Cosmology

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Cosmology, Winter Semester 2009/10

The outline of the course

- The observed Universe
- Brief "Course" of General Relativity and the Friedmann-Lemaître-Robertson-Walker Metric
- Cosmological Models
- Distances in cosmology
- Elementary Particles and Fundamental Interactions
- Thermodynamics of the Early Universe
- Thermal Relics from the Big Bang
- Gravitational Lensing
- Dark Matter and Dark Energy

- Inflation
- Baryogenesis

The Course Grading

- written exam
- oral exam

Recommended Texbooks

1. L Bergström and A. Goobar, "Cosmology and Particle Asrophysics"
2. A. Liddle, "Wprowadzenie do kosmologii wspolczesnej"
3. E. Kolb and M. Turner, "The Early Universe"
4. S. Weinberg, "Gravitation and Cosmology"
5. A. Dodelson, "Modern Cosmology"

The observed Universe

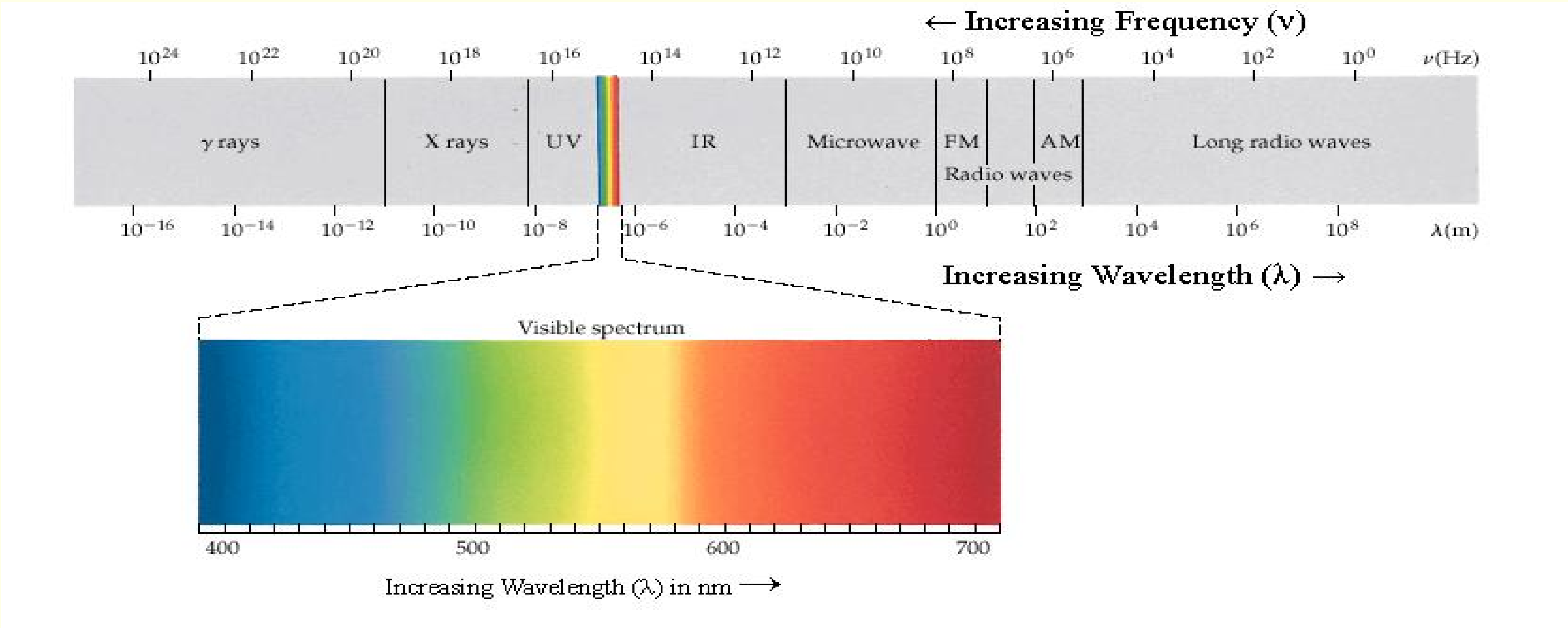

Figure 1: *The electromagnetic spectrum*

- Optical Astronomy:
Professional astronomers today hardly ever actually look through telescopes. Instead, a telescope sends an objects light to a photographic plate or to an electronic light-sensitive computer chip called a charge-coupled device, or CCD. CCDs are about 50 times more sensitive than film. Telescopes may use either lenses or mirrors to gather visible light. The Hubble Space Telescope (HST) , a reflecting telescope that orbits Earth, has returned the clearest images of any optical telescope. The main mirror of the HST is only 94 in (2.4 m) across, far smaller than that of the largest ground-based reflecting telescopes. Turbulence in the atmosphere makes observing objects as clearly as the HST can see impossible for ground-based telescopes. HST images of visible light are about five times finer than any produced by ground-based telescopes. Giant telescopes on Earth, however, collect much more light than the HST can. Examples of such giant telescopes include the twin 32-ft (10-m) Keck telescopes in Hawaii and the four 26-ft (8-m) telescopes in the Very Large Telescope array in the Atacama Desert in northern Chile (the nearest city is Antofagasta, Chile). Often astronomers use space- and ground-based telescopes in conjunction.

- Gamma-Ray and X-Ray Astronomy:
Gamma rays have the shortest wavelengths. Special telescopes in orbit around Earth, such as the National Aeronautics and Space Administration's (NASA's)

Compton Gamma-Ray Observatory, gather gamma rays before Earth's atmosphere absorbs them. X rays, the next shortest wavelengths, also must be observed from space. NASA's Chandra X-Ray Observatory (CXO) is a school-bus-sized spacecraft that began studying X rays from orbit in 1999.

- Ultraviolet Astronomy:
  Ultraviolet light has wavelengths longer than X rays, but shorter than visible light. Ultraviolet telescopes are similar to visible-light telescopes in the way they gather light, but the atmosphere blocks most ultraviolet radiation. Most ultraviolet observations, therefore, must also take place in space. Most of the instruments on the Hubble Space Telescope (HST) are sensitive to ultraviolet radiation. Humans cannot see ultraviolet radiation, but astronomers can create visual images from ultraviolet light by assigning particular colors or shades to different intensities of radiation.

- Infrared Astronomy:
  Infrared astronomers study parts of the infrared spectrum, which consists of electromagnetic waves with wavelengths ranging from just longer than visible light to 1,000 times longer than visible light. Earth's atmosphere absorbs infrared radiation, so astronomers must collect infrared radiation from places where the atmosphere is very thin, or from above the atmosphere. Observatories for these wavelengths are located on certain high mountaintops or in space. Most infrared wavelengths can be observed only from space. Every warm object emits some infrared radiation. Infrared astronomy is useful because objects that are not hot enough to emit visible or ultraviolet radiation may still emit infrared radiation. Infrared radiation also passes through interstellar and intergalactic gas and dust more easily than radiation with shorter wavelengths. Further, the brightest part of the spectrum from the farthest galaxies in the universe is shifted into the infrared.

- Radio Astronomy:
  Radio waves have the longest wavelengths. Radio astronomers use giant dish antennas to collect and focus signals in the radio part of the spectrum. These celestial radio signals, often from hot bodies in space or from objects with strong magnetic fields, come through Earth's atmosphere to the ground. Radio waves penetrate dust clouds, allowing astronomers to see into the center of our galaxy and into the cocoons of dust that surround forming stars.

