$$B_0 = \mu^{4-D}\int \frac{d^Dk}{(2\pi)^D}\frac{1}{(k^2-M_1^2)((p+k)^2-M_2^2)}$$ $p^2 = M_1^2 = M^2$, $M_2^2 = m^2$ (1)

$$= \frac{i}{(4\pi)^2}\left[\frac{2}{4-D} - \gamma + \ln 4\pi - \int_0^1 \ln\frac{M_1^2(1-x)+M_2^2x-p^2x(1-x)}{\mu^2}\,dx\right]$$

IF NECESSARY, PUT $M_i^2 \to M_i^2 - i\varepsilon$

ASSUME $p^2 = M_1^2$, $\Delta = \frac{2}{4-D} - \gamma + \ln 4\pi\mu^2$

THEN

$$B_0 = \frac{i}{(4\pi)^2}\left(\Delta - \int_0^1 \ln\frac{M_1^2(1-x)+M_2^2x-M_1^2x(1-x)}{}\,dx\right)$$

$$I = \int_0^1 \ln\frac{M_1^2(1-x)^2+M_2^2x}{}\,dx = x\ln\left(M_1^2(1-x)^2+M_2^2x\right)\Big|_0^1$$

$$-\int_0^1 x\,\frac{1}{M_1^2(1-x)^2+M_2^2x}\left(M_2^2+M_1^2\,2(x-1)\right)dx$$

$$= \ln M_2^2 - \int_0^1 \frac{x\left[M_1^2\,2(x-1)+M_2^2\right]}{M_1^2(1-x)^2+M_2^2x}$$

DEFINE $\eta = \frac{M_2^2}{M_1^2}$. THAN:

$$I = \ln M_2^2 - \int_0^1 \frac{x(2x-2+\eta)}{(1-x)^2+\eta x}\,dx = \ln M_2^2 - \int_0^1 \frac{2x^2+(\eta-2)x}{x^2+(\eta-2)x+1}$$

$$I = \ln m_2^2 - \int_0^1 \frac{2x^2 + 2(\eta-2)x + 2 - (\eta-2)x - 2}{x^2 + (\eta-2)x + 1}\,dx$$

$$= \ln m_2^2 - 2\int_0^1 dx + \int_0^1 \frac{(\eta-2)x + 2}{x^2 + (\eta-2)x + 1}\,dx$$

$$x^2 + (\eta-2)x + 1 = \left(x + \frac{\eta-2}{2}\right)^2 - \frac{(\eta-2)^2}{4} + 1$$

$$= \left(x + \frac{\eta}{2} - 1\right)^2 - \frac{\eta^2}{4} + \eta + 2$$

NEW VARIABLE $y = x + \frac{\eta}{2} - 1$; $x = y + 1 - \frac{\eta}{2}$

$$I = \ln m_2^2 - 2 + \int_{\frac{\eta}{2}-1}^{\frac{\eta}{2}} \frac{(\eta-2)\left(y - \frac{\eta-2}{2}\right) + 2}{y^2 + 1 - \frac{(\eta-2)^2}{4}}\,dy$$

$$x_\eta = \frac{m_2^2}{m_1^2} < 1 \Rightarrow a^2 = 1 - \frac{(\eta-2)^2}{4} > 0$$

$$I = \ln m_2^2 - 2 + (\eta-2)\int_{\frac{\eta}{2}-1}^{\frac{\eta}{2}} \frac{y - \frac{\eta-2}{2} + \frac{2}{\eta-2}}{y^2 + 1 - \frac{(\eta-2)^2}{4}}\,dy$$

$$= \ln m_2^2 - 2 + (\eta-2)\int_{\frac{\eta}{2}-1}^{\eta/2} \frac{y + \frac{4-(\eta-2)^2}{2(\eta-2)}}{y^2 + \frac{4-(\eta-2)^2}{4}}\,dy$$

$$\int \frac{y\,dy}{y^2+a^2} = \frac{1}{2}\int \frac{d(y^2+a^2)}{y^2+a^2} = \frac{1}{2}\ln(y^2+a^2)$$

$$\int \frac{dy}{y^2+a^2} = \frac{1}{a}\int \frac{d(y/a)}{(y/a)^2+1} = \frac{1}{a}\,\mathrm{arctg}\,\frac{y}{a}$$

