Lessons learnt

The chiral symmetry — basic property of the QCD Lagrangian for massless quarks — must be obeyed by all composite fields made of quarks and gluons. Had it been unbroken in Nature, all observed fields would have appeared in pairs of opposite parities, which is obviously not the case; hence it must be spontaneously broken. The so-called σ model provides a simple illustration of the breaking mechanism, whereupon the four-dimensional chiral-invariant real fields become separated into the radial field, which spontaneously acquires non-zero value in the vacuum — the chiral condensate — and three orthogonal fields that represent the massless Goldstone bosons related to the broken chiral symmetry — the pions. The mass of physical pions results from the explicit breaking of the chiral symmetry caused by non-zero masses of quarks. Differences in masses of the three pions result from the coupling to the electromagnetic field.

Nucleon-pion Lagrangian

$$\mathcal{L}_{N\pi} = -\bar{N}\left(\gamma^\mu\partial_\mu + g_\phi[\phi_4 + 2i\gamma_5\vec{t}\cdot\vec{\phi}]\right)N$$

In the angle-radial variables this reads:

$$\mathcal{L}_{N\pi} = -\bar{\widetilde{N}}\left(\gamma^\mu\partial_\mu + g_\phi\sigma + 2i\vec{t}\cdot(\vec{z}\times\gamma^\mu\vec{\mathcal{D}}_\mu)\right.$$
$$\left. + 2ig_A\gamma_5\vec{t}\cdot\gamma^\mu\vec{\mathcal{D}}_\mu\right)\widetilde{N}$$

- Nucleon iso-spinor: $N = \begin{pmatrix} p \\ n \end{pmatrix}$
- Transformed nucleon field: $\widetilde{N} = (1 + 2i\gamma_5\vec{t}\cdot\vec{z})N/\sqrt{1+\vec{z}^2}$
- Nucleon mass results from the chiral symmetry breaking: $m_N = g_\phi\sigma$
- Axial-vector coupling constant g_A introduced "by hand", because the last term is separately chiral invariant.
- Nucleons must couple through the covariant derivatives $\vec{\mathcal{D}}_\mu = \partial_\mu + 2i\vec{t}\cdot(\vec{z}\times\vec{D}_\mu)$
- In the lowest order the nucleons interact by a one-pion-exchange potential (OPEP).

Effective Lagrangians

Instead of solving simple Lagrangians to all orders one may solve corrected Lagrangians to low orders:

$$\mathcal{L}^{\text{eff}}_{\pi\pi} = -\tfrac{1}{2}F^2\vec{D}_\mu\vec{D}^\mu - \tfrac{1}{4}c_4\left(\vec{D}_\mu\vec{D}^\mu\right)^2 - \tfrac{1}{4}c'_4\left(\vec{D}_\mu\vec{D}_\nu\right)\left(\vec{D}^\mu\vec{D}^\nu\right) + \ldots$$

- Pion coupling constant: $F = 2\sigma$
- $\mathcal{L}^{\text{eff}}_{\pi\pi}$ gives a good description of the $\pi\pi$ scattering lengths.

$$\mathcal{L}^{\text{eff}}_{N\pi} = -\bar{\tilde{N}}\left(\gamma^\mu\vec{\mathcal{D}}_\mu + m_N + 2ig_A\gamma_5\vec{t}\cdot\gamma^\mu\vec{D}_\mu\right)\tilde{N}$$

$$-c_{2\alpha\beta}\left(\bar{\tilde{N}}\Gamma_\alpha\tilde{N}\right)\left(\bar{\tilde{N}}\Gamma_\beta\tilde{N}\right) + \ldots$$

- Projectors on different spin-isospin channels: Γ_α
- $\mathcal{L}^{\text{eff}}_{N\pi}$ gives a good description of the NN scattering lengths.
- Supplemented by the next order gives a good description of the NN phase shifts: D.R. Entem and R. Machleidt, Phys. Lett. B524, 93 (2002).

Emission of long electromagnetic waves

(I) EXACT

For an arbitrary current $\vec{J}(\vec{x}', t')$:

$$\vec{A}(\vec{x},t) = \frac{1}{c}\int \mathrm{d}^3\vec{x}' \int \mathrm{d}t' \, \frac{\delta(t' - (t - \frac{1}{c}|\vec{x}' - \vec{x}|))}{|\vec{x}' - \vec{x}|} \vec{J}(\vec{x}', t')$$

For harmonic currents (or a single Fourier component),

$$\vec{J}(\vec{x},t) = \vec{J}(\vec{x})e^{-i\omega t},$$

the fields are also harmonic,

$$\vec{A}(\vec{x},t) = \vec{A}(\vec{x})e^{-i\omega t},$$

and the amplitudes outside the sources read

$$\vec{A}(\vec{x}) = \frac{4\pi i}{c}\sum\nolimits_{lm} k^{l+1} h_l^{(1)}(kr) Y_{lm}(\theta,\phi) \vec{M}_{lm}(k)$$

for

$$\vec{M}_{lm}(k) = \frac{1}{k^l}\int \mathrm{d}^3\vec{x}' j_l(kr') Y^*_{lm}(\theta',\phi') \vec{J}(\vec{x}')$$

and $k = \omega/c$.

Emission of long electromagnetic waves

(II) APPROXIMATE

Details of the current distribution become totally invisible when a long wave, $kr'{\ll}1$, is recorded at a large distance, $kr{\gg}1$:

$$\vec{A}(\vec{x}) = \frac{4\pi}{c}\frac{e^{ikr}}{r}\sum\nolimits_{lm}(-ik)^l Y_{lm}(\theta,\phi)\vec{M}_{lm}$$

for

$$\vec{M}_{lm} = \frac{1}{(2l+1)!!}\int \mathrm{d}^3\vec{x}' r'^l Y^*_{lm}(\theta',\phi')\vec{J}(\vec{x}')$$

Within the long wavelength approximation, only a few numbers (the multipole moments $\vec{M}_{lm}$) are needed to fully describe the emitted radiation. Details of current distribution inside the source become irrelevant.