- Other Emissions:
  - neutrinos:
    Most neutrino telescopes consist of huge underground tanks of liquid. These tanks capture a few of the many neutrinos that strike them, while the vast majority of neutrinos pass right through the tanks.
  - cosmic rays:
    Cosmic rays are electrically charged particles that come to Earth from outer space at almost the speed of light. They are made up of electrons and protons. Astronomers do not know where most cosmic rays come from, but they use cosmic-ray detectors to study the particles. Cosmic-ray detectors are usually grids of wires that produce an electrical signal when a cosmic ray passes close to them.
  - gravitational waves:
    Gravitational waves are a predicted consequence of the general theory of relativity. No gravitational waves have yet been detected. Gravitational waves should be very weak, and the instruments are probably not yet sensitive enough to register them. In the 1970s and 1980s American physicists Joseph Taylor and Russell Hulse observed indirect evidence of gravitational waves by studying systems of double pulsars. A new generation of gravitational-wave detectors, developed in the 1990s, uses interferometers to measure distortions of space that would be caused by passing gravitational waves.

Astronomical units

- The astronomical unit:

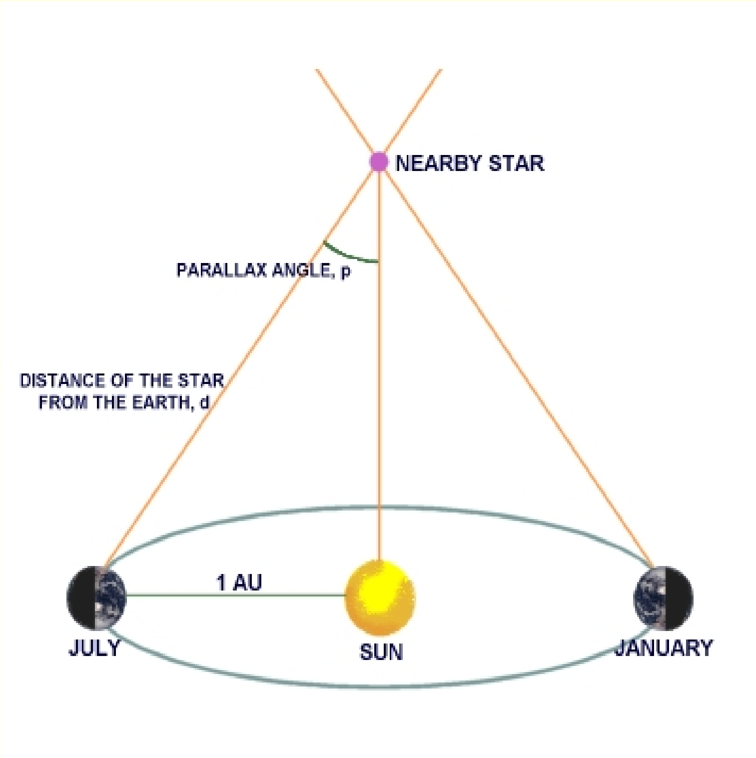

Figure 2: *The astronomical unit*

The astronomical unit (AU or au or a.u. or sometimes ua) is a unit of length approximately equal to the distance from the Earth to the Sun. The currently accepted value of the AU is 1 AU $= 149\,597\,870\,691 \pm 30$ m (nearly 150 million kilometers).

- The parsec:
  The parsec (symbol pc) is a unit of length used in astronomy. The length of the parsec is based on the method of trigonometric parallax, one of the oldest methods for measuring the distances to stars. The name parsec stands for "parallax of one second of arc", and one parsec is defined to be the distance from the Earth to a star that has a parallax of 1 arcsecond. The actual length is 1 pc $\simeq 3.086 \times 10^{16}$ m, or about $\simeq 3.262$ ly.

- Light-year:
  A light-year or light year (symbol: ly) is a unit of measurement of length, specifically the distance that light travels in a vacuum in one year, 1 ly $= 9.461 \times 10^{15}$ m. While there is no authoritative decision on which year is used, the International Astronomical Union (IAU) recommends the Julian year (365.25 days).

Observed structures

- Stars:
  - Sun mass $M_\odot \simeq 2 \times 10^{30}$ kg
  - Distance to next closest stars $\simeq 1$ ly (1 ly $= 9.461 \times 10^{15}$ m)

- Galaxies:
  A galaxy (from the Greek root galaxias [$\gamma\alpha\lambda\alpha\xi\iota\alpha\zeta$], meaning "milky," a reference to the Milky Way) is a massive, gravitationally bound system consisting of stars, an interstellar medium of gas and dust, and dark matter. Typical galaxies range from dwarfs with as few as ten million (10^7) stars up to giants with one trillion (10^{12}) stars, all orbiting a common center of mass. There are probably more than 100 billion (10^{11}) galaxies in the observable universe. Most galaxies are 1,000 to 100,000 parsecs in diameter and are usually separated by distances on the order of millions of parsecs (or megaparsecs).
  - Spirals (e.g. Milky Way)
  - Ellipticals
  - Lenticular galaxies

- Clusters of Galaxies:
Their mutual gravity can draw galaxies together into a cluster that is several millions of light years across. Some clusters have only a handful of galaxies and are called poor clusters. Other clusters with hundreds to thousands of galaxies are called rich clusters. The low mass of a poor cluster prevents the cluster from holding onto its members tightly. The poor cluster tends to be a bit more irregular in shape than a rich cluster. Our Milky Way is part of a poor cluster called the Local Group. The Local Group has two large spirals, one small spiral, two ellipticals, at least 19 irregulars, at least 17 dwarf ellipticals and at least 5 dwarf spheroidals. There may be more irregular and dwarf galaxies. The Local Group is about 3 million light years across with the two large spirals, the Milky Way and Andromeda Galaxy, dominating the two ends. Each large spiral has several smaller galaxies orbiting them. The proportions of the different types of galaxies in the Local Group probably represents the number of the different types of galaxies in the rest of the universe. The small galaxies can be seen in the Local Group because they are close enough to us. But the dwarf galaxies are hard to see in far away clusters.

- Superclusters of Galaxies:
The clustering phenomenon does not stop with galaxies. Galaxy clusters attract each other to produce superclusters of tens to hundreds of clusters. Their mutual

gravity binds them together into long filaments (thin, stringlike structures) 300 to 900 million light years long, 150 to 300 million light years wide, and 15 to 30 million light years thick on average. The discovery of these huge structures was made recently from years of taking doppler shifts of thousands of galaxies.

- Voids, sheets & filaments:
  Deep redshift surveys reveal a very bubbly structure to the universe with galaxies primarily confined to sheets and filaments. Voids are the dominant feature and have a typical diameter of about 25Mpc. They fill about 90% of space and the largest observed, Bootes void, has a diameter of about 124Mpc. Other features that have been observed are the 'Great Wall', an apparent sheet of galaxies 100Mpc long at a distance of about 100Mpc.

