Contents

1 Interactions

Description of interactions in terms of string diagrams. Weyl symmetry, conformal symmetry. Coupling constant. Interaction between branes, brane massess.

1.1 Euclidian picture of world-sheets: $\tau = -i\tau_E,\ z = \tau_E + i\sigma$

Examples of conformal transformations:$\Sigma \rightarrow\ \Sigma'$. ♣ *These are of different type then considered in the context of charges: they change Σ.* ♣

$\boxed{\star}$

1.a. *tube* $\rightarrow\ S^2 - \{0, \infty\}$*:* $z = \tau + i\sigma \rightarrow\ z' = e^z$

1.b. *strip* $\rightarrow\ H - \{0, \infty\}$*:* $z = \tau + i\sigma \rightarrow\ z' = e^z$ ♣ the important point here is that $\tau = -\infty \rightarrow\ z' = 0$ ♣

1.c. $H - \{r_1, r_2\} \rightarrow$ *strip:* $z' = \ln((z - r_1)/(z - r_2))$

1.d. $H - \{0, \infty\} \rightarrow\ D^2$*:* $z \rightarrow\ (iz + 1)/(z + i)$

1.e. $H - \{r_1, r_2, r_3\} \rightarrow$ *two-strip joint with one:* $z' = \ln((z - r_1)(z - r_2)/(z - r_3)^2)$

In order to see the picture it is enough to map the boundary:the map is determined by the sign and the magnitude of $(z - r_1)(z - r_2)/(z - r_3)^2$ []

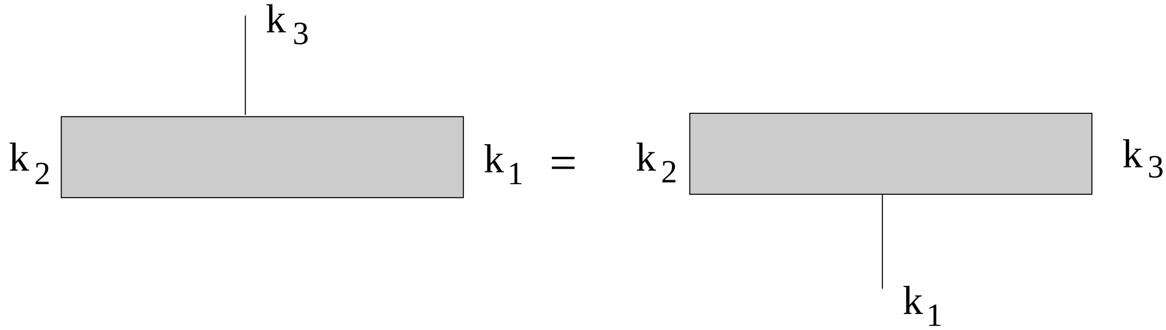
From now on we shall work on $\Sigma = C$ (ⓒ) or H_+ ($\boxed{\sim}$).

so e.g. $X = x + ip\ln|z|^2 + i\sum \frac{1}{n}(\alpha_n z^{-n} + \tilde{\alpha}_n \bar{z}^{-n})$ but $\partial X = pz^{-1} - i\sum(\alpha_n z^{-n} + \tilde{\alpha}_n \bar{z}^{-n})$.

Reality: $X^*(1/z, 1/\bar{z}) = X(z, \bar{z})$ where $*$ does not act on $z, \bar{z}$.

1.2 Amplitudes

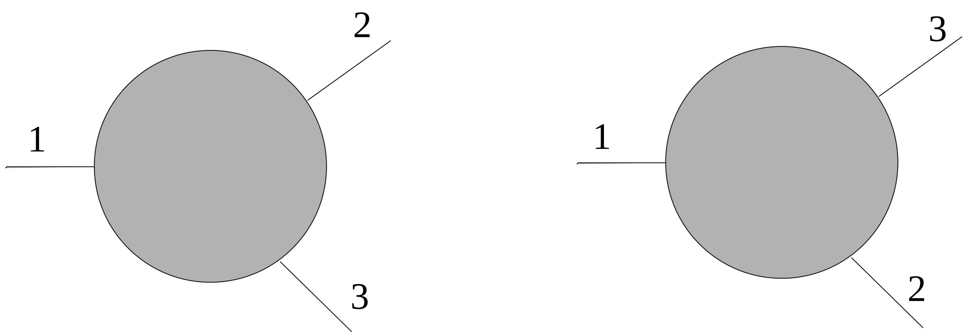
1.2.1 MINKOWSKI PICTURE Inequivalent diagrams for the orientable surfaces.



Two cyclicly equivalent Minkowski diagram. Explicitely

$$\langle -k_2|V^{k_3}|k_1\rangle = \langle -k_2|V^{k_1}_{\pi}|k_3\rangle \tag{1.1}$$

using two vertex operators for two boundaries. We shall often use their Euclidean version as below on the right picture.