Blue-sky problem - Compton scattering

(I) CLASSICAL EM: A charge q confined by a potential with eigen-frequency of ω_0, shaken by an external force F with frequency ω, radiates a wave with power P:

$$P = \frac{q^2F^2}{3c^3m^2}\frac{\omega^4}{(\omega^2-\omega_0^2)^2}$$

(II) QED: Sum of three 2nd order diagrams.
(III) EFT: The energy density H^0_{eff} of an atom in state Ψ reads:

$$H^0_{\text{eff}} = \Psi^*\left(\frac{p^2}{2m}+e\phi\right)\Psi$$

When the atom is placed in an EM field it acquires additional energy density H^1_{eff} that must be a scalar, T-even, and P-even function of fields, i.e., for sufficiently weak fields:

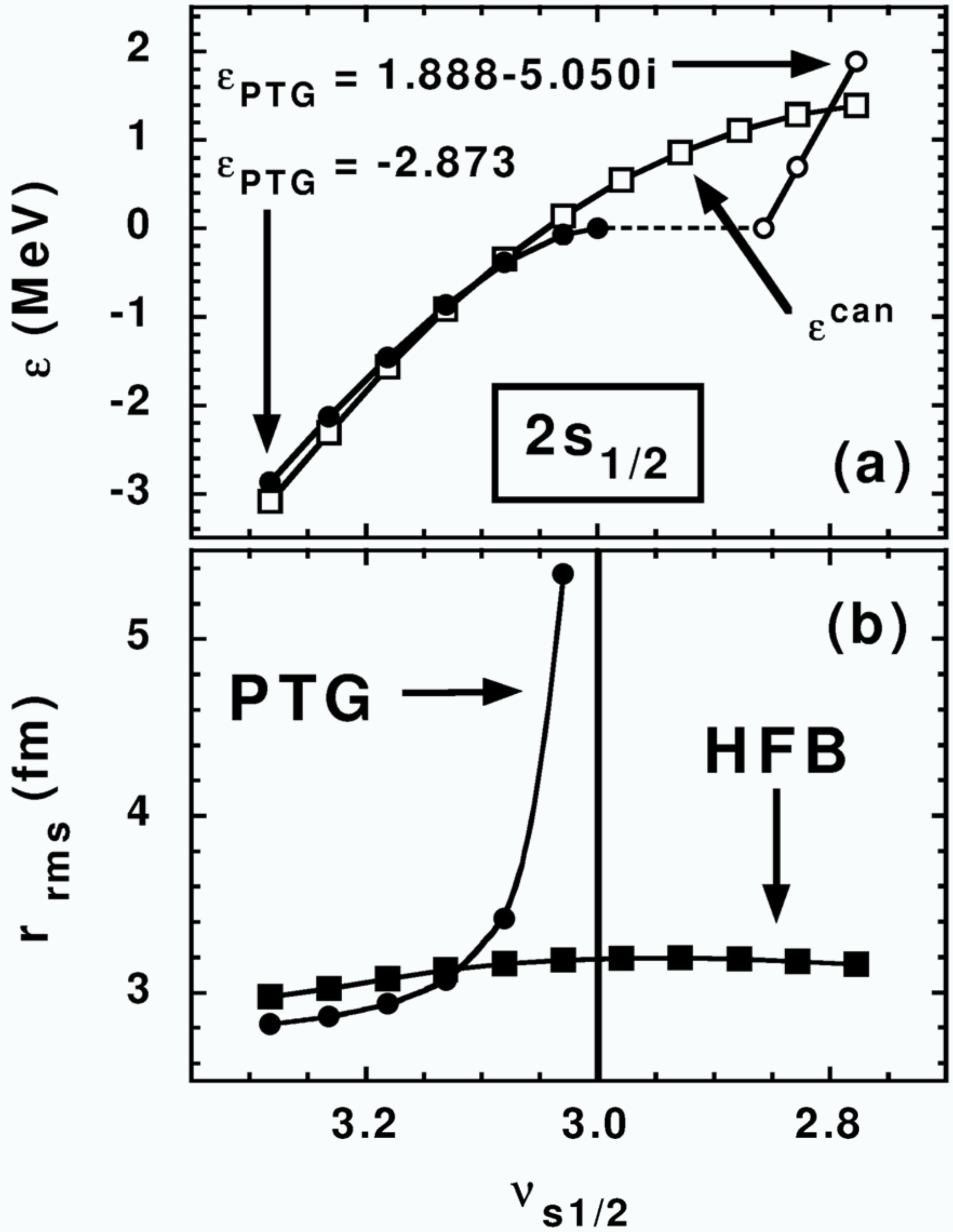
$$H^1_{\text{eff}} = -\frac{1}{2}\Psi^*\Psi\left(c_E\vec{E}^2+c_B\vec{B}^2\right)$$

Since the coupling constants c_E and c_B have dimensions of a volume, they must be related to the volume of the atom a_0^3 by: $c_E = \chi_E a_0^3$, $c_B = \chi_B a_0^3$ with dimensionless coupling constants χ_E and χ_B of the order of 1. Finaly, for the EM wave, $|\vec{E}|{\sim}\omega$ and $|\vec{B}|{\sim}\omega$ we obtain:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = |\langle f|H^1_{\text{eff}}|i\rangle|^2 \sim \omega^4 a_0^6.$$

