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Analysis of EEG Transients by Means of Matching Pursuit

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Abstract—Matching pursuit (MP), a new technique of time-frequency signal analysis, was applied to simulated signals and the awake and sleep EEG. With the MP algorithm, waveforms from a very large class of functions were fitted to the local signal structures in a recursive procedure. By means of this technique, sleep spindles were localized in the time-frequency plane with high precision, and their intensities and time spans were found. The MP technique makes following the temporal evolution of transients and their propagation in brains possible. It opens up new possibilities in EEG research providing a means of investigation of dynamic processes in brains in a much finer time-frequency scale than any other method available at present.

Keywords—Matching Pursuit, Wavelet analysis, EEG, Sleep spindles.

INTRODUCTION

We would like to introduce to physiological signal analysis a new method called matching pursuit (MP) which can be considered a generalization of the wavelet transform. MP was proposed by Mallat and Zhang (1).

The complex structure of physiological time series and the rapidly varying characteristics of the patterns embedded in them suggest their decomposition over large classes of waveforms. Linear expansion in a single Fourier or wavelet basis is not always sufficient. A Fourier basis gives a poor representation of functions well localized in time, whereas wavelet bases are not well adapted to represent functions whose Fourier transforms have a narrow frequency of support. In both above-mentioned methods, the detection and identification of signal transients from their expansion coefficients may not be easy since the information can be diluted across the whole basis.

Matching pursuit relies on the decomposition of signals into linear expansion of waveforms belonging to a very broad class of functions. These waveforms are adaptively matched to the local signal patterns. This kind of analysis is especially suitable for characterizing transients appearing randomly in the signal, such as sleep spindles, K-complexes, epileptic spikes. We have demonstrated the performance of MP on simulated signals and experimental EEG series; special attention was paid to sleep spindles.

METHOD

In matching pursuit, the repertoire of functions used for the decomposition of signals is very broad and redundant. From this large dictionary of possible functions, a subfamily of time-frequency atoms is chosen in such a way as to best match the local signal structures. The family of time-frequency atoms is created by scaling, translating, and modulating a window function $g(t)$:

$$g_I(t) = \frac{1}{\sqrt{s}} g\left(\frac{t-u}{s}\right) e^{i\xi t}, \tag{1}$$

where $s > 0$ is scale, ξ is frequency modulation, and u is translation.

Index $I = (s,\xi,u)$ describes the set of parameters. $g(t)$ is usually even and its energy is mostly concentrated around u in a time domain proportional to s. In the frequency domain energy is mostly concentrated around ξ with a spread proportional to $1/s$. The minimum of time-bandwidth product is obtained when $g(t)$ is Gaussian. The windowed Fourier transform (FT) and wavelet transform (WT) can be considered as a particular cases of MP corresponding to the certain restrictions concerning the choice of parameters. In the case of FT, scale s is constant—equal to window length and parameters ξ and u are uniformly sampled; therefore, FT is not appropriate for describing structures much smaller or much larger than the window. The wavelet transform overcomes this limitation since it allows for the change of scale, decomposing the signal over the atoms of varying time-frequency coordinates. Nevertheless, in the case of WT the frequency modulation is limited by the restriction on the frequency parameter which remains inversely proportional to the scales. Therefore, as was pointed out above, the wavelet frame does not provide precise estimates of the frequency content of waveforms well localized in the time domain.

In matching pursuit all parameters defined in Eq. 1 vary

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freely; the time-frequency atoms remain to be chosen in an optimal way. The method of adaptive selection of functions best approximating the signal was proposed by Mallat and Zhang (7). It relies basically on finding a linear expansion of f over a selected subset of vectors g_I.

In the first step of iteration procedure, the function g_{Io} is chosen to give the biggest product with signal $f(t)$. Then, the residual vector R_1, obtained after approximating f in the direction g_{Io} is decomposed in similar way. The procedure is iterated on subsequently obtained residues:

$$R^n f = \langle R^n f, g_{I_n} \rangle g_{I_n} + R^{n+1} \mathrm{f}. \qquad (2)$$

In this way the signal f is decomposed into a sum of time-frequency atoms that are chosen to best match its residues:

$$f = \sum_{n=0}^{m} \langle R^n f, \mathrm{g}_{\mathrm{I}_n} \rangle \mathrm{g}_{\mathrm{I}_n} + \mathrm{R}^{\mathrm{n}+1}\mathrm{f}. \qquad (3)$$

Although the procedure converges to $f(t)$, we have to stop it at some point. We can define a magnitude $\lambda(n)$ as a ratio of energy of residue $R_n f$, explained by g_{I_n}:

$$\lambda(n) = \frac{\langle R^n(f), g_{I_n} \rangle}{R^n(f)\|}. \qquad (4)$$

It converges to a constant value λ^e depending on the size of a signal. Asymptotic value λ^e corresponds to a situation in which there are no more structures coherent with a dictionary in the residuum. Recent research shows that residua converge to a chaotic attractor of a process called ''dictionary noise'' (1).

