D) SOLVING THE RGE

EXAMPLE : $\lambda \phi^4$ AGAIN:

$$\left[\mu \frac{\partial}{\partial \mu} + \beta \frac{\partial}{\partial \lambda} + \gamma_m m \frac{\partial}{\partial m} - n\gamma\right] \Gamma^{(n)} = 0$$

SUBSTITUTE VARIABLES $\mu = \mu_0 e^{-t} \Rightarrow \mu \frac{\partial}{\partial \mu} = -\frac{\partial}{\partial t}$

FIND RUNNING COUPLING AND MASS

$$\int_{\lambda}^{\bar{\lambda}(t,\lambda)} \frac{dx}{\beta(x)} = t \qquad \left[\frac{\partial \lambda}{\partial t} = \beta(\lambda)\right]$$

$$\bar{m}(t,\lambda,m) = m \exp\left[\int_{\lambda}^{\bar{\lambda}(t,\lambda)} \frac{\gamma_m(x)}{\beta(x)} dx\right] = m \exp\left[\int_0^t \gamma_m(\lambda(t)) dt\right]$$

$$\left[\frac{\partial m}{\partial t} = m \gamma_m(\lambda(t))\right]$$

$$\begin{cases} \bar{\lambda}(0,\lambda) = \lambda \\ \bar{m}(0,\lambda,m) = m \end{cases}$$

THEN

$$\Gamma^{(n)}(p,\lambda,m,\mu) = \Gamma^{(n)}(p, \bar{\lambda}(t), \bar{m}(t), \mu e^{t}) \exp\left[-n \int_0^t \gamma(\bar{\lambda}(t')) dt'\right]$$

MAIN BEHAVIOUR DEPENDS ON $\beta(\lambda)$, THEN γ, γ_m !

RESCALING THE MOMENTA:

$p_i \to \rho p_i$

LET'S TAKE GREEN'S FUNCTION OF DIM D:

$$\Gamma^{(n)} \sim [M]^D$$

THEN $\Gamma^{(n)}(\rho p_i, m, \lambda, \mu) = \mu^D f\left(\rho^2 \frac{p_i p_j}{\mu^2}, \frac{m}{\mu}, \lambda\right)$

— $\Gamma^{(n)}$ IS HOMOGENOUS FUNCTION OF m, ρ, μ OF THE ORDER D. FOR SUCH FUNCTIONS:

$$\left(\rho \frac{\partial}{\partial \rho} + m \frac{\partial}{\partial m} + \mu \frac{\partial}{\partial \mu} - D\right) \Gamma^{(n)} = 0$$

DENOTE $t = \ln \rho$, $\rho \frac{\partial}{\partial \rho} = \frac{\partial}{\partial t}$, EVALUATE $\mu \frac{\partial}{\partial \mu}$ FROM RGE

$$\left[-\frac{\partial}{\partial t} + \beta \frac{\partial}{\partial \lambda} + (\gamma_m - 1) m \frac{\partial}{\partial m} - n\gamma + D\right] \Gamma^{(n)} = 0$$

$$\Gamma^{(n)}(\rho p_i, \lambda, m, \mu) \underset{\mu \text{ CHANGE}}{=} \Gamma^{(n)}(\rho p_i, \bar{\lambda}(t), \bar{m}(t), \mu e^{t}) e^{-n\int \gamma dt}$$

$$\underset{\text{HOMOGENITY}}{=} \rho^D \Gamma^{(n)}\left(p_i, \bar{\lambda}(t), \frac{\bar{m}(t)}{\rho}, \mu\right) e^{-n\int \gamma dt}$$

VERY IMPORTANT!! GREEN'S FUNCTION AT GIVEN MOMENTUM IS GIVEN IN TERMS OF EFF. COUPLING AND MASS AT THIS MOMENTUM

CRUCIAL KNOWLEDGE - BEHAVIOUR OF THE EFFECTIVE COUPLING

(3)

$$\mu \frac{d}{d\mu} \tilde{\lambda}(\mu) = \beta(\tilde{\lambda}(\mu))$$

LET'S USE $t = \ln \frac{\mu}{\mu_0}$ $\tilde{\lambda}(\mu = \mu_0) = \lambda$

$$\cancel{\frac{d}{d\mu}} = dt = \frac{d\mu}{\mu}$$

$$\frac{d}{d\mu} = \frac{dt}{d\mu}\frac{d}{dt} = \frac{1}{\mu}\frac{d}{dt}$$

$$\frac{d\tilde{\lambda}}{dt} = \beta(\tilde{\lambda})$$

$$(*) \quad \int_{\lambda}^{\tilde{\lambda}(t,\lambda)} \frac{dx}{\beta(x)} = t$$