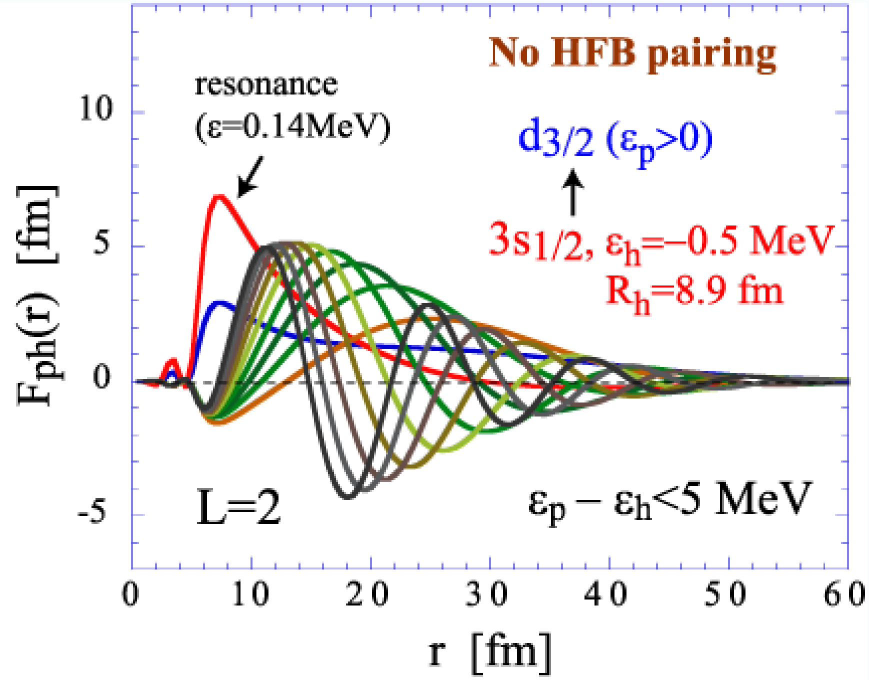
Pairing anti-halo effect

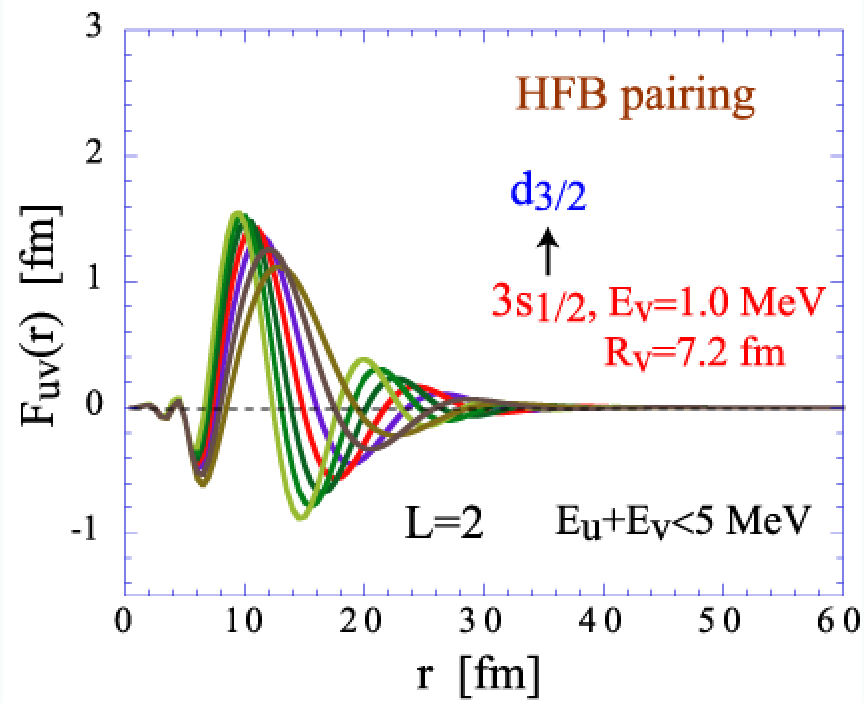
Parameter $\nu_{s1/2}$ of the Poschel-Teller-Ginocchio model is varied in such a way that the $2s_{1/2}$ single-particle state crosses the threshold of zero binding $\varepsilon=0$ and becomes a virtual state and then a resonance. When approaching the threshold, its rms radius increases to infinity. With the pairing correlations included, the HFB rms radius of the canonical $2s_{1/2}$ state remains always finite.



K. Bennaceur, *et al.*, Phys.Lett. B496, 154 (2000)

Roles of pairing anti-halo effect





Restoration of spatial localization by pairing anti-halo effect in HFB

This figure was presented by M. Yamagami at ENAM'04

Asymptotic properties of mean-field single-particle wave functions

Single-particle mean-field wave function $\phi(r)$ is a solution of the Schrödinger equation:

$$\left(-\frac{\hbar^2}{2m}\Delta + V(r)\right)\phi(r) = \epsilon\phi(r).$$

For ℓ=0 and potential $V(r)$ vanishing at infinity the asymptotic solution reads:

$$\phi(r) \simeq \exp(-\kappa r)/r \quad \text{for} \quad r \to \infty$$

where

$$\kappa = \sqrt{-2m\epsilon}/\hbar$$

For small binding energy ϵ, the root-mean-squared radius

$$R^2_{\text{rms}} = \frac{\int 4\pi r^2 dr r^2 \phi^2(r)}{\int 4\pi r^2 dr \phi^2(r)}$$

is determined by the asymptotic solution, and hence:

$$R^2_{\text{rms}} \simeq \frac{\int dr r^2 \exp(-2\kappa r)}{\int dr \exp(-2\kappa r)}$$

The integrals are elementary:

$$\int dr \exp(-2\kappa r) \sim \frac{1}{2\kappa}$$

$$\int dr r^2 \exp(-2\kappa r) \sim \frac{1}{4}\frac{d^2}{d\kappa^2}\frac{1}{2\kappa} = \frac{1}{16\kappa^3},$$

and hence

$$R_{\text{rms}} \simeq \frac{1}{\sqrt{8}\kappa} = \frac{\hbar}{4\sqrt{m}}(-\epsilon)^{-1/2}$$

while for $\ell \neq 0$

$$R_{\text{rms}} \simeq \frac{\hbar}{4\sqrt{m}}(-\epsilon)^{-1/2+\ell/4}$$

Asymptotic properties of paired quasiparticle wave functions

Quasiparticle two-component wave function $(\phi_1(r),\phi_2(r))$ is a solution of the HFB equation:

$$\begin{aligned}\left(-\frac{\hbar^2}{2m}\Delta + V(r) - \lambda\right)\phi_1(r) + \Delta(r)\phi_2(r) &= E\phi_1(r) \\ \left(-\frac{\hbar^2}{2m}\Delta + V(r) - \lambda\right)\phi_2(r) - \Delta(r)\phi_1(r) &= -E\phi_2(r)\end{aligned}.$$

For ℓ=0 and potentials $V(r)$ and $\Delta(r)$ vanishing at infinity the asymptotic solution for the lower component $\phi_2(r)$ (which defines the particle density) reads:

$$\phi_2(r) \simeq \exp(-\kappa r)/r \quad \text{for} \quad r \to \infty$$

where

$$\kappa = \sqrt{2m(E-\lambda)}/\hbar$$

For small Fermi (λ) and quasiparticle (E) energies, the root-mean-squared radius behaves as

$$R_{\rm rms} \simeq \frac{\hbar}{4\sqrt{m}}(E-\lambda)^{-1/2+\ell/4}$$

i.e., even at the drip line ($\lambda = 0$) the radius stays finite for paired orbitals (pairing gap in the quasiparticle spectrum requires $E>\Delta$).