Energy conservation rule:

$$f^2 = \sum_{n=0}^{m-1} |< R^{nf}, g_{v_n} > |^2 + R^{mf\,2} \qquad (5)$$

allows us to conveniently visualize the energy density in time-frequency plane in a form of the Wigner distribution. Unlike the Wigner or Cohen class distributions, MP representation does not include interference terms (as shown in 7) and thus provides a clear picture in the time-frequency space.

RESULTS AND DISCUSSION

The properties of MP approach can be illustrated by a simulation study. In simulations, as well as in the EEG time series analysis, the window function in Eq. 1 was Gauss. Resulting g_I were Gabor functions (Gauss modulated by sinus), which gave best time-frequency localization. Fig. 1 shows a time-frequency map (*i.e.*, Wigner distribution of a signal containing different data structures).

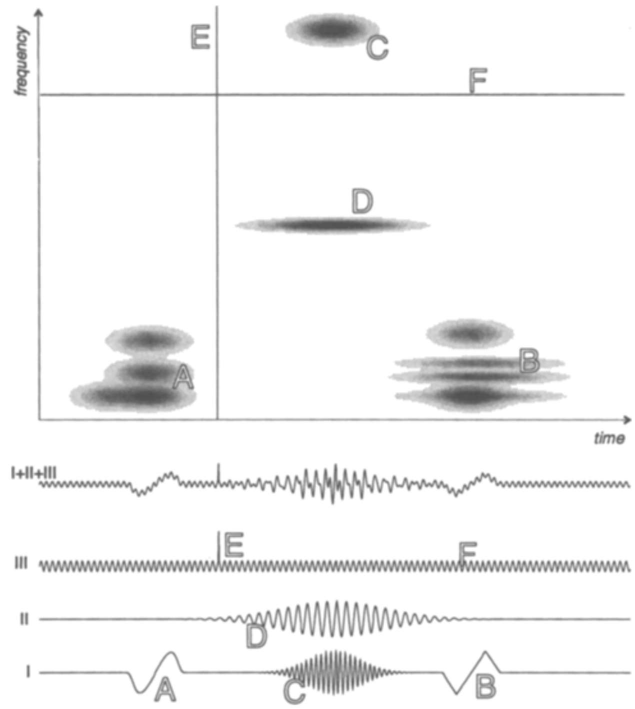

FIGURE 1. Time-frequency maps obtained by means of MP from the simulated signal shown directly below (summation of signals I, II, and III). (A–F) signal structures and corresponding atoms or groups of atoms obtained by means of MP. F = sinusoid; E = delta function; D and C = sinusoids modulated by Gauss. Horizontal axis = time in seconds; vertical axis = frequency in Hz.

Sinusoid, Dirac's delta, and Gabor functions, belonging to the chosen dictionary, are expressed by one atom each. Other structures are described by few atoms. Obviously, Gabor functions are especially suitable for the description of spindles. Spindles can be easily distinguished even if two of them occur at the same time (providing that their frequencies differ).

We have applied MP to the awake and sleep EEG. In Fig. 2, a time-frequency map of the awake EEG is pre-

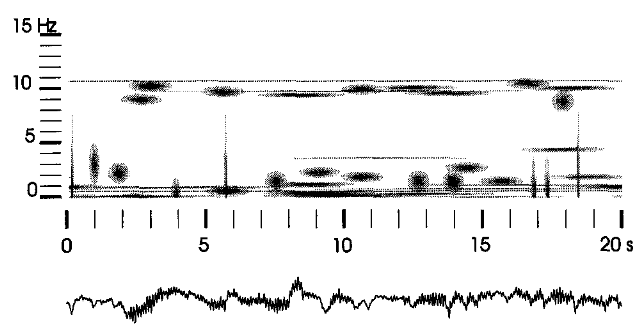

FIGURE 2. Time-frequency map for the awake EEG shown below. The intensities shown as shades of gray. The dynamic changes of signals in alpha band and low frequency components are clearly visible. Coordinates as in Fig. 1.

sented. One can see that the dynamics of signal features is reflected with great fidelity. The changes of activity in alpha band and low-frequency components are easy to follow.

Each atom is characterized quantitatively by the following parameters: time and frequency coordinates, energy, and scale s proportional to the spread in time (Eq. 1). The number of atoms taken into account or, in another words, the percentage of energy accounted for in the recursive procedure can be chosen.

In Fig. 3a, an example of a time-frequency map for EEG signal (sleep stage 2) is shown. It is easy to observe that several rhythmic components of frequency below 7 Hz are present. Sharp, spike-like structures of a signal are visible as vertical lines. Circular structure at the frequency of 12 Hz corresponds to the sleep spindle. In Fig. 3a, 50 waveforms corresponding to 94.25% of the energy of signal are shown. In Fig. 3 b and c, 100 atoms (97.7%) of energy and 200 atoms (99.32% of energy) for the same data segment are presented. In Fig. 3b, compared with Fig. 3a some higher frequency rhythmical components and more spindle-like atoms appear.