LET'S ASSUME $(*)$ VALID FOR $t \in (-\infty, \infty)$ (OR CUTOFF NEEDED)

$\beta(x) \neq 0$ ALWAYS $\Rightarrow$ $\lambda \to \infty$ FOR $t \to \pm\infty$

WHAT IF $\beta(\lambda_0) = 0$?

$$\beta(\lambda) \underset{\lambda\to\lambda_0}{\simeq} \beta^{(0)} (\lambda - \lambda_0)^N$$

$$\int_{\lambda}^{\tilde{\lambda}} \frac{dx}{(x-\lambda_0)^N} = \beta^{(0)} t$$

$N=1$ ~~IN GENE~~

$$\int_{\lambda}^{\bar\lambda} \frac{dx}{x-\lambda_0} = \ln \frac{\bar\lambda-\lambda_0}{\lambda-\lambda_0} = p^{(0)} t$$

$$\bar\lambda - \lambda_0 = (\lambda-\lambda_0)\, e^{p^{(0)}t}$$

$$\bar\lambda = \lambda_0 + (\lambda-\lambda_0)\, e^{p^{(0)}t}$$

$N>1$

$$\int_{\lambda}^{\bar\lambda} \frac{dx}{(x-\lambda_0)^N} = \frac{1}{1-N} (x-\lambda_0)^{1-N}\Big|_{\lambda}^{\bar\lambda} = p^{(0)} t$$

$$(\bar\lambda-\lambda_0)^{-N+1} = (\lambda-\lambda_0)^{-N+1} - \frac{p^{(0)}t}{N-1}$$

$$\bar\lambda = \lambda_0 + \frac{1}{\left[(\lambda-\lambda_0)^{-N+1} - \frac{p^{(0)}t}{N-1}\right]^{N-1}}$$

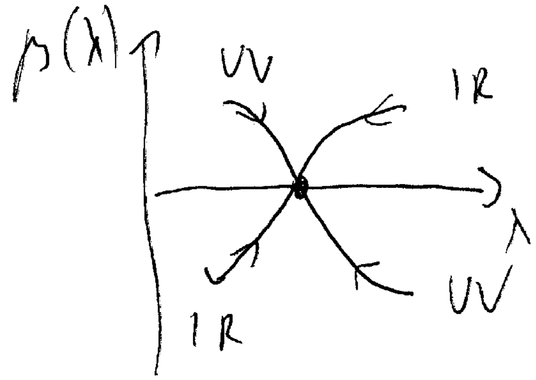
IN GENERAL, BOTH $N=1$, $N>1$

$p^{(0)} > 0 \Longrightarrow$ FOR $t \to -\infty$ $\quad \lambda \to \lambda_0 \equiv \lambda_-$

$[p'(\lambda_-) > 0]$ IR STABLE POINT

$p^{(0)} < 0 \Longrightarrow$ FOR $t \to +\infty$ $\quad \lambda \to \lambda_0 \equiv \lambda_+$

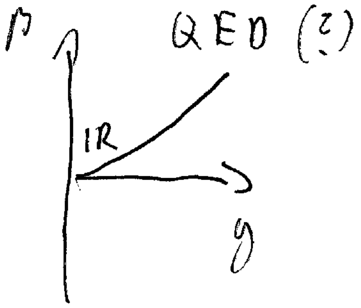
$[p'(\lambda^+) < 0]$ UV STABLE POINT

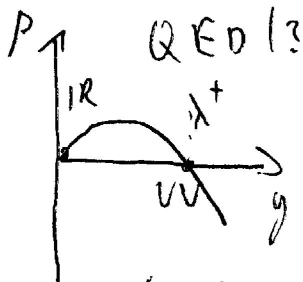


ALWAYS (PERTURBATIVELY) $\beta(0)=0$.