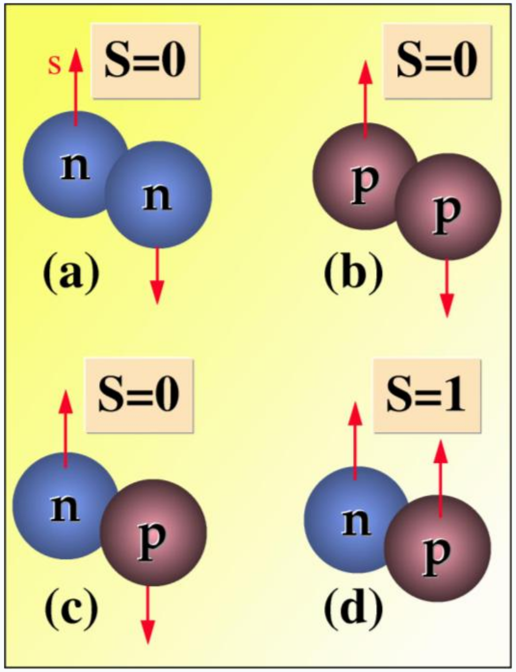
Asymptotic properties

Lessons learnt

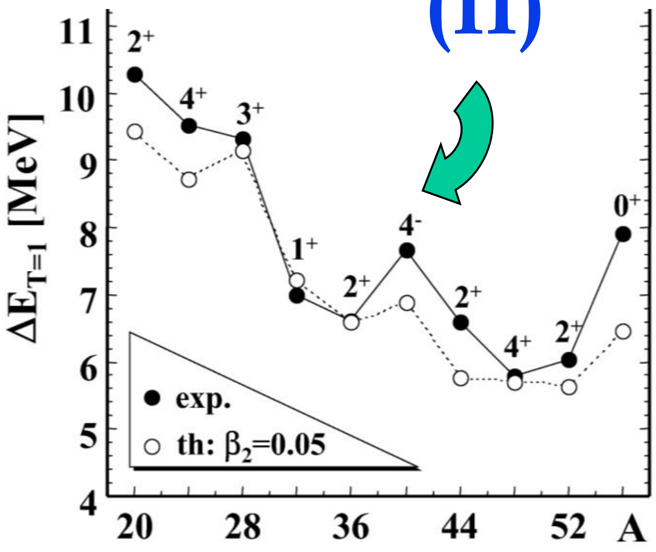
Asymptotic properties of nuclear wave functions determine the size and characteristics of nuclear halos. By assuming that neutrons are bound only by the mean-field potential, one concludes that weakly-bound s and p waves induce halos of infinite size. Nuclear correlations induce additional binding, which qualitatively modifies this conclusion. In particular, pairing correlations lead to additional binding of a weakly-bound neutron within the pair, and hence always lead to finite halos (the pairing anti-halo effect). Nevertheless, halos obtained by occupying weakly-bound s and p states are larger than those corresponding to higher orbital momenta. Since the pairing correlations always induce some non-zero occupation of weakly-bound s and p states, the halo phenomenon may, in fact, occur more often than one might have guessed by using the pure mean-field picture without correlations.

Neutron-proton pairing collectivity

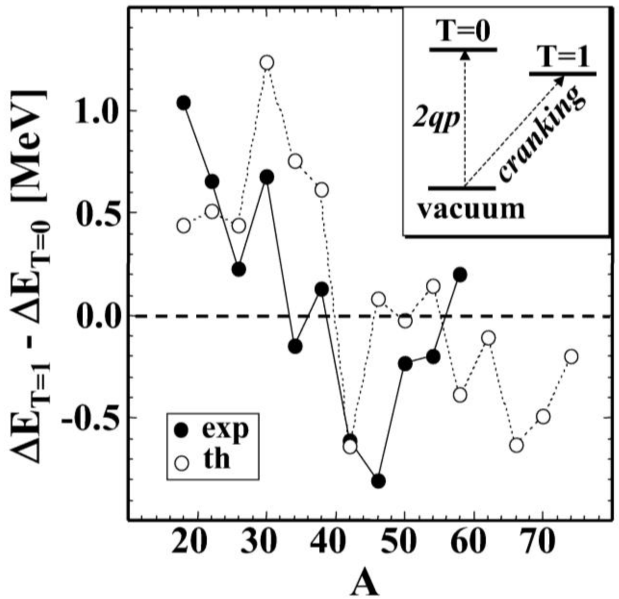
(a fit plus three easy steps)