Formation of structures

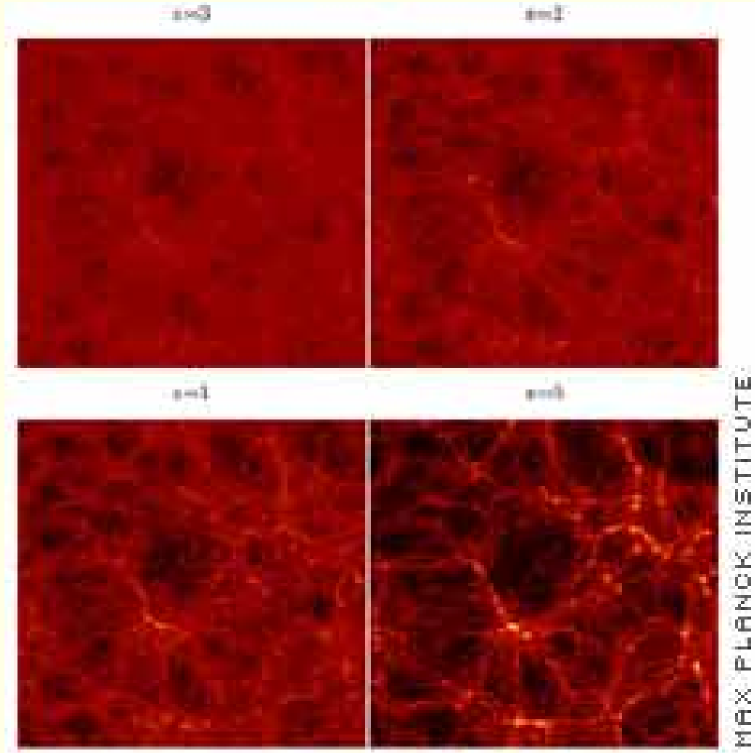

Figure 3: *The history of the Universe.*

History of the universe were simulated on a Cray supercomputer at the Max Planck Institute in Germany. Matter condenses gradually (from top left to bottom right) out of a near-uniform soup. Each square represents a slice of the universe 550 million light-years on a side. Bright spots indicate regions of greater density of matter.

Observed structures

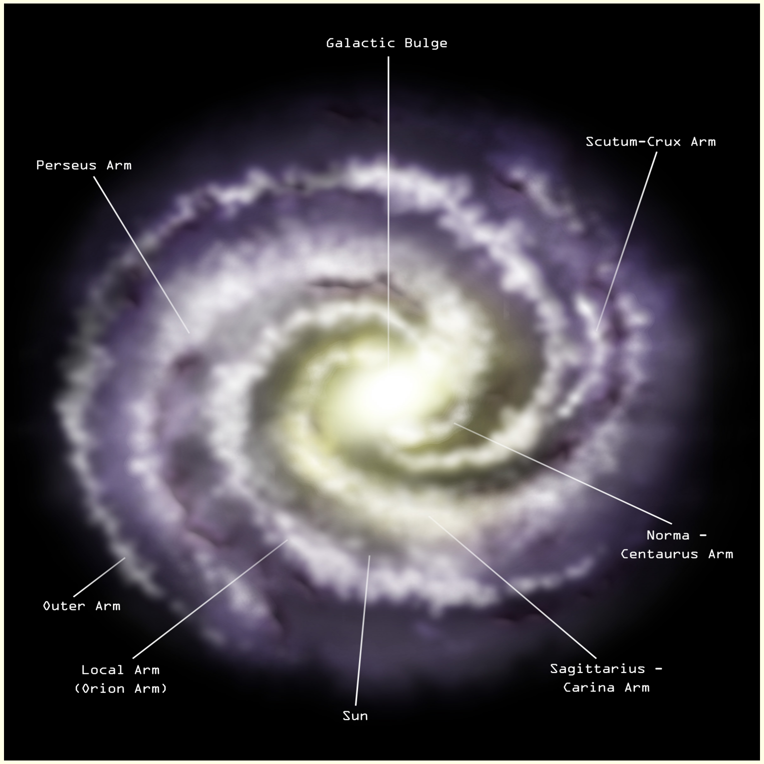

Figure 4: *Milky Way Galaxy, Face-on View*

The Milky Way is a spiral galaxy. It is the home of our Solar System together with at least 200 billion other stars (more recent estimates have given numbers around 400 billion). For mere details see:
http://chandra.harvard.edu/resources/illustrations/milkyWay.html

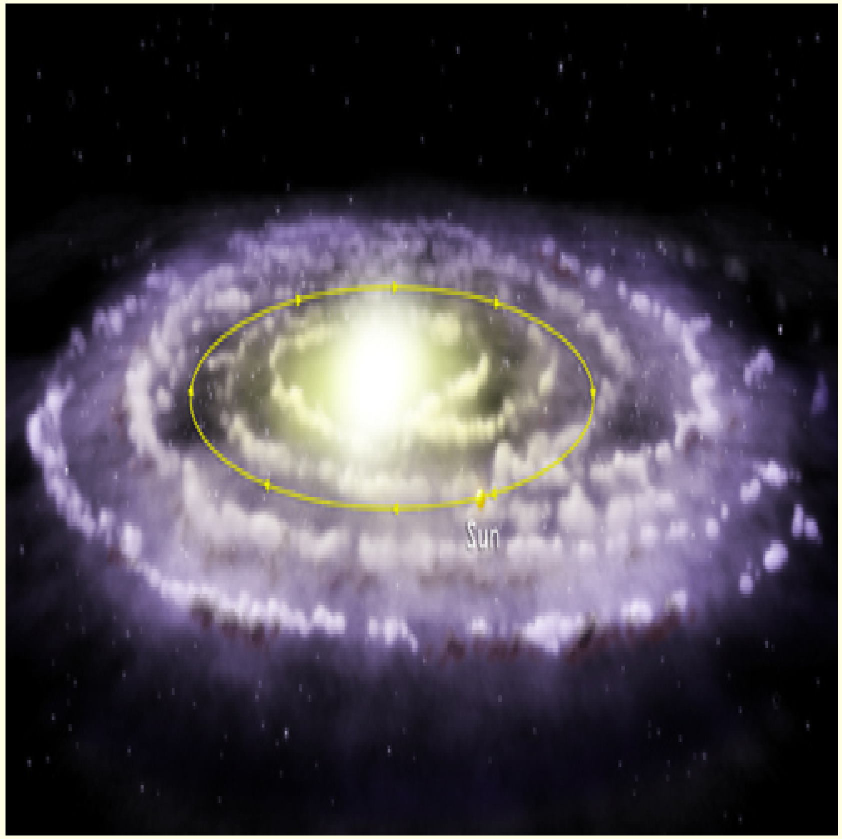

Figure 5: *Illustration of Solar System's Orbit*

It takes the Solar System about 225,250 million years to complete one orbit of the galaxy (a galactic year), so it is thought to have completed 2025 orbits during the lifetime of the Sun and 1/1250th of a revolution since the origin of humans. The orbital speed of the Solar System about the center of the Galaxy is approximately 220 km/s. At this speed, it takes around 1400 years for the Solar System to travel a distance of 1 light-year, or 8 days to travel 1 AU.