$$I = \ln M_2^2 - 2 + \frac{(\eta-2)}{2} \ln\left(\frac{\frac{\eta^2}{4} + \frac{4-(\eta-2)^2}{4}}{\frac{(\eta-2)^2}{4} + \frac{4-(\eta-2)^2}{4}} \right)$$ ③

$$+ \frac{\eta-2}{\sqrt{\frac{4-(\eta-2)^2}{4}}} \left[\arctan \frac{\eta/2}{\sqrt{}} - \arctan \frac{\frac{\eta-2}{2}}{\sqrt{}} \right] \otimes$$

$$\otimes \; \frac{4-(\eta-2)^2}{2(\eta-2)}$$

$$= \ln M_2^2 - 2 + \frac{\eta-2}{2} \ln \frac{\cancel{4}+4\eta}{4} + \frac{\sqrt{4-(\eta-2)^2}}{2} \left[\arctan \frac{\eta/2}{a} - \arctan \frac{\frac{\eta-2}{2}}{a} \right]$$

$$= \ln M_2^2 - 2 + \frac{\eta-2}{2} \ln \frac{M_2^2 \cancel{+M_1^2}}{M_1^2} + \frac{\sqrt{4-(\eta-2)^2}}{2} \left[\; \dots \; \right]$$

$$2a = \sqrt{4-(\eta-2)^2}$$

DEFINE $\sqrt{4-(\eta-2)^2} = \sqrt{4\eta - \eta^2} \equiv A$

$$I = \ln M_2^2 - 2 + \frac{\eta-2}{2} \ln(\eta) + \frac{A}{2}\left[\arctan \frac{\eta}{A} - \arctan \frac{\eta-2}{A} \right]$$

$$B_0 = \frac{i}{(4\pi)^2} \left[\frac{2}{4-D} - \gamma + \ln 4\pi - \ln \frac{M_2^2}{\mu^2} + 2 - \frac{\eta-2}{2} \ln \eta \right.$$
$$\left. - \frac{A}{2} \left[\arctan \frac{\eta}{A} - \arctan \frac{\eta-2}{A} \right] \right]$$

$$\eta = \frac{M_2^2}{M_1^2} = \frac{m^2}{M^2} \quad A = \sqrt{4-(\eta-2)^2}$$

[IN A.D. SYMBOLS
$m^2 = M_2^2$
$M^2 = p^2 = M_1^2$]

$$0 < \eta = \frac{m_2^2}{m_1^2} < 1$$

$$A = \sqrt{4-(\eta-2)^2} < 2$$

$$\frac{\eta}{A} = \frac{\eta}{\sqrt{4\eta-\eta^2}} = \frac{1}{\sqrt{4/\eta - 1}} < \frac{1}{\sqrt{3}}$$

$$\frac{\eta-2}{A} = \frac{\eta-2}{\sqrt{4-(\eta-2)^2}} = \frac{-1}{\sqrt{\frac{4}{(\eta-2)^2}-1}}$$

$$\frac{\eta(\eta-2)}{A^2} = \frac{\eta(\eta-2)}{4-(\eta-2)^2} = \frac{\eta^2-2\eta}{4\eta-\eta^2} = \frac{-2\eta+\frac{\eta^2}{2}+\frac{\eta^2}{2}}{4\eta-\eta^2}$$

$$= -\frac{1}{2} + \frac{1}{2}\,\frac{1}{4/\eta-1} > -1$$

$$\operatorname{arctg}\frac{\eta}{A} - \operatorname{arctg}\frac{\eta-2}{A} = \operatorname{arctg}\frac{\frac{\eta-\eta+2}{A}}{1+\frac{\eta(\eta-2)}{A^2}} =$$

$$= \operatorname{arctg}\frac{2A}{4\eta-\cancel{\eta^2}+\cancel{\eta^2}-2\eta} = \operatorname{arctg}\frac{2A}{2\eta} = \operatorname{arctg}\frac{A}{\eta}$$

FINALLY

$$B_0 = \frac{i}{(4\pi)^2}\left[\frac{2}{4-D} - \gamma + \ln 4\pi - \ln\frac{m_2^2}{\mu^2} + 2 + \frac{\eta-2}{2}\ln\eta - \frac{A}{2}\operatorname{arctg}\frac{A}{\eta}\right]$$

$$\eta = \frac{m_2^2}{m_1^2} = \frac{m^2}{M^2}$$

$$A = \sqrt{4-(\eta-2)^2} = \sqrt{4\eta-\eta^2}$$