We have two cyclecly inequivalent diagrams if strings are oriented

♣ *In the CFT language this means that*

$$< 0|V_1(z_1)V_2(z_2)V_3(z_3)|0> + < 0|V_3(z_1)V_2(z_2)V_1(z_3)|0> \tag{1.2}$$

i.e. $V_3 \leftrightarrow V_1$ ♣

1.2.2 EUCLIDEAN PICTURE OF THE INTERACTION We take $\tau \rightarrow -i\tau_E$, $\sigma^+ \rightarrow z = e^{i\sigma+\tau_E} \in H_+$

$$\langle 0|TV_2V_3V_1|0\rangle \tag{1.3}$$

There is an $SL(2,R)$ symmetry which allows to fix 3 points on the boundary so we can drop integration. We take $\tau_E^1 \rightarrow -\infty$ $(z_1 \rightarrow 0)$, $\tau_E^2 \rightarrow \infty$ $(z_2 \rightarrow \infty)$. In this way one gets the previous expressions.

The advantage of this method is:

- symmetry and possibility of changing world-sheets
- easily generalizes
- one can use Wick theorrem to caculate vev of the chronological order

Finally our prescription is

$$\langle state|V|state'\rangle \tag{1.4}$$

1.3 Vertex operators

1.3.1 CONFORMAL FIELDS: THEORY WITH BOUNDARY Classical boundary conformal field of dim. h:

$$\Phi'(\tau) = \left(\frac{\partial\tau'}{\partial\tau}\right)^h \Phi(\tau')$$
$$\delta\Phi(\tau) = [h\partial_\tau\epsilon(\tau) + \epsilon(\tau)\partial_\tau]\Phi(\tau) = \sum_m i\epsilon_m\, e^{im\tau}[-i\partial_\tau + h\, m]\Phi(\tau) \tag{1.5}$$

♣ *Recall quantum symmetries* ♣. Quantum analog should read:

$$\delta\Phi(\tau) = i\sum_m \epsilon_m[L_m, \Phi(\tau)] \Rightarrow [L_m, \Phi(\tau)] = e^{im\tau}[-i\partial_\tau + h(\Phi)m]\Phi(\tau) \tag{1.6}$$

Vertex operator

$$\boxed{V = \int d\tau\, V(\tau)\,, \qquad h(V(\tau)) = 1}$$

is invariant (up to boundary terms – see below) $[L_m, V] = 0$ because $[L_m, V(\tau)] = \partial_\tau(sth),\ sth = e^{im\tau}V(\tau)$.

- Conformal invariance gives decoupling of null states $(m > 0)$: $\langle phys|VL_{-m}|any'\rangle = \langle phys|[V, L_{-m}]|any'\rangle = 0$
- **Vertex operators attached at** $\sigma = \pi$ my differ from those attached at $\sigma = 0$ by a phase. These former we shall denote by $\mathbf{V}_\pi$

We assume that all operators are functions of X.

1.3.2 Tachyon

$$V_T(\tau,\sigma)|_{\sigma=0,\pi} = \eta_T(\sigma) : e^{ikX(\tau,\sigma)} := \prod_{n>0} \underbrace{\exp\{\epsilon(\sigma) k \frac{2\alpha_{-n}}{n} e^{in\tau}\}}_{v_{-n}(k)} v_0(k,\tau) \prod_{n>0} v_n \tag{1.7}$$

$$v_0(k,\tau) = \text{"} : e^{ik(x+2\alpha_0\tau)} : \text{"} = e^{ik\alpha_0\tau} e^{ikx} e^{ik\alpha_0\tau}, \quad \leftarrow \text{!!!!!} \tag{1.8}$$

$$\epsilon(\sigma) = cos(\sigma), \ \rightarrow \ \epsilon(0) = 1, \ \epsilon(\pi) = -1 \tag{1.9}$$

With this $h(V(\tau)) = 2k^2 \Rightarrow (h = 1 \Rightarrow 2k^2 = 1)$ - mass shell. [1]

1.3.3 Vector

$$V_V(\tau) = \frac{1}{2} : \xi_\mu \partial_\tau X^\mu \, e^{ikX(\tau,0)} : , \quad h(V_V(\tau)) = 2k^2 + 1 \tag{1.11}$$