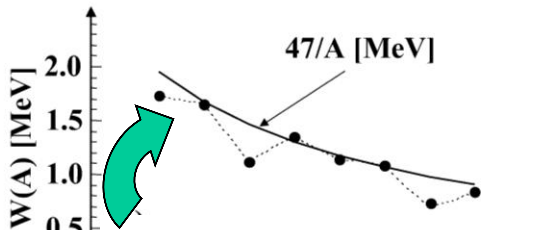


$E_{T=1} - E_{T=0}$ (even-even) **(II)**



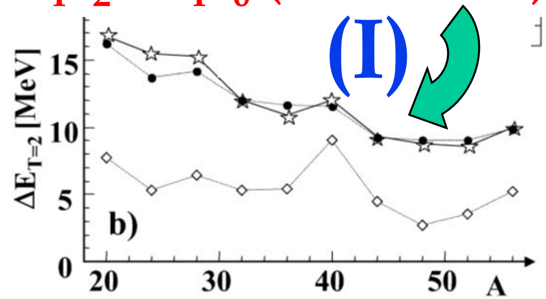
$E_{T=1} - E_{T=0}$ (odd-odd) **(III)**





Fit of $G_{T=0}/G_{T=1}$

$E_{T=2} - E_{T=0}$ (even-even) **(I)**

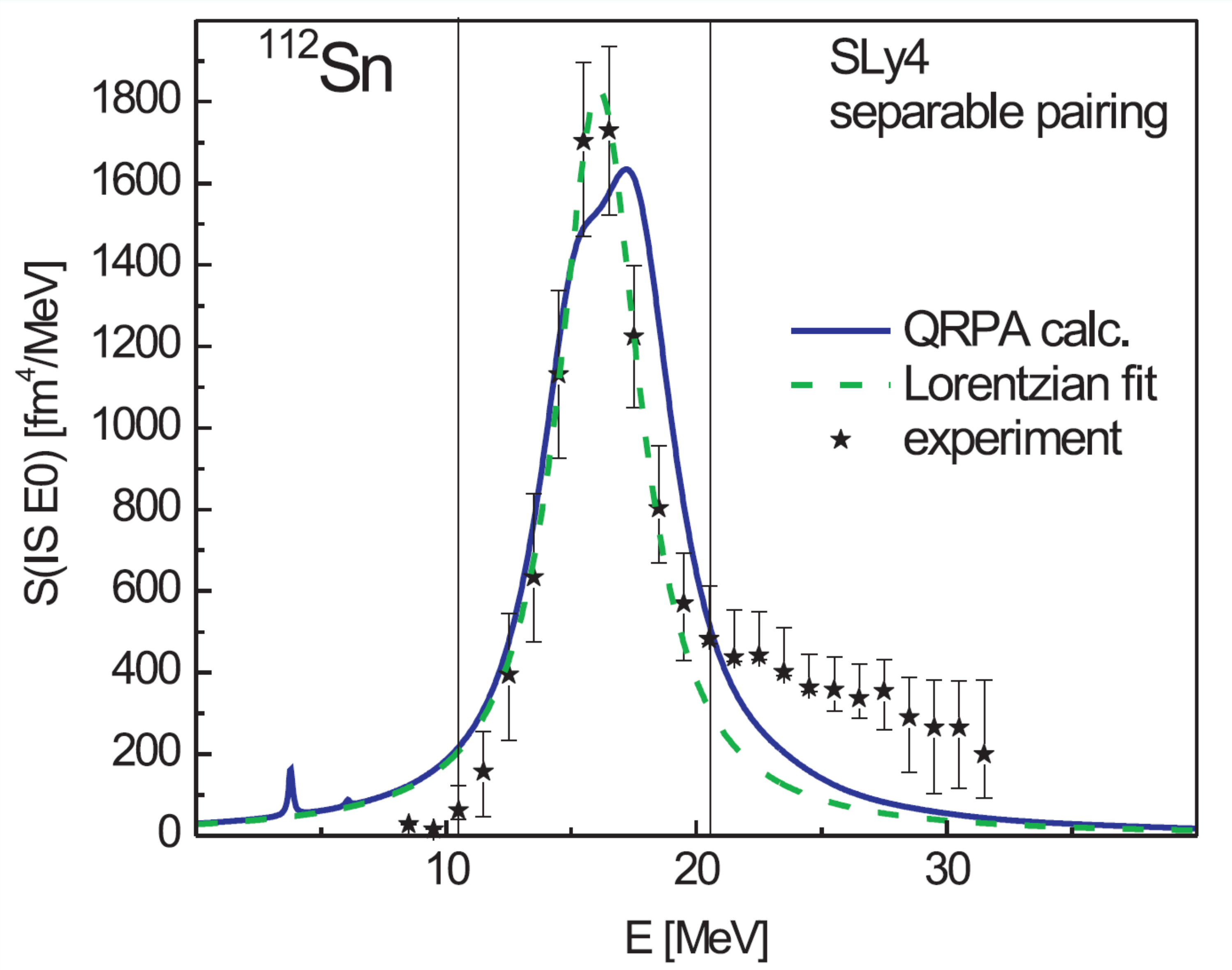


- Wigner energy linked to the n-p pairing collectivity
- T=2 states in even-even nuclei obtained from isocranking
- T=1 states in even-even nuclei obtained as 2qp excitations
- T=1 states in odd-odd nuclei obtained from isocranking
- T=0 states in odd-odd nuclei obtained as 2qp excitations

W. Satuła & R. Wyss Phys. Rev. Lett., 86, 4488 (2001); Phys. Rev. Lett., 87, 052504 (2001)

Giant monopole resonances

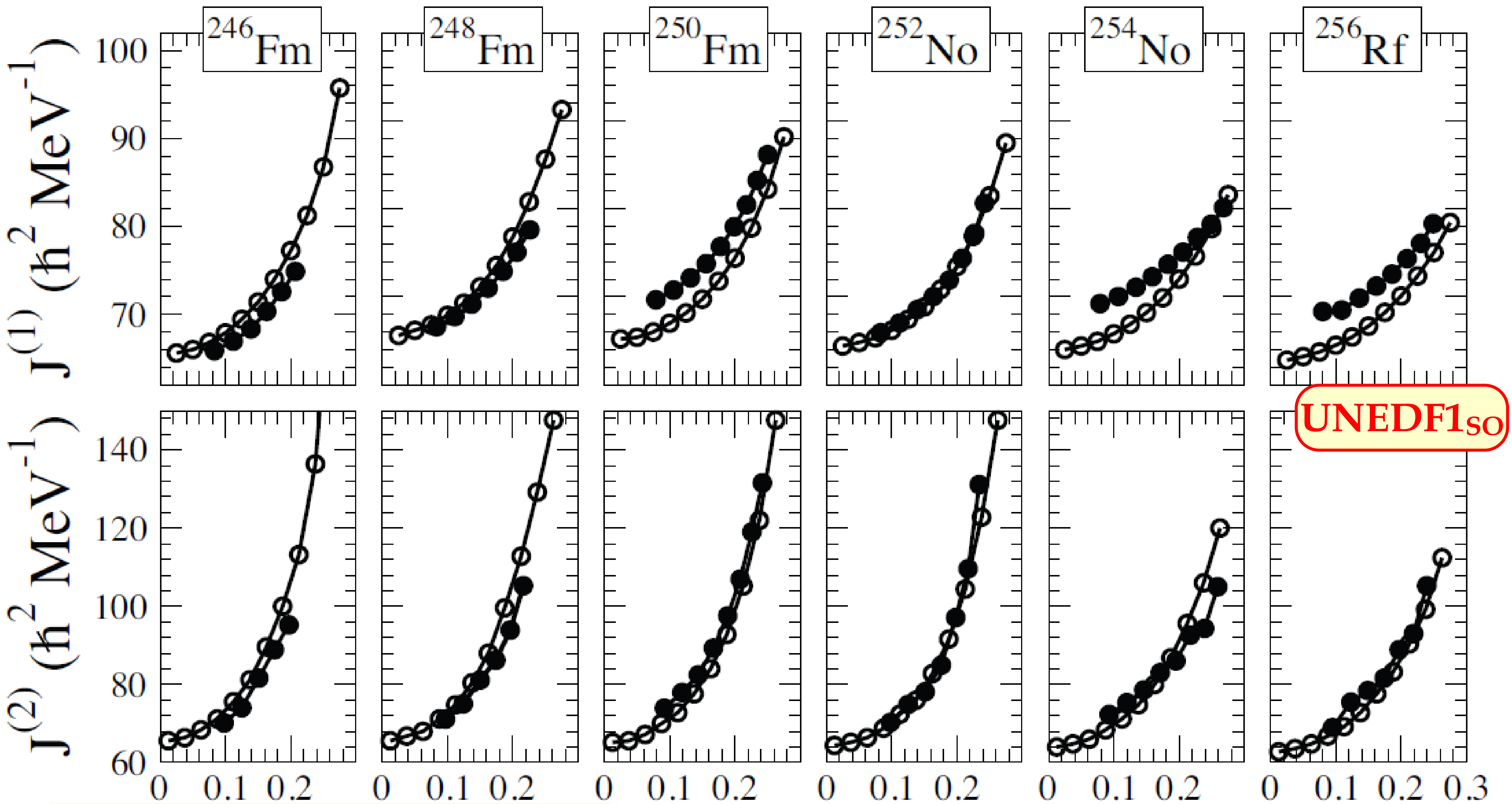
P. Veselý, *et al.*, C 86, 024303 (2012)



Experimental data from:
T. Li, U. Garg, Y. Liu, et al.,Phys. Rev. Lett. 99, 162503 (2007);
Phys. Rev. C 81, 034309 (2010))

Spectroscopy in the nobelium region

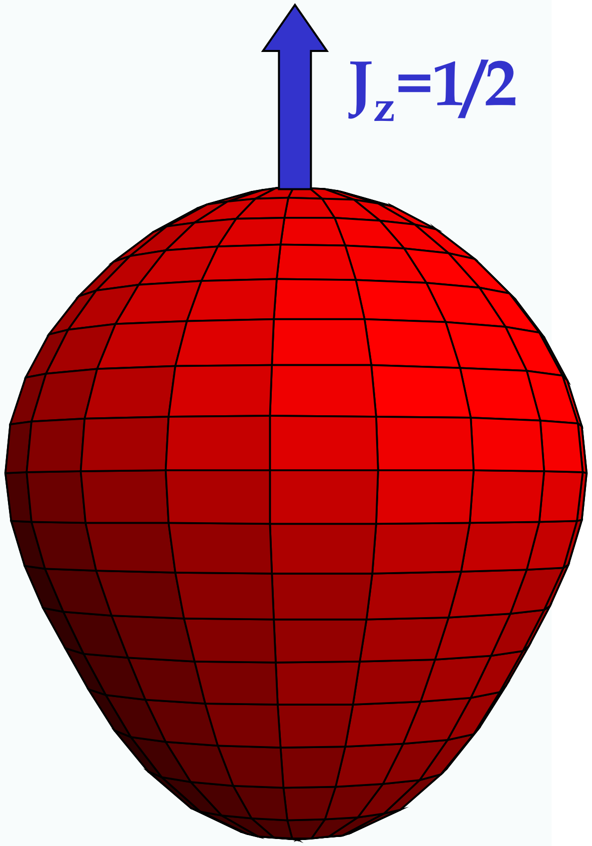
Y. Shi, J.D., P.T. Greenleees, Phys. Rev. C89, 034309 (2014)



full=experiment, open=theory

Skyrme-Hartree-Fock
J. Dobaczewski, J. Engel,
Phys. Rev. Lett. 94, 232502 (2005)