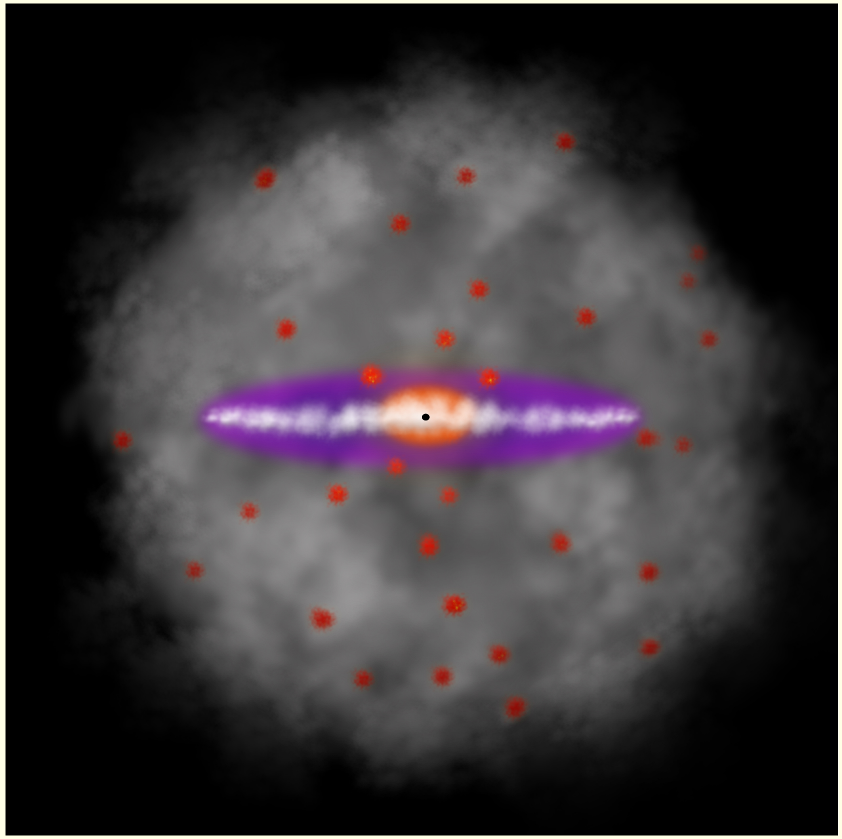

Figure 6: *Milky Way Halo*

Schematic of Milky Way showing the dark matter halo (gray), globular clusters (a spherical collection of stars that orbits a galactic center as a satellite – red circles), the thick disk (old stars – dark purple), the stellar disk (white), the stellar bulge (tightly packed group of stars – red-orange), and the central black hole. The stellar disk is about 100,000 light years in diameter. The dark halo extends to a diameter of at least 600,000 light years.

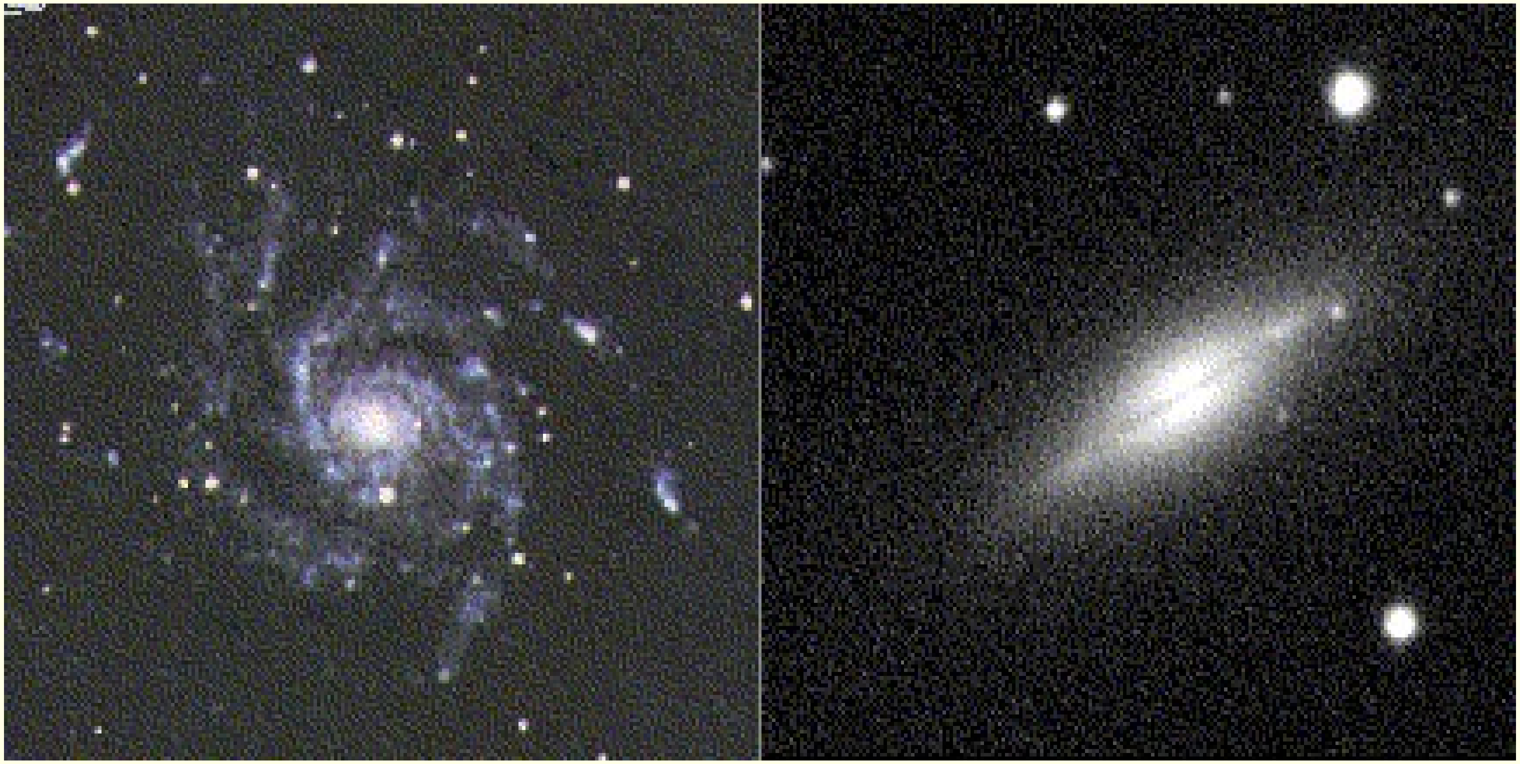

Figure 7: *Lenticular Galaxy M102*

A lenticular galaxy is a type of galaxy which is intermediate between an elliptical galaxy and a spiral galaxy in galaxy morphological classification schemes. Lenticular galaxies are disc galaxies (like spiral galaxies) which have used up or lost their interstellar matter (like elliptical galaxies). Because of their ill-defined spiral arms, if they are inclined face-on it is often difficult to distinguish between them and elliptical galaxies.

Figure 8: *Elliptical Galaxy M87 (NGC 4486), type E1*

Elliptical galaxies have smooth, featureless light-profiles and range in shape from nearly spherical to highly flattened, and in size from hundreds of millions to over one trillion (10^{12}) stars. In the outer regions, many stars are grouped into globular clusters. Most elliptical galaxies are composed of older, low-mass stars, with a sparse interstellar medium and minimal star formation activity. Elliptical galaxies are believed to make up approximately 10-15 % of galaxies in the local Universe.

Figure 9: *Irregular Galaxy LMC, the Large Magellanic Cloud*

Synergy of X-ray and optical observations!

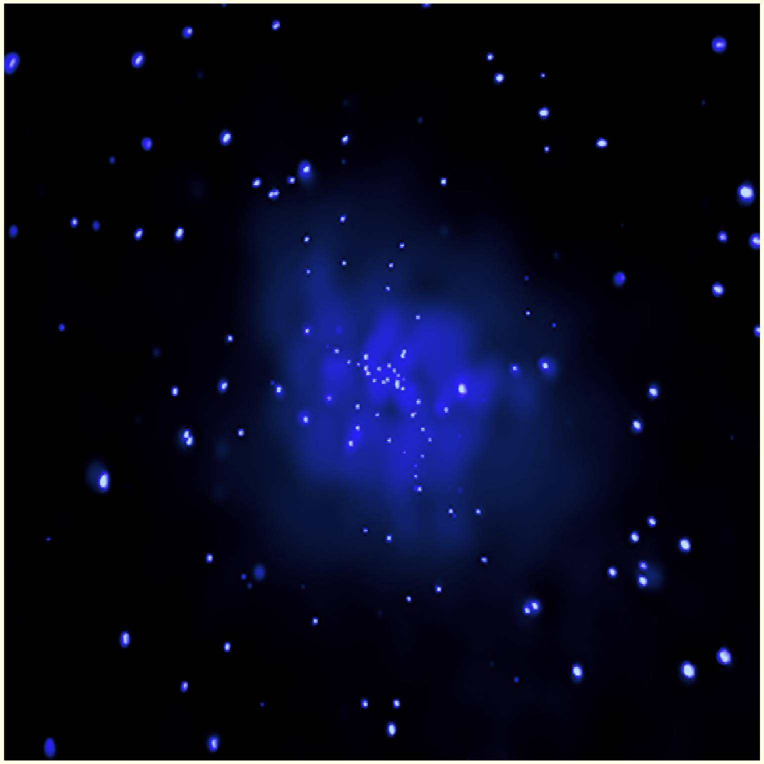

Figure 10: *Chandra X-ray Image of Andromeda Galaxy (M31)*

Figure 11: *Optical Image of Andromeda Galaxy (M31)*

Synergy of infrared and optical observations!