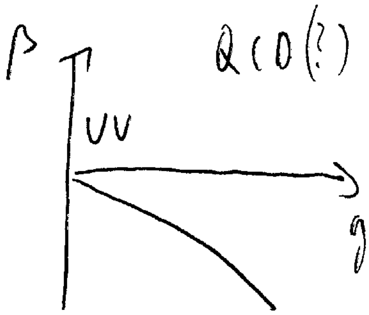
FOR SINGLE-COUPLINGS THEORIES ALWAYS $g=0$ IS IR FIXED POINT ("IR STABLE THEORY" — $\lambda\Phi^4$, QED, $g\to g_0$ FOR $q^2\to 0$) OR UV FIXED POINT ("ASYMPTOTICALLY FREE THEORY" — QCD, $g\to 0$ FOR $q^2\to\infty$)

VARIOUS SITUATIONS POSSIBLE:

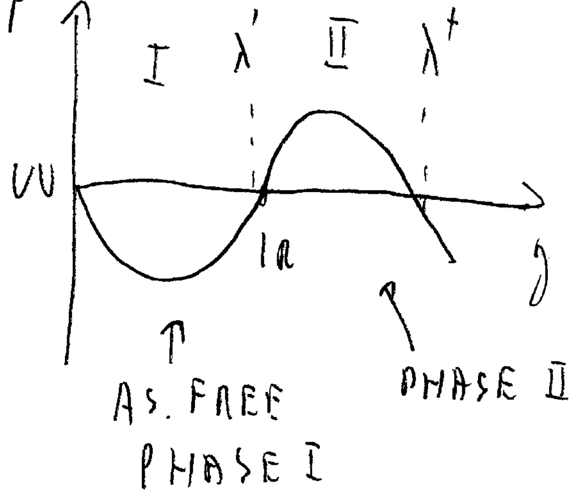




(**)



(*)



CONSIDER FIGURE (*) - QCD HOPEFULLY

$$\int_{g}^{\tilde{g}(t=-\infty,g)} \frac{dx}{\beta(x)} = -\infty \Rightarrow \hat{g}(t\to-\infty)\to\infty$$

"IR SLAVERY" $\not\Rightarrow$ "QUARK CONFINEMENT"

NOT PROVEN YET!

LANDAU POLE

FIRST ORDER QED:

$$\beta(e) = \frac{4}{3}\frac{e^3}{16\pi^2} + O(e^5) \quad b_2 = \frac{1}{i} > 0 \qquad e_0 = e(q^2 = m_e^2)$$

$$\int_{e_0}^{e(t)} \frac{dx}{\frac{4}{3}\frac{x^3}{16\pi^2}} = t$$

$$\int_{e_0}^{e(t)} \frac{dx}{x^3} = \frac{t}{12\pi^2} = -\frac{1}{2x^2}\Big|_{e_0}^{e(t)} = -\frac{1}{2}\left(\frac{1}{e^2} - \frac{1}{e_0^2}\right)$$

$$\frac{1}{e^2} = \frac{1}{e_0^2} - \frac{t}{6\pi^2}$$

$$e^2 = \frac{e_0^2}{1 - \frac{e_0^2 t}{6\pi^2}} \to \infty \quad \text{FOR} \quad t = \frac{6\pi^2}{e_0^2} = \ln\frac{q^2}{m_e^2}$$

$$q_\infty^2 = m_e^2 e^{\frac{3\pi}{2\alpha_0}} > M_{PLANCK}$$

CUTOFF NECESSARY, OR HIGHER ORDER TERMS LEAD TO (**)! $\lambda\Phi^4$ - SIMILAR.

HOW STABLE ARE FIXED POINTS?

THEY ARE!