Experiment
R.G. Helmer *et al.*, Nucl. Phys. A474 (1987) 77



$\beta_{10} = 0.023$

$\beta_{20} = 0.161$

$\beta_{30} = -0.128$

$\beta_{40} = 0.091$

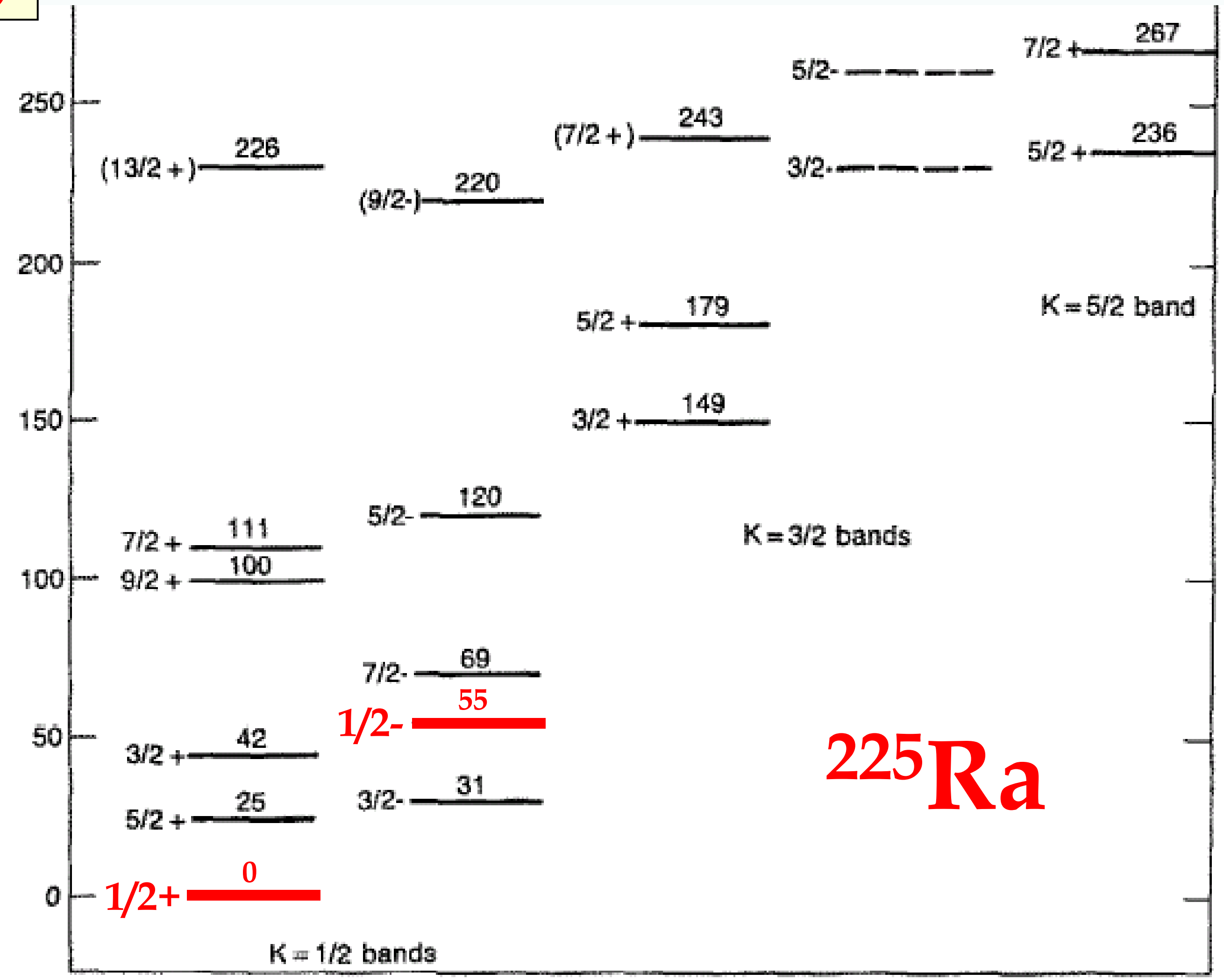
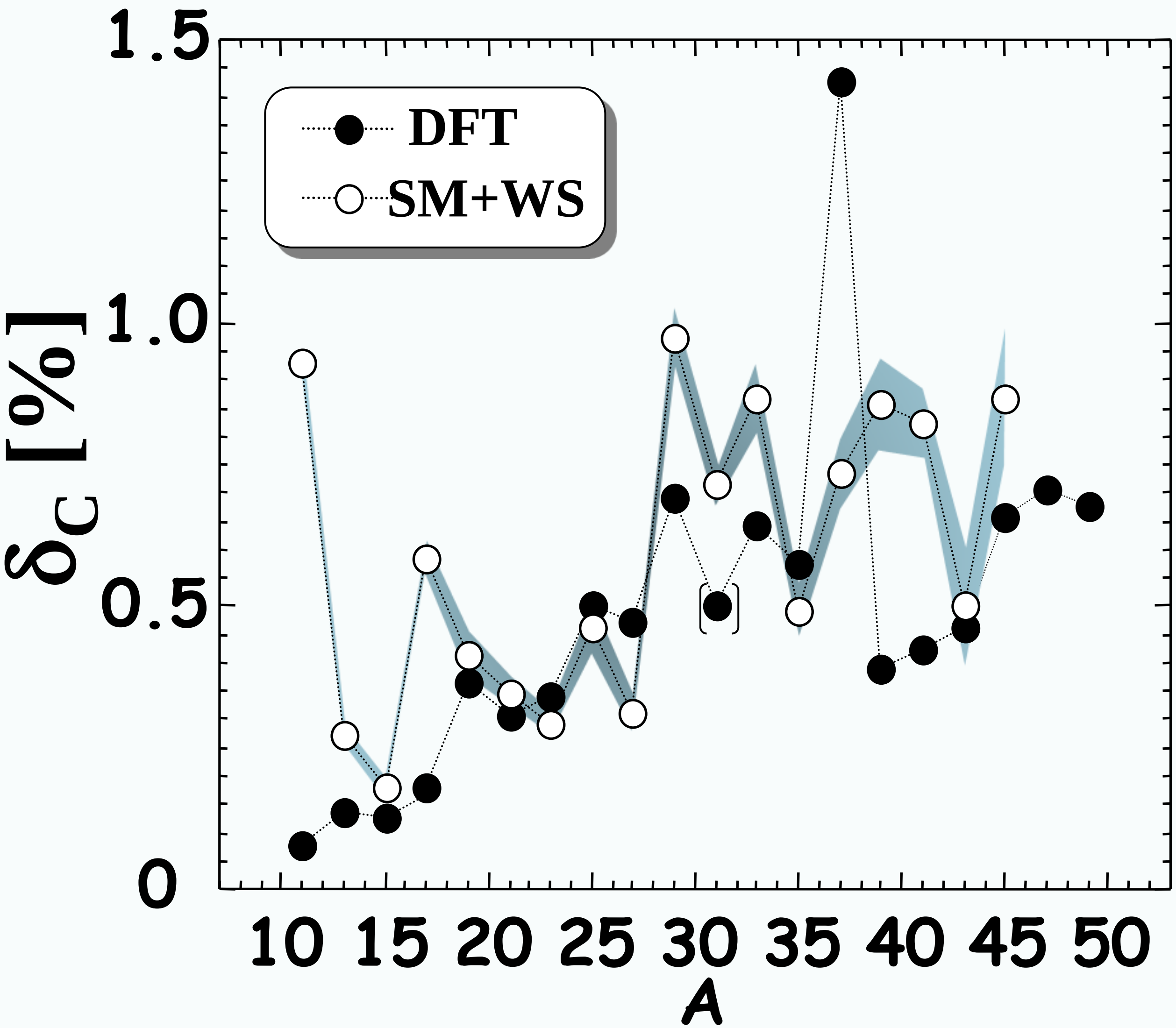