Figure 12: *The Sombrero Galaxy in Infrared*

This floating ring is the size of a galaxy. It is part of Sombrero Galaxy (M104), one of the largest galaxies in the nearby Virgo Cluster of Galaxies. The dark band of dust that obscures the mid-section of the Sombrero Galaxy in optical light actually glows brightly in infrared light. The above image shows the infrared glow, recently recorded by the orbiting Spitzer Space Telescope, superposed in false-color on an existing image taken by NASA's Hubble Space Telescope in optical light.

Cosmological Principle

On large spatial scales, the Universe is homogeneous and isotropic.

Isotropy and Homogeneity:

- If the Universe is isotropic then this means you will see no difference in the structure of the Universe as you look in different directions. When viewed on the largest scales, the Universe looks the same to all observers and the Universe looks the same in all directions as viewed by a particular observer.

- Homogeneity, when viewed on the largest scales, means that the average density of matter is about the same in all places in the Universe and the Universe is fairly smooth on large scales.

Notice that this is clearly not true for the Universe on small scales such as the size of the Earth, the size of the Solar System and even the size of the Galaxy. Terms such as look the same and smooth in density are applied only on very large scales. For cosmology, we only consider the isotropy and homogeneity of the Universe on scales of millions of light-years.

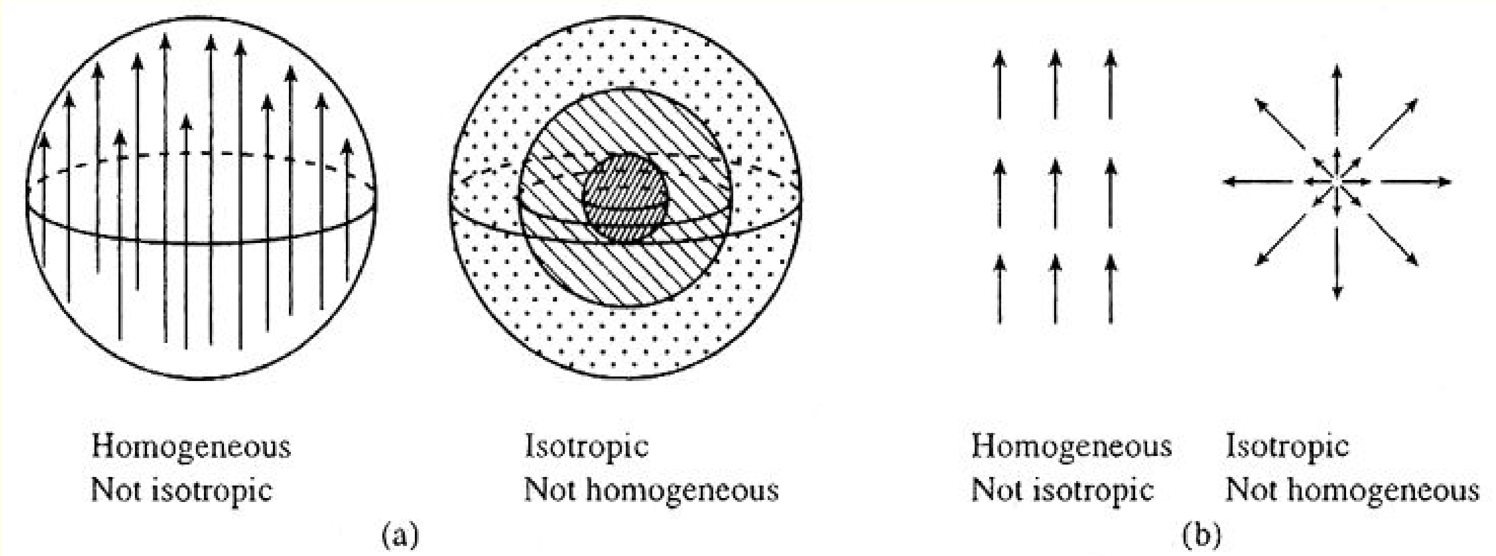

Figure 1.5 Illustrations of how homogeneity and isotropy are not equivalent in (a) three dimensions and (b) two dimensions. In the first example of each, a unique direction is picked out but translation invariance is maintained. In the second example of each, all directions are the same (rotation invariance) but a radial gradient exists.

Figure 13: *Isotropy and Homogeneity*

Notice that isotropy for all observers (all places in the Universe) implies homogeneity for all observers. It is possible to construct universes that are homogeneous but anisotropic; the reverse, however, is not possible. Consider an observer who is surrounded by a matter distribution that is perceived to be isotropic; this means not only that the mass density is a function of radius only, but that there can be no preferred axis for other physical attributes such as the velocity field.

An isotropic Universe also means that there is no 'center' to the Universe. This is an important point when we consider the origin of the Universe known as the Big Bang. Due to isotropy, there is no ‘place’ where the Big Bang occurred, there is no center point.

Olbers' Paradox and the Dark Night Sky

Another simple observation is that the visible night sky is dark. If the universe is infinite, eternal, and static, then the sky should be as bright as the surface of the Sun all of the time! Heinrich Olbers (lived 1758–1840) popularized this paradox in 1826, but he was not the first to come up with this conclusion. Thomas Digges wrote about it in 1576, Kepler stated it in 1610, and Edmund Halley and Jean Philippe de Cheseaux talked about it in the 1720's, but Olbers stated it very clearly, so he was given credit for it. This problem is called Olbers' Paradox.
If the universe is uniformly filled with stars, then no matter which direction you look, your line of sight will eventually intersect a star (or other bright thing). Now it is known that stars are grouped into galaxies, but the paradox remains: your line of sight will eventually intersect a galaxy.

$$\Phi \propto \int d\Omega \, dr \, r^2 \, n \, I(r)$$

where n is the density of stars while $I(r)$ the energy flux observed at the distance r

from the star. Since $I(r) \propto r^{-2}$, therefore

$$\Phi \propto \int d\Omega\, dr\, r^2 n\, I(r) \propto 4\pi\, n \int dr = \infty$$