A) REN. SCHEME DEPENDENCE. EXAMPLE QED/QCD

$$g' = F(g) = g + O(g^3) \qquad R \to R'$$

$$\left.\begin{aligned} Z'_m(g') &= Z_m(g)\, F_m(g) \\ Z'_3(g') &= Z_3(g)\, F_3(g) \end{aligned}\right\} \quad F_i(g) = 1 + O(g^2)$$

FOR SMALL g $\quad \frac{dF}{dg} \neq 0, \quad F_i \neq 0$

$$\beta'(g') = \mu \frac{d}{d\mu} g' = \frac{dF}{dg}\, \beta(g)$$

$$\gamma'(g') = \frac{1}{2}\mu \frac{d}{d\mu} \ln Z'_3(g') = \gamma(g) + \frac{1}{2} \frac{d \ln F_3}{dg}\, \beta(g)$$

$$\gamma'_m(g') = \gamma_m(g) + \frac{1}{2} \frac{d \ln F_m}{dg}\, \beta(g)$$

TO BE PROVED:

i) $$\frac{d\beta'}{dg'} = \frac{dg}{dg'} \frac{d}{dg}\left(\frac{dF}{dg}\beta(g)\right) = \frac{1}{F'(g)}\left[F''(g)\beta + F'(g)\frac{d\beta}{dg}\right]$$

$$\frac{d\beta'}{dg'} = \frac{d\beta}{dg} + \frac{F''}{F'}\beta$$

FOR FIXED POINT $\beta = 0$, $\frac{d\beta'}{dg'} = \frac{d\beta}{dg} \to$ IR/UV CHARACTER PRESERVED

ii) $\beta(g) = -2b_0 g^3 - 2b_1 g^5$

$$\Rightarrow -2b_0 g'^3 - 2b_1 g'^5 = \beta(g')$$

(MASS-IND. REN. SCHEMES)

iii) FOR FIXED POINT

$$\gamma'(g') = \gamma(g) \qquad \gamma_m'(g') = \gamma_m(g)$$

iv) $\gamma(g) = \gamma_0 g^2 + O(g^4) \Rightarrow \gamma'(g') = \gamma_0 g'^2 + O(g'^4)$

THE SAME FOR γ_m

i, ii) $\rightarrow$ UV, IR STABILITY

iii, iv) $\rightarrow$ SCALING BEHAVIOUR AND MASSES PRESERVED, THE SAME FOR LEADING BEHAVIOUR $t \rightarrow \pm\infty$

SIMILARLY, GAUGE PARAMETER CHANGE LEAD TO $\alpha(\mu) \rightarrow \alpha'(\mu) = \alpha(\mu) + O(\alpha^2)$, BUT b_0 REMAINS <u>UNCHANGED</u>

EXERCISE – PROVE $\beta(\alpha)$ IS GAUGE-INDEP.

IN MS SCHEME:

$$0 = \left.\frac{d\alpha_B}{da}\right|_{\varepsilon \text{ FIXED}} = \frac{da_B}{da}\frac{d\alpha_B}{da_B} = 0$$

$$\alpha_B = \mu^{\varepsilon} Z_\alpha \alpha$$

$$0 = \left.\frac{d}{da}\left(Z_\alpha \alpha\right)\right|_{\alpha_B, \varepsilon \text{ FIXED}} = Z_\alpha \frac{d\alpha}{da} + \alpha \frac{dZ_\alpha}{da} \qquad (*)$$

IN MS: $Z_\alpha = 1 + \frac{Z_\alpha^{(1)}(\alpha, a)}{\varepsilon} + \frac{Z_\alpha^{(2)}(\alpha, a)}{\varepsilon^2} + \dots \qquad (**)$

INSERTING $(**)$ INTO $(*)$

$$\frac{d\alpha}{da} + \frac{1}{\varepsilon}\left(\frac{d\alpha}{da} Z_\alpha^{(1)} + \alpha \frac{dZ_\alpha^{(1)}}{da}\right) + O\left(\frac{1}{\varepsilon^2}\right) = 0$$

THUS: $\frac{d\alpha}{da} = 0$ – 0-th ORDER

$\frac{dZ_\alpha^{(1)}}{da} = 0$ – $\frac{1}{\varepsilon}$ TERM

~~THUS~~ SO: $\alpha(\mu, a) = \alpha(\mu)$

$\beta(\cancel{\alpha}(a)) = \beta(\alpha)$

SOLVING THE RGE FOR RUNNING COUPLING

$$\beta(\alpha) = \alpha \sum_{n=1}^{\infty} b_n \left(\frac{\alpha}{\pi}\right)^n$$

$$\frac{d\alpha}{dt} = \beta(\alpha) \qquad (*)$$

LET'S EXPAND:

$$\alpha(t) = \sum_{m=1}^{\infty} a_m(t)\, \alpha^m \qquad a_1 = 1 \qquad \alpha = \alpha(t=0)$$