For $\frac{1}{2}$ see (1.16). Proof: Notice that $\xi_\mu \partial_\tau X^\mu \ : e^{ikX(\tau,0)} :=: \xi_\mu \partial_\tau X^\mu \, e^{ikX(\tau,0)} :$ +terms with $[2\xi\alpha_n, v_{-n}(k)]$. The latter are proportional to (1.10) $\xi_\mu k^\mu ... \rightarrow 0$ i.e. THERE IS NO NEED FOR NORMAL ORDERING BETWEEN THE TWO PARTS OF THE VERTEX. Also $[L_m, \xi_\mu \partial_\tau X^\mu] = e^{im\tau}(-i\partial_\tau + m)\xi_\mu \partial_\tau X^\mu$ ♣ *notice that this holds only for $\epsilon(\sigma = 0, \pi) = 1$, so we can not have $\epsilon(\pi) = -1$.* ♣ thus $[L_m, V_V(\tau)] = [L_m, \xi_\mu \partial_\tau X^\mu \ : e^{ikX(\tau,0)} :] = [L_m, \xi_\mu \partial_\tau X^\mu] : e^{ikX(\tau,0)} : + \xi_\mu \partial_\tau X^\mu \, [L_m, : e^{ikX(\tau,0)} :] = e^{im\tau}(-i\partial_\tau + (2k^2+1)m) V_V(\tau)$

1.3.4 Vertices at $\sigma = \pi$

The diagrams as they stand has 2 discoint boundaries thus 2 set of V: for $\sigma = 0$ and for $\sigma = \pi$. (?)$\rightarrow$ [The dependence should be through $\epsilon(\sigma) = \cos(\sigma)$]$\leftarrow$ (?)[NO]
The important one:
(a) Vertex-state relation (b) 3-point functions and cyclicity relations (1.1) – see also below.

In the presented calculations the two boundaries of the strip are visible only through requirement of adding cycliny independent configurations.

[1] Usefull relation

$$[\alpha_n^\mu, v_{-n}(k)] = 2\epsilon(\sigma) k^\mu v_{-n}(k) \tag{1.10}$$

We assume that all operators are functions of X.

Tachyon

$$V_T(\tau,\sigma)|_{\sigma=0,\pi} = \eta_T(\sigma) : e^{ikX(\tau,\sigma)} := \prod_{n>0} \underbrace{\exp\{\epsilon(\sigma)k\frac{2\alpha_{-n}}{n}e^{in\tau}\}}_{v_{-n}(k)} v_0(k,\tau) \prod_{n>0} v_n$$

$$v_0(k,\tau) = \text{"} : e^{ik(x+2\alpha_0\tau)} : \text{"} = e^{ik\alpha_0\tau}e^{ikx}e^{ik\alpha_0\tau}, \quad \leftarrow \text{!!!!!} \tag{1.12}$$

Vector

$$V_V(\tau) = \frac{1}{2} : \xi_\mu \partial_\tau X^\mu \, e^{ikX(\tau,0)} : , \quad h(V_V(\tau)) = 2k^2 + 1 \tag{1.13}$$

1.3.5 VERTEX-STATE RELATION Notice that for Euclidean $\tau_E \rightarrow -\infty$, $(\tau = -i\tau_E)$ i.e. $\tau \rightarrow i\infty$

$$V_T(\sigma)|0\rangle \rightarrow e^{i2k^2\tau}|k\rangle = \eta_T e^{iL_0\tau}|k\rangle = e^{i\tau}|k\rangle, \rightarrow \eta_T = 1 \tag{1.14}$$

$$V_V(\sigma)|0\rangle \rightarrow \tfrac{1}{2}\,(\xi\alpha_0 + \xi\alpha_{-1}e^{i\tau})e^{i2k^2\tau}|k\rangle \tag{1.15}$$

$$= 0 + \tfrac{1}{2}\,\epsilon\xi\alpha_{-1}e^{i\tau}|k\rangle = e^{iL_0\tau}\xi\alpha_{-1}|k\rangle, \tag{1.16}$$

For $\tau_E \rightarrow \infty$, $\langle 0|V_T \rightarrow \langle -k|e^{-i2k^2\tau} = \langle -k|e^{-iL_0\tau}$.

1.3.6 VERTICES FOR BRANES

1.4 3-point functions

1.4.1 τ INDEPENDENCE ♣ *For any states 3-point scattering the amplitude $\langle in|V^k(\tau)|out\rangle$ is τ independent.* ♣ Proof: $\langle in|V^k(\tau)|out\rangle \overset{(1.6)}{=} \langle in|e^{-iL_0\tau}V^k(0)e^{iL_0\tau}|out\rangle = \langle in|V^k(0)|out\rangle$.

$$g\langle -k_2|V^k|k_1\rangle = g\langle -k_2|V^k(0)|k_1\rangle \int d\tau \tag{1.17}$$

The crucial is kinematics: $k_1 + k + k_2 = 0$ plus mass-shell.