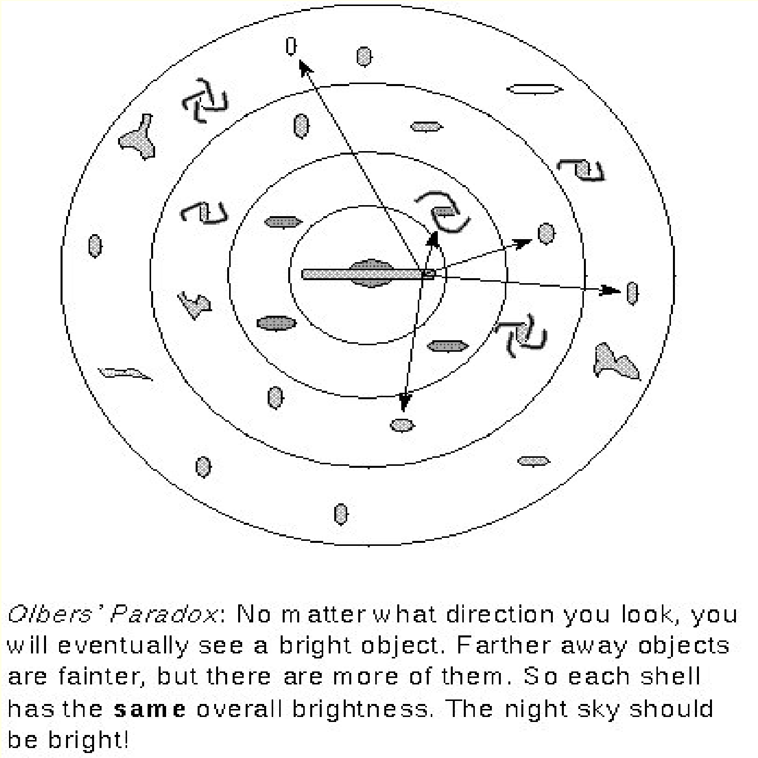

Figure 14: *The Universe and the Olbers' paradox*

The brightnesses of stars does decrease with greater distance (remember the inverse square law) BUT there are more stars further out. The number of stars within a spherical shell around us will increase by the same amount as their brightness decreases. Therefore, each shell of stars will have the same overall luminosity and because there are a lot of ever bigger shells in an infinite universe, there is going to be a lot of light!

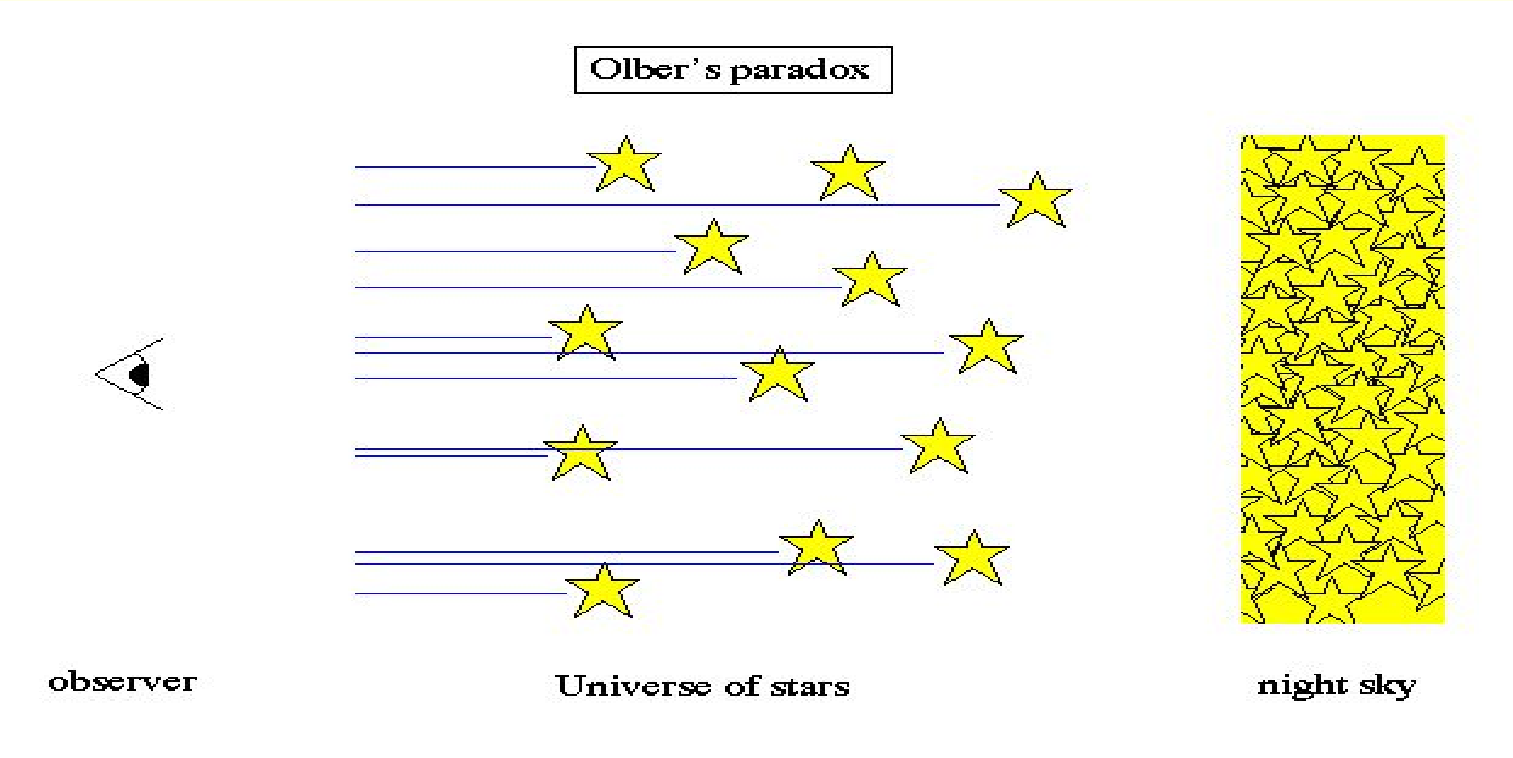

Figure 15: *The Universe and the Olbers' paradox*

Alternative View:
Another way to think about the problem is to compare the brightness of the nightsky

to the brightness of the surface of the Sun. Just as obviously, we know that the surface of the Sun blazes away at a temperature of 5,800 Kelvin. The nightsky is substantially less bright. To see why this is a paradox consider the following:

- A star (like the Sun) of radius R covers an area of size, $A = \pi R^2$. The fraction of the surface area of a sphere of radius r covered by such a star is then

$$f = \frac{\pi R^2}{4\pi r^2} = \left(\frac{R}{2r}\right)^2 .$$

  The total fraction of the shell covered by all of the stars in the shell is then the fraction due to one star $\times$ total number of stars

$$F = \left(\frac{R}{2r}\right)^2 \times (n \times 4\pi r^2 \times \Delta r) \simeq 5 \times 10^{-16} \times n \times \Delta r$$

  where n denotes the star number density and Δr is the shell width. In the last step I assumed the stellar density n as the number of stars per cubic parsec and the thickness of the shell measured in parsecs. ($R_\odot = 7 \cdot 10^8$ m, 1 pc $\simeq 3 \cdot 10^{16}$ m) These are convenient units because in our Galaxy, there is roughly 1 star per cubic parsec and the average separation between stars is on the order of 1 pc.

- The fraction of the shell blocked out by the stars in the shell does not depend upon the radius of the shell (how far away the shell lives) $\Longrightarrow$ Olbers's Paradox if the Universe is big enough.

Possible Resolutions of Olbers's Paradox

- Obscuration by dust:
  Distant stars are blocked out and appear fainter. Turns out this won't work because dust, if it absorbs energy will heat up and re-radiate the energy. This means that the Universe will still be filled with the same amount of radiation, the dust acts simply as a go-between so to speak.

- Expansion of the Universe:
  Redshift z $(1+z \equiv \lambda_{\rm obs}/\lambda_{\rm emit})$ of photons implies $\lambda_{\rm obs}$ is larger than $\lambda_{\rm emit}$ $\Longrightarrow$ we absorb lower energy photons than are produced by the distant stars. Distant objects in an expanding universe have apparent brightnesses which fall off faster than the inverse square law. This decreases the contributions from distant shells. The expanding universe effects partially explain Olbers's Paradox.