FROM $(*)$

$$\sum_{m=1}^{\infty} \frac{da_m}{dt} \alpha^m = \cancel{\alpha} \sum_{n=1}^{\infty} \frac{b_n}{\pi^n} \left[\alpha + \sum_{m=1} a_m(t)\, \alpha^m\right]^n$$

COMPARING α^k COEFFICIENT ONE GET'S:

$$a_2(t) = \frac{b_1 t}{\pi}$$

$$a_3(t) = \left(\frac{b_1 t}{\pi}\right)^2 + \frac{b_2 t}{\pi}$$

$$\vdots$$

$$a_n(t) = \left(\frac{b_1 t}{\pi}\right)^{n-1} + O\left(t^{n-1}\right)$$

SUMMING

$$\alpha(t) = \alpha\left[1 + \sum_{n=1}^{\infty}\left[\left(\frac{b_1 t \alpha}{\pi}\right)^n + O\left(\alpha^n t^{n-1}\right)\right]\right]$$

USUALLY $t = \frac{1}{2} \ln \frac{\mu^2}{\mu_0^2}$ OR $t = \frac{1}{2} \ln \frac{p^2}{p_0^2}$

LEADING-LOG APPROXIMATION – NEGLECT TERMS $O(t^{n-1}\alpha^n)$ COMPARING TO $O(t^n \alpha^n)$

$= \alpha\, O(t^{n-1}\alpha^{n-1})$

IN LLA:

$$\alpha(t) = \frac{\alpha}{1 - \frac{\alpha b_1}{\pi} t} \qquad \alpha = \alpha(t=0)$$

GENERAL SOLUTION

$$b_1 t = \frac{\pi}{\alpha} - \frac{\pi}{\alpha(t)} + B_1 \ln \frac{\alpha}{\alpha(t)} + \sum_{n=1} \frac{C_n}{n} \left[\left(\frac{\alpha(t)}{\pi}\right)^n - \left(\frac{\alpha}{n}\right)^n \right]$$

$$B_n = \frac{b_{n+1}}{b_0}$$

$$C_n = (-1)^{n+1} \det \begin{vmatrix} B_1 & B_2 & \cdots & B_{n+1} \\ 1 & \ddots & \ddots & \vdots \\ & \ddots & \ddots & B_2 \\ 0 & & 1 & B_1 \end{vmatrix}$$

NLLA:

$$\alpha_{NLO}(t) = \frac{\alpha}{1 - \frac{\alpha b_1}{\pi} t} \left[1 - \frac{\alpha}{\pi} \frac{1}{1 - \frac{\alpha b_1}{\pi} t} \frac{b_2}{b_1} \ln\left(1 - \frac{\alpha b_1 t}{\pi}\right) \right]$$

$$= \alpha_{LO}(t) \left[1 - \frac{\alpha_{LO}(t)}{\pi} \frac{b_2}{b_1} \ln \frac{\alpha}{\alpha_{LO}(t)} \right]$$

SMALL SUMMARY

IN QCD/QED

$$\frac{\bar{g}^2(Q^2)}{16\pi^2} = \frac{\bar{g}^2(\mu^2)}{16\pi^2} \frac{1}{1 - \frac{\bar{g}^2(\mu^2)}{16\pi^2} b_1 \ln \frac{Q^2}{\mu^2}}$$

$$\bar{g}^2(Q^2) \xrightarrow[Q^2 \to \infty]{} 0 \qquad b_1 < 0 \quad (QCD \to ASF)$$

$$\to \infty \qquad b_1 > 0 \quad (QED \to \text{LANDAU POLE})$$

$$b_1 = \begin{cases} -\left(\frac{11}{3} N_{COLOR} - \frac{2}{3} N_{FLAVOUR}\right) & QCD \\ -\frac{4}{3} \sum_i Q_i^2 & QED \end{cases}$$

DEFINE $\Lambda^2_{QCD} = \mu^2 \exp\left[\frac{16\pi^2}{b_1 \bar{g}^2(\mu)}\right] \quad \left[\simeq 200 \pm 100 \text{ MeV}\right]$

THEN:

$$\alpha_s(Q^2) = \frac{4\pi}{(-b_1) \ln \frac{Q^2}{\Lambda^2_{QCD}}} = \frac{12\pi}{(33-2n_F) \ln \frac{Q^2}{\Lambda^2_{QCD}}}$$

"DIMENSIONAL TRANSMUTATION" - DIMENSIONFUL PARAMETER Λ_{QCD} APPEARS IN QCD DEFINED BY DIMENSIONLESS CONSTANT g_s!