Fig. 5. Proposed grouping of the low-lying states of ^{225}Ra into rotational bands. The two members of the $K^\pi = \frac{3}{2}^-$ band have been reported in a study of the ^{225}Fr decay [20]); they are not observed in the present study.

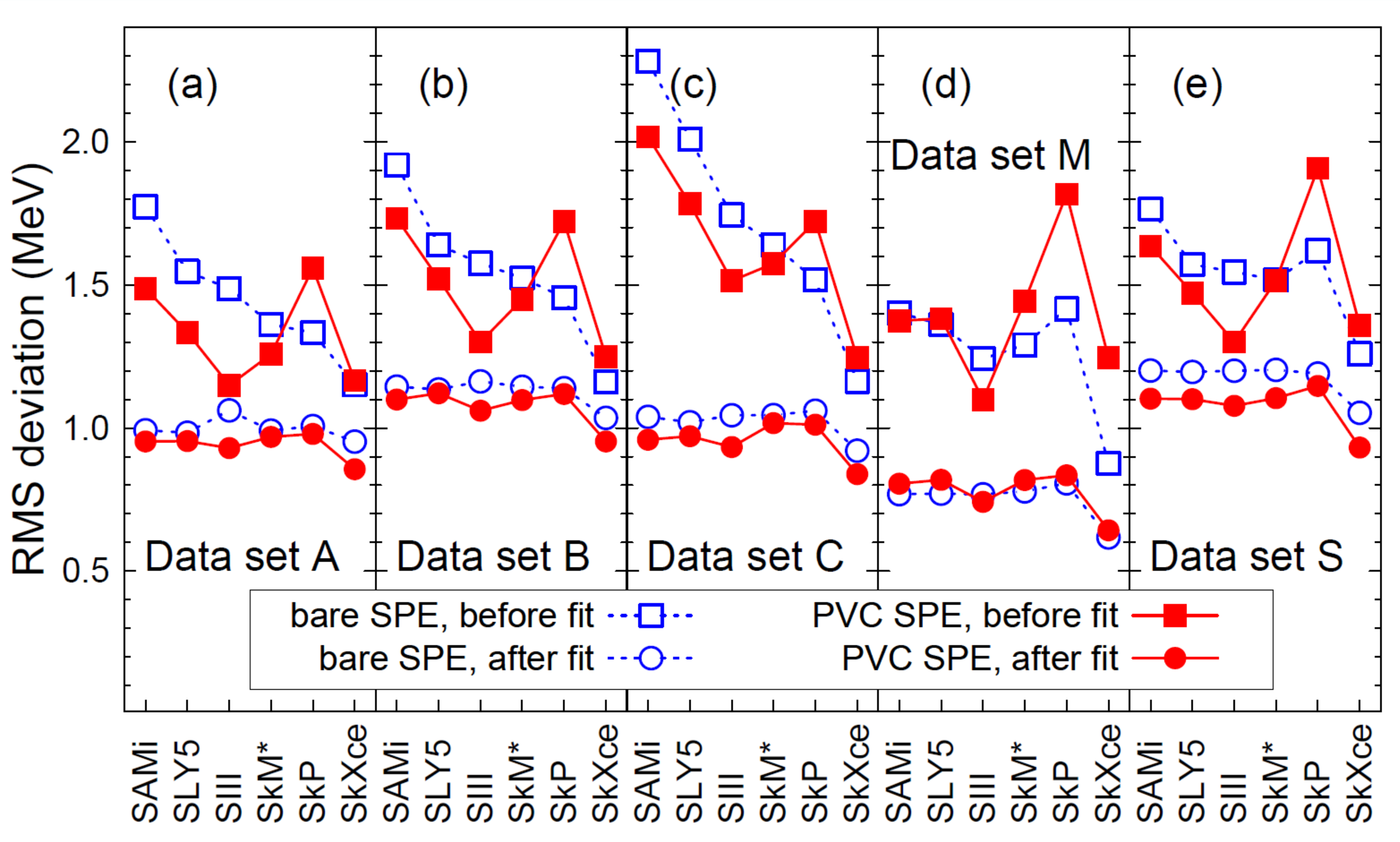
ISB corrections to the Fermi transitions in T=1/2 mirrors



DFT results from:
W. Satuła, J. Dobaczewski, W. Nazarewicz, and M. Rafalski, Phys. Rev. C86, 054314(2012).

SM+WS results from:
N. Severijns, M. Tandecki, T. Phalet, and I. S. Towner, Phys. Rev. C **78**, 055501 (2008).

Particle-vibration-coupling (PVC) corrections



D. Tarpanov *et al.*, arXiv:1405.4823

Quasilocal EDF up to N^{3}LO

Local (primary) densities are defined by four quantum numbers $nLvJ$ as

$$\rho^t_{nLvJ}(\vec{r}) = \left\{[K_{nL}\rho^t_v(\vec{r},\vec{r}\,')]_J\right\}_{\vec{r}\,'=\vec{r}},$$

where the nth-order and rank-L relative derivative operators K_{nL} act on the scalar ($v=0$) or vector ($v=1$) isoscalar ($t=0$) or isovector ($t=1$) nonlocal densities.