- Finite size and age of the Universe:
  One shell of stars covers a fraction $5 \times 10^{-16} \times n \times \Delta r$ of the sky. So, to make

the night sky as bright as a star, we would like to make the stars cover most of the observable sky:

$$5 \times 10^{-16} \times n \times \Delta r \times N \simeq 1$$

where N is the number of shell needed. To calculate N, we note that there is roughly 1 star per cubic parsec in our galaxy, so average distance between stars in our Galaxy is ~ 1 pc, so we choose the shell thickness $\Delta r \simeq 1$ pc. Then

$$N \simeq 2 \times 10^{15}$$

Because each shell is ~ 1 pc thick therefore the Universe needs to be at least 2×10^{15} pc in radius. So the Universe must be at least 6.6×10^{15} ly in size in order to make the night sky as bright as the surface of the Sun. The current Universe is ~ 13.7 billion years old and has an observable size of ~ 45 billion light years. This is much less than needed to produce Olbers's Paradox. The fact that the Universe has a finite age is the principal explanation of Olbers's Paradox.

Hubble law and the expanding Universe

The "redshift" z:

$$z = \frac{\lambda_{\rm observ}}{\lambda_{\rm emitt}} - 1$$

Hubble's law is a statement in physical cosmology which states that the redshift in light coming from distant galaxies is proportional to their distance:

$$z \propto r$$

The law was first formulated by Edwin Hubble and Milton Humason in 1929 after nearly a decade of observations. It is considered the first observational basis for the expanding space paradigm and today serves as one of the most often cited pieces of evidence in support of the Big Bang. The most recent calculation of the proportionality constant, using the satellite WMAP began in 2003, yielding a value of 71 ± 4 (km/s)/Mpc. In August, 2006, a less accurate figure was obtained independently using data from NASA's orbital Chandra X-ray Observatory: 77 (km/s)/Mpc with an uncertainty of $\pm 15\%$.

If the redshift is interpreted as a non-relativistic Doppler effect:

$$\frac{\lambda_{\rm observ}}{\lambda_{\rm emitt}} = 1 + \frac{v}{c} \quad \Longrightarrow \quad \frac{v}{c} = z$$

then the redshift-distance relation yields a straightforward mathematical expression for Hubble's Law as follows:

$$\vec{v} = H_0 \vec{r}$$

where $\vec{v}$ is the recessional velocity, typically expressed in km/s, H_0 is Hubble's constant and corresponds to the value of H (often termed the Hubble parameter which is a value that is time dependent) in the Friedmann equations taken at the time of observation denoted by the subscript $_0$. This value is the same throughout the universe for a given comoving time, and $\vec{r}$ is the comoving distance from the galaxy to the observer, measured in Mpc, in the 3-space defined by given cosmological time.

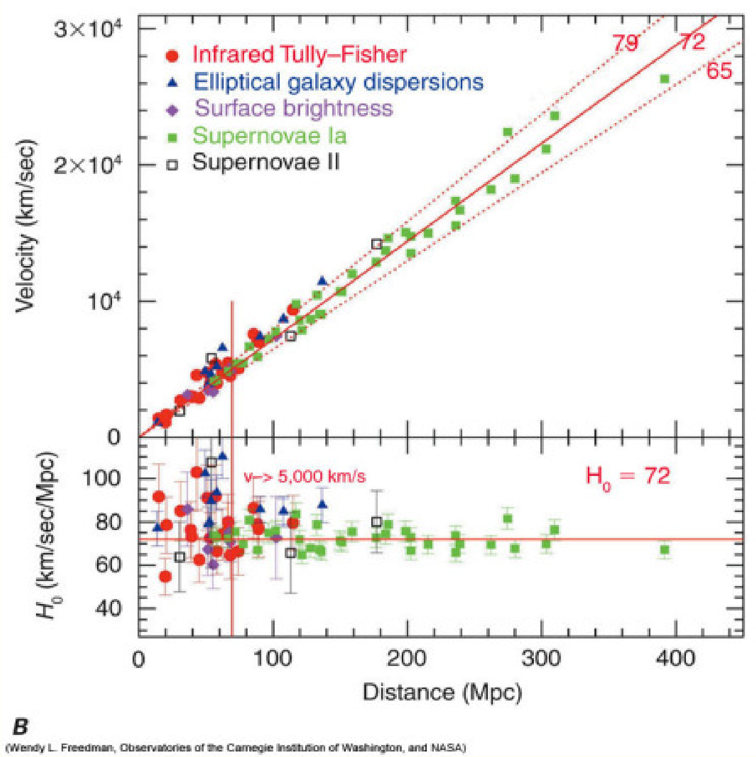

Figure 16: *The Hubble diagram for the redshift interpreted as a non-relativistic Doppler effect. The Hubble's law works very well out to distances of many hundred Mpc.*

"Models" of the expanding Universe:

- The famous balloon analogy:
  To visualise the expanding universe one can compare 3d space with the surface of an expanding balloon. This analogy was used by Arthur Eddington as early as 1933 in his book "The Expanding Universe".
  One must remember that:
  - The 2d surface of the balloon is analogous to the 3 dimensions of space.
  - The 3d space in which the balloon is embedded is not analogous to any higher dimensional physical space.
  - The centre of the balloon does not correspond to anything physical.
  - The universe may be finite in size and growing like the surface of an expanding balloon but it could also be infinite.
  - Galaxies move apart like points on the expanding balloon but the galaxies themselves do not expand because they are gravitationally bound.

- The raisin bread analogy:
  There is a very common misconception about the expansion. Many people envision this expansion as analogous to an explosion. In an explosion matter flies out to fill in space that is already there. This analogy is misleading. The raisin bread is the better analogy. The yeast filled dough is analogous to the space in the universe.

The space in the universe, like the dough, is expanding causing the galaxies, or raisins, to move farther apart. The galaxies, or raisins, are not rushing out to fill in space, or dough, that is already there.

Questions:

- Is the expansion of the Universe consistent with the cosmological principle?
- Where is the centure of the Universe?
- How large was the Universe at the Big Bang?

Cosmic microwave background radiation

The cosmic microwave background radiation (most often abbreviated CMB but occasionally CMBR, CBR or MBR, also referred to as relic radiation) is a form of electromagnetic radiation discovered in 1965 that fills the entire Universe. It has a thermal black body spectrum at a temperature of 2.725 K. Thus the spectrum peaks in the microwave range at a frequency of 160.2 GHz, corresponding to a wavelength of 1.9 mm. Most cosmologists consider this radiation to be the best evidence for the Big Bang model of the universe.