SOLUTION FOR MASSES

$$\bar{m}(t,m,\lambda) = m\, e^{\int_{\lambda}^{\bar{\lambda}} \frac{\gamma_m(x)}{\beta(x)} dx}$$

EXERCISE: ASSUMING GRAND UNIFICATION IN THE SM AND MSSM, FIND RELATION BETWEEN α_{EM}, $\alpha_S(M_Z)$ AND $\sin^2\theta_W$. (IN LO)

$\boxed{\text{SM FIRST}}$

$$\frac{dg_i}{dt} = b_i g_i^3$$

IN SM $16\pi^2 b_Y = \frac{41}{6}$ IN MSSM $16\pi^2 b_Y = 11$

$16\pi^2 b_2 = -19/6$ $16\pi^2 b_2 = 1$ (NOT AS-FREE!)

$16\pi^2 b_3 = -7$ $16\pi^2 b_3 = -3$

UNIFICATION $g_1 = g_2 = g_3$, $g_1 = \sqrt{\frac{5}{3}}\, g_Y$

SOLUTION OF RGE:

$$\frac{dg_i}{g_i^3} = b_i\, dt \Rightarrow -\frac{1}{2}\left(\frac{1}{g_i^2(GUT)} - \frac{1}{g_i^2(M_Z)}\right) = b_i(t_{GUT} - t_{M_Z}) = b_i \ln\frac{M_{GUT}}{M_Z}$$

$$\equiv t_0 b_i$$

$$\cancel{\frac{1}{g^2_{GUT}} - \frac{1}{g^2(M_Z)}}$$

$$\frac{1}{g_i^2(M_Z)} = \frac{1}{g^2_{iGUT}} + 2b_i t_0$$

$$g_2^2(M_Z) = \frac{g^2_{2GUT}}{1+2b_i t_0 g^2_{2GUT}}$$, THE SAME FOR g_3^2, g_Y^2

UNIFICATION $g_2^2(GUT) = g_3^2(GUT) = g_0^2$, $g_Y^2(GUT) = \frac{3}{5} g_0^2$

SO $\left(\alpha_0 = \frac{g_0^2}{4\pi}\right)$

$$\alpha_2(M_Z) = \frac{\alpha_0}{1+8\pi b_2 \alpha_0 t_0}$$

$$\alpha_3(M_Z) \equiv \alpha_s = \frac{\alpha_0}{1+8\pi b_3 \alpha_0 t_0}$$

$$\alpha_Y(M_Z) = \frac{3/5\, \alpha_0}{1+8\pi b_Y \, 3/5\, \alpha_0 t_0}$$

NOW

$$\alpha_Y(M_Z) = \frac{\alpha_{em}(M_Z)}{\cos^2\theta_W} \equiv \frac{\alpha_{em}}{1-s^2} \equiv \frac{\alpha}{1-s^2}$$

$$\alpha_2(M_Z) = \frac{\alpha_{em}}{s^2} = \frac{\alpha}{s^2}$$

$$\cancel{\alpha_0}\quad \begin{cases} \dfrac{1}{\dfrac{1}{\alpha_0} + 8\pi b_z t_0} = \dfrac{\alpha}{s^2} \\[2ex] \dfrac{1}{\dfrac{5}{3\alpha_0} + 8\pi b_y t_0} = \dfrac{\alpha}{1-s^2} \end{cases}$$

$$\begin{cases} \dfrac{1}{\alpha_0} + 8\pi b_z t_0 = \dfrac{s^2}{\alpha} \\[2ex] \dfrac{5}{3\alpha_0} + 8\pi b_y t_0 = \dfrac{1-s^2}{\alpha} \end{cases}$$

$$8\pi\left(b_z - \frac{3}{5} b_y\right) t_0 = \frac{s^2}{\alpha} - \frac{3}{5}\,\frac{1-s^2}{\alpha} = \frac{1}{5\alpha}\left(8s^2 - 3\right)$$