We act on each of the local primary densities with mth-order and rank-I derivative operator D_{mI}, and then couple ranks I and J to the total rank J', which gives the local secondary densities, $\left[D_{mI}\rho^t_{nLvJ}(\vec{r})\right]_{J'}$. From primary and secondary densities we build terms of the EDF:

$$T^{n'L'v'J',t}_{mI,nLvJ}(\vec{r}) = [\rho^t_{n'L'v'J'}(\vec{r})[D_{mI}\rho^t_{nLvJ}(\vec{r})]_{J'}]_0,$$

Then, the total energy density reads

$$\mathcal{H}(\vec{r}) = \sum_{\substack{n'L'v'J',t\\ mI,nLvJ,J'}} C^{n'L'v'J',t}_{mI,nLvJ}\, T^{n'L'v'J',t}_{mI,nLvJ}(\vec{r}),$$

where $C^{n'L'v'J',t}_{mI,nLvJ}$ are coupling constants and the summation again runs over all allowed indices.

B.G. Carlsson *et al.*, Phys. Rev. C 78, 044326 (2008)

Regularized pseudopotentials

We regularize the zero-range delta interaction using the Gaussian function,

$$\delta(\vec{r}) = \lim_{a\to 0} g_a(\vec{r}) = \lim_{a\to 0} \frac{e^{-\frac{\vec{r}^2}{a^2}}}{\left(a\sqrt{\pi}\right)^3}.$$

Then, the resulting central two-body regularized pseudopotential reads,

$$V(\vec{r}_1\vec{r}_2;\vec{r}\,'_1\vec{r}\,'_2) = \sum_{i=1}^{4} \hat{P}_i\hat{O}_i(\vec{k},\vec{k}\,')\delta(\vec{r}_1 - \vec{r}\,'_1)\delta(\vec{r}_2 - \vec{r}\,'_2)g_a(\vec{r}_1 - \vec{r}_2),$$

where $\vec{k} = \frac{1}{2i}(\vec{\nabla}_1 - \vec{\nabla}_2)$ and $\vec{k}' = \frac{1}{2i}(\vec{\nabla}'_1 - \vec{\nabla}'_2)$ are the standard relative-momentum operators, and the Wigner, Bartlett, Heisenberg, and Majorana terms are given by the standard spin and isospin exchange operators, $\hat{P}_1 \equiv 1$, $\hat{P}_2 \equiv \hat{P}_\sigma$, $\hat{P}_3 \equiv -\hat{P}_\tau$, $\hat{P}_4 \equiv -\hat{P}_\sigma\hat{P}_\tau$.

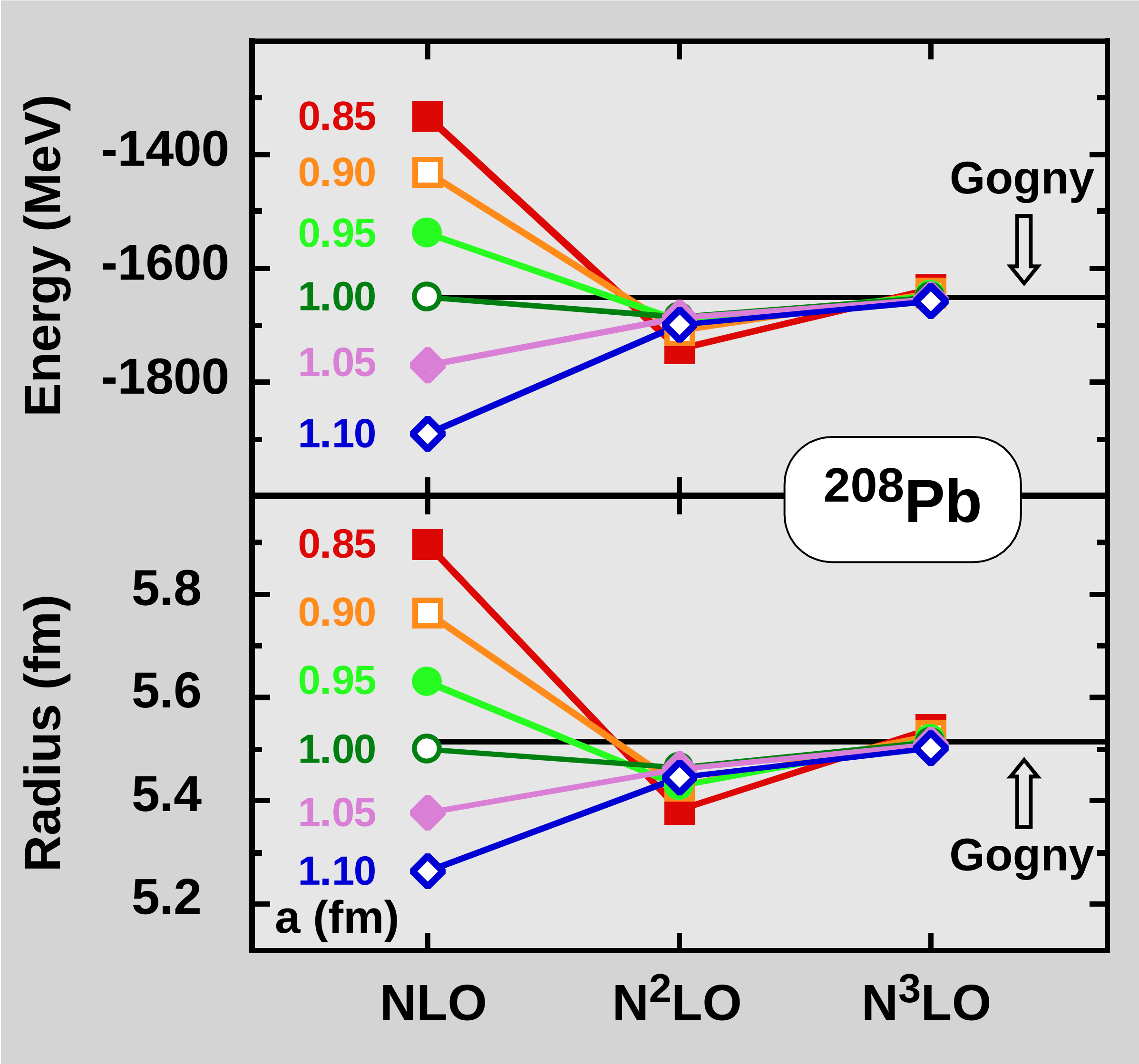
To give a specific example, up to the second-order, that is, up to the next-to-leading-order (NLO) expansion, operators $\hat{O}_i(\vec{k},\vec{k}')$ read

$$\hat{O}_i(\vec{k},\vec{k}') = T_0^{(i)} + \frac{1}{2}T_1^{(i)}\left(\vec{k}'^{*2} + \vec{k}^2\right) + T_2^{(i)}\vec{k}'^{*}\cdot\vec{k},$$

where $T_k^{(i)}$ are the channel-dependent coupling constants.

J.D, K. Bennaceur, F. Raimondi, J. Phys. G. 39, 125103 (2012)

Regularized pseudopotentials vs. Gogny



J.D, K. Bennaceur, F. Raimondi, J. Phys. G. 39, 125103 (2012)

Two-body a=1.4, Three-body zero range