1.4.2 3-TACHYONS

$$g\langle -k_2|V_T^k(0)|k_1\rangle = g\eta_T\langle -k_2|e^{ik\alpha_0\tau}e^{ikx}e^{ik\alpha_0\tau}|k_1\rangle|_{\tau=0} = g\,\delta(k_1+k+k_2) \tag{1.18}$$

Below we shall suppress $\delta(k_1 + k + k_2)$ and $\boxed{g}$

1.4.3 2-TACHYONS, VECTOR

$$\langle -k_2|V_V^k(0)|k_1\rangle = \eta_V(\sigma)\langle -k_2|\tfrac{1}{2} :(\xi\partial_\tau X)\, e^{ikX(\tau,0)}: |k_1\rangle = \eta_V\langle -k_2|\xi\cdot\alpha_0 e^{ikx}|k_1\rangle = \eta_V\xi\cdot(k_1-k_2) \tag{1.19}$$

For cyclicly inequivalent diagram $2 \leftrightarrow 1$ and the sum is zero. By cyclicity we also have

$$\langle -k_2|V_V^k|k_1\rangle = \langle -k_2|V_T^{k_1}\xi_\mu\alpha_{-1}^\mu|k\rangle = -\xi\cdot(k_1-k_2) \tag{1.20}$$

$$\langle -k_2|V_T^{k_2}\xi_\mu\alpha_{-1}^\mu|k\rangle = \langle -k_2|V_V^k|k_1\rangle \;\; ????? \tag{1.21}$$

Non-abelian 2-tachyon-vector

$$\begin{gathered} 2(k_1-k_2)^\mu \mathrm{tr}\{T_2^\dagger\xi_\mu T_1 - T_2^\dagger T_1\xi_\mu\} \\ \downarrow \\ \mathrm{tr}\{-i\partial_\mu T^\dagger[A_\mu, T] + h.c\} \end{gathered} \tag{1.22}$$

Notice that now the amplitude is symmetric $2 \leftrightarrow 1$.

1.4.4 3-VECTORS From (GSW p.372). Below $k_3 \equiv k$.

$$\begin{aligned} \langle -k_2|V_V^k(\tau)|k_1\rangle &= \langle -k_2|\xi_2\cdot\alpha_1 : \tfrac{1}{2}\,\xi_\mu\partial_\tau X^\mu\, e^{ikX(\tau,0)} : \xi_1\cdot\alpha_{-1}|k_1\rangle \\ &= \langle -k_2|(\xi_2\alpha_1)\,[(\xi_3\alpha_{-1})v_{-1}v_0v_1 + \quad v_{-1}(\xi_3\alpha_0)v_0v_1 + \quad v_{-1}v_0v_1(\xi_3\alpha_1)]\,(\xi_1\alpha_{-1})|k_1\rangle \\ &\qquad\qquad \downarrow \qquad\qquad\qquad\qquad \downarrow \qquad\qquad\qquad\qquad \downarrow \\ &\qquad (\xi_2\xi_3)(\xi_1(-2k_3)) \qquad (\xi_2\xi_1)(\xi_3(-2k_2)) + O(k^3) \qquad (\xi_3\xi_1)(\xi_2(2k_3)) \\ &= 2\,[(\xi_1k_2)(\xi_2\xi_3) + cycl.[= (\xi_2k_3)(\xi_3\xi_1) + (\xi_3k_1)(\xi_1\xi_2)]] + 4(\xi_1k_2)(\xi_2k_3)(\xi_3k_1) \end{aligned} \tag{1.23}$$

Notice that the last term is $\sim \alpha'$. If we add the second cyclicly inequivalent diagram then the sum is zero.

Non-abelian 3-vector As above but $\xi \in Mat(N)$ for N-branes. Then the amplitude contain the trace.

$$2\mathrm{tr}\{[(\xi_1k_2)(\xi_2\xi_3) + cycl.] + 4(\xi_1k_2)(\xi_2k_3)(\xi_3k_1)\} \tag{1.24}$$

Adding the second cyclicly inequivalent diagram produces the commentator and going to the position representation we get

$$-\,ig\;2\mathrm{tr}\{[\quad \xi_1^\mu[\partial_2^\mu\xi_2^\nu, \xi_3^\nu] \quad + cycl.]\} + \ldots \tag{1.25}$$

$$\downarrow$$

$$-ig\;2\mathrm{tr}\{\quad A_\mu[\partial_\mu A_\nu, A_\nu] \qquad \} \tag{1.26}$$

1.5 Effective free Lagrangians

$$\mathcal{L} = \mathrm{tr}\{-\frac{1}{4}F_{\mu\nu}^2 - |D_\mu T|^2 - m_T^2|T|^2\} \tag{1.27}$$

1.5.1 DIMENSIONAL REDUCTION

and the lagrangian for brane modes.

1.5.2 SSB AS VEV FOR BRANE SCALARS

For two Dp-branes shifted by d^m physical conditions for $\xi_\mu \alpha_{-1}^\mu |k\rangle$ are