$$t_0 = \frac{1}{40\pi\alpha}\,\frac{3 - 8s_w^2}{\frac{3}{5} b_y - b_z} = \frac{1}{8\pi\alpha}\,\frac{3 - 8s^2}{3b_y - 5b_z}$$

$$\frac{1}{\alpha_0} = \frac{s^2}{\alpha} - 8\pi b_z t_0 = \frac{s^2}{\alpha} - \frac{b_z}{5\alpha}\,\frac{3 - 8s^2}{\frac{3}{5} b_y - b_z}$$

$$= \frac{1}{\alpha\,(3b_y - 5b_z)}\left[3b_y s^2 - 5b_z s^2 - 3b_z + 8b_z s^2\right]$$

$$= \frac{3\left[b_y s^2 + b_z s^2 - b_z\right]}{\alpha\,(3b_y - 5b_z)} = \frac{3\left(b_y s^2 - b_z c^2\right)}{\alpha\,(3b_y - 5b_z)}$$

$$\begin{cases} \alpha_0 = \alpha\, \dfrac{b_y - \frac{5}{3} b_z}{b_y s^2 - b_z c^2} \\[2ex] t_0 = \dfrac{1}{8\pi\alpha}\, \dfrac{1 - \frac{8}{3} s^2}{b_y - \frac{5}{3} b_z} \end{cases}$$

FINALLY, α_s:

$$\alpha_s = \frac{1}{\frac{1}{\alpha_0} + 8\pi b_3 t_0} = \frac{1}{\frac{b_Y s^2 - b_2 c^2}{\alpha(b_Y - \frac{5}{3} b_2)} + \frac{b_3}{\alpha} \frac{1 - \frac{8}{3} s^2}{b_Y - \frac{5}{3} b_2}}$$

$$= \frac{\alpha\left(b_Y - \frac{5}{3} b_2\right)}{b_Y s^2 - b_2 c^2 + b_3\left(1 - \frac{8}{3} s^2\right)}$$

$$\alpha_s(M_Z) = \frac{\alpha_{em}(M_Z)\left(b_Y - \frac{5}{3} b_2\right)}{b_Y s_w^2 - b_2(1 - s_w^2) + b_3\left(1 - \frac{8}{3} s_w^2\right)}$$

$\alpha_{em}(M_Z) = \frac{1}{128.8}$

$s_w^2 = 0.23$

$\alpha_s^{SM}(M_Z) = 0.072$ [EXPERIMENT $\alpha_s(M_Z) = 0.118 \pm 0.005$]

$$\begin{cases} \alpha_0 = 0.023 \\ t_0 = 25.8 \\ M_{GUT} = 1.5 \cdot 10^{13}\ \text{GeV} \end{cases}$$

NOT VERY GOOD...

MSSM – BETTER, BUT COMPLICATION $\rightarrow$

$\rightarrow$ (DIMREG $\rightarrow$ DRED)

MSSM

$\alpha_0 = 0.041$

$t_0 = 33.5$

$M_{GUT} = 3.3 \cdot 10^{16}$

$\alpha_s(M_Z) = 0.12$ (!!)

IN PRACTICE, RGE FOR MSSM ARE OBTAINED IN DRED, NOT DIMREG (DRED DOES NOT BREAK SUSY)

$$\text{DIMREG} = g^{\mu\nu} g_{\mu\nu} = n$$

$$\text{DRED} = g^{\mu\nu} g_{\mu\nu} = 4$$

OTHER DEFINITIONS OF DIRAC ALGEBRA IN n-dim etc.

CONVERSION:

$$\begin{cases} \alpha_Y^{DRED} = \alpha_Y^{DIMREG} \\ \alpha_2^{DRED} = \dfrac{\alpha_2^{DIMREG}}{1 - \dfrac{\alpha_2^{DIMREG}}{6\pi}} \\ \alpha_3^{DRED} = \dfrac{\alpha_3^{DIMREG}}{1 - \dfrac{\alpha_3^{DIMREG}}{4\pi}} \end{cases}$$

IMPORTANT AT HIGHER ORDERS, BUT $\alpha_s \to 0.118$!