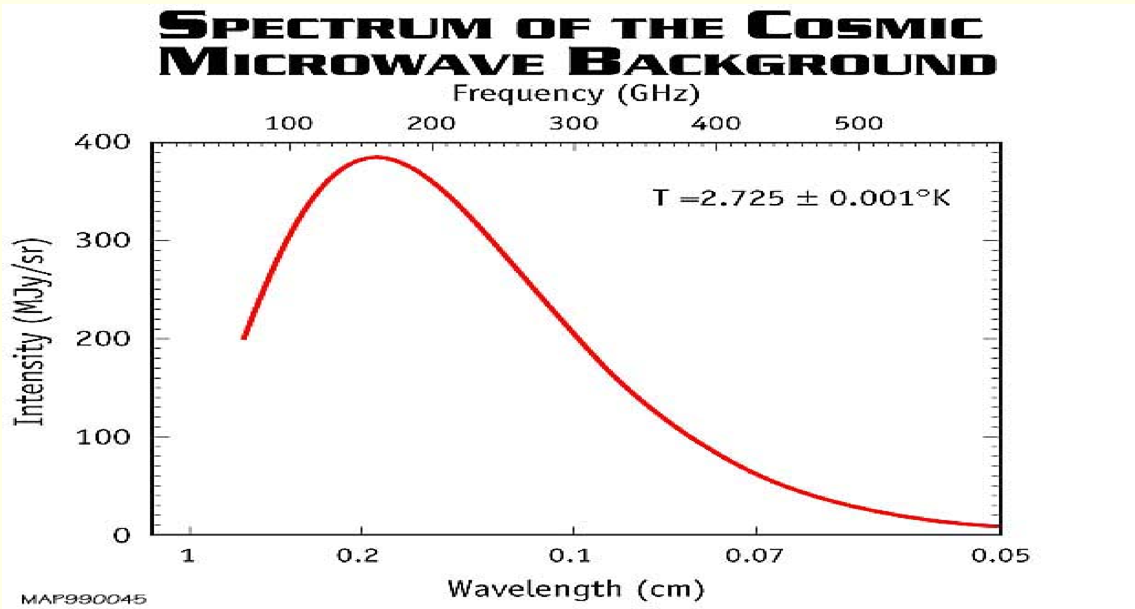

Figure 17: *The spectrum of the CMBR.*

The cosmic microwave background was predicted in 1948 by George Gamow and Ralph Alpher, and by Alpher and Robert Herman. Moreover, Alpher and Herman were able to estimate the temperature of the cosmic microwave background to be 5 K, though two years later, they re-estimated it at 28 K. In 1965, Arno Penzias and Robert Woodrow Wilson at the Crawford Hill location of Bell Telephone Laboratories had built a radiometer that they intended to use for radio astronomy and satellite communication experiments. Their instrument had an excess 3.5 K antenna temperature which they could not account for. The spectral energy density

$$\varepsilon(\nu, T) = \frac{8\pi h}{c^3} \frac{\nu^3}{e^{\frac{h\nu}{kT}} - 1}$$

which has units of energy per unit volume per unit frequency (joule per cubic meter per hertz). The total energy density and the number density

$$\varepsilon_{\rm tot}(T) = \int_0^\infty \varepsilon(\nu, T) d\nu = \frac{8\pi^5 k^4}{15 h^3 c^3} T^4 \qquad n_{\rm tot}(T) = \int_0^\infty n(\nu) d\nu = \frac{16\zeta(3)\pi k^3}{c^3 h^3} T^3$$

where $n(\nu) \equiv \varepsilon(\nu)/(h\nu)$. The average photon energy reads:

$$\langle \varepsilon \rangle = \frac{\varepsilon_{\rm tot}(T)}{n_{\rm tot}(T)} = \frac{\pi^4}{30\zeta(3)} kT \simeq 2.7 kT$$

Galactic rotation curves - Dark Matter

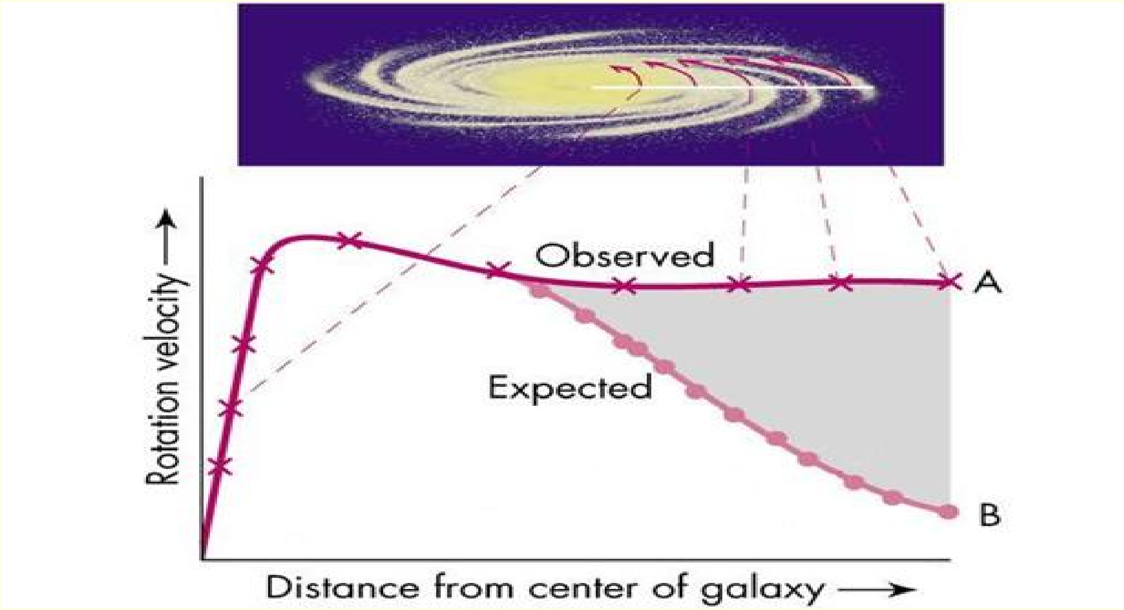

Figure 18: *A typical rotation curve.*

For nearly 40 years after Zwicky's initial observations, no other observations indicated that the mass to light ratio was anything other than unity (a high mass-to-light ratio indicates the presence of dark matter). Then, in the late 1960s and early 1970s, Vera Rubin, a young astronomer at the Department of Terrestrial Magnetism at the Carnegie Institution of Washington presented findings based on a new sensitive spectrograph that could measure the velocity curve of edge-on spiral galaxies to a greater degree of accuracy than had ever before been achieved. Together with fellow staff-member Kent Ford, Rubin announced at a 1975 meeting of the American Astronomical Society the astonishing discovery that most stars in spiral galaxies orbit at roughly the same speed. This result suggests that:

- Either Newtonian gravity does not apply universally, see MOND.

- Or that, conservatively, upwards of 50% of the mass of galaxies was contained in the relatively dark galactic halo.

MOND (Modified Newtonian Dynamics) was proposed by Mordehai Milgrom in 1983 as a way to model observed flat rotational curves. Milgrom noted that Newton's law for gravitational force has been verified only where gravitational acceleration is large, and suggested that for extremely low accelerations the theory may not hold. MOND theory posits that acceleration is not linearly proportional to force at low values:

$$G_N \frac{Mm}{r^2} = ma \quad \Longrightarrow \quad G_N \frac{Mm}{r^2} = m\mu\left(\frac{a}{a_0}\right)a$$

where

$$\mu(x) = \begin{cases} 1 & \text{for} \quad x \gg 1 \\ x & \text{for} \quad x \ll 1 \end{cases}$$

Then assuming $a \ll a_0$

$$G_N \frac{Mm}{r^2} = m\mu\left(\frac{a}{a_0}\right)a = m\frac{v^2}{r} \quad \Longrightarrow \quad G_N \frac{M}{r^2} = \frac{a}{a_0}a = \frac{1}{a_0}\left(\frac{v^2}{r}\right)^2$$

$$\Downarrow$$

$$v = \sqrt[4]{G_N M a_0}$$

Typical value of a_0: $a_0 \sim 1.2 \cdot 10^{-8}\mathrm{cm\ s}^{-2}$.

MOND stands in contrast to the more widely accepted theory of dark matter. Dark matter theory suggests that each galaxy contains a halo of yet unidentified type of matter that provides an overall mass distribution different from the observed distribution of visable matter. This dark matter modifies gravity so as to cause the flat rotational curves.

Tensor-Vector-Scalar gravity (TeVeS) proposed by Jacob Bekenstein in 2004 is a relativistic theory that is equivalent to MOND in the non-relativistic limit, which explains the galaxy rotation problem without invoking dark matter.

The break-through of TeVeS over MOND is that it can also explain the phenomenon of gravitational lensing, a cosmic phenomenon in which nearby matter bends light, which has been confirmed many times.