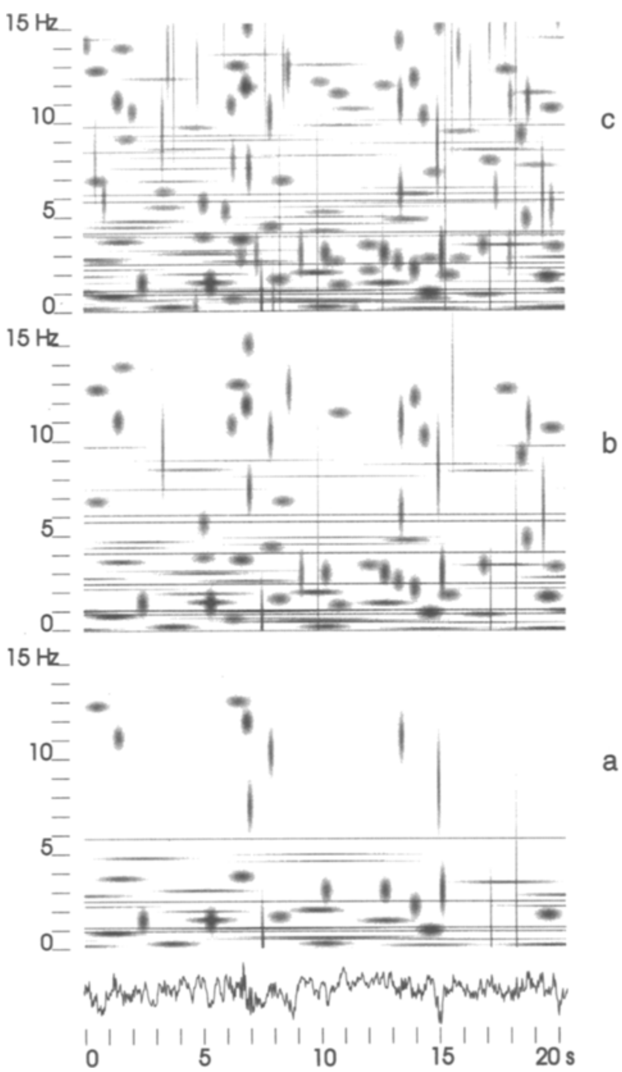

FIGURE 3. Time-frequency maps of the signal shown below; 2048 data points sampled at 102.4 Hz. Number of atoms shown: (a) 50, (b) 100, (c) 200. Coordinates as in Fig. 1.

In the next Panel (Fig. 3c), the picture becomes more complicated. While inspecting Figure 3 a–c, we have encountered the problem of how many atoms are sufficient to characterize the signal?

Where should the threshold be set in order to discriminate against a noise without information loss? The λ function (Eq. 4) can provide some help in this respect. It characterizes coherence of the signal with respect to the dictionary of functions used in approximation. In Fig. 4, a plot of λ value as a function of number of atoms (*i.e.*, algorithm's iterations) is shown. We can observe that for >100 atoms, decrease of λ becomes very slow, and for 150 atoms, $\lambda \approx 0.1$. If we denote by λ^e an average level of λ (Eq. 4) for Gabor dictionary of functions fitted to the white noise, then for $N = 2048$ we get $\lambda^e \approx 0.1$.

These considerations indicate that in order to obtain clear Wigner plot, for this particular time series (consisting of 2048 points) 100–150 atoms should be taken into account. Addition of more atoms increases mainly noise component. If the very weak components of the signal are not of particular interest, it is better not to use too many atoms when constructing of time-frequency representations.

A good example of the MP application to the EEG analysis is the study of sleep spindles. This phenomenon remained elusive to conventional EEG analysis, since spindle identification requires a simultaneous information on the time and frequency coordinates. The techniques used for automatic spindles detection include: phase

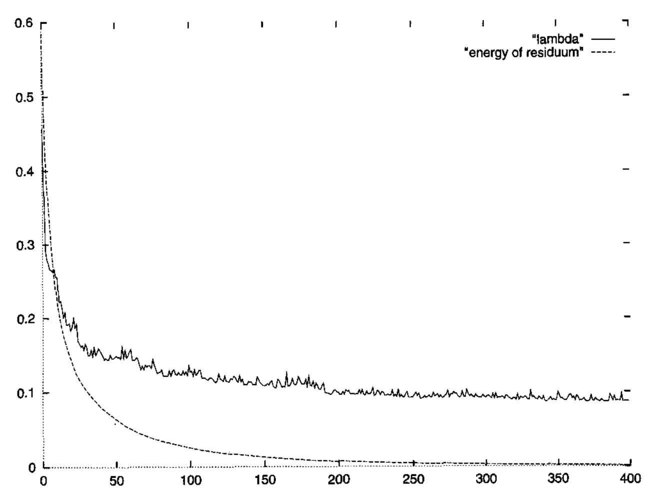

FIGURE 4. Solid line = λ as a function of number of atoms taken into account; broken line = the amount of energy not accounted for (total energy normalized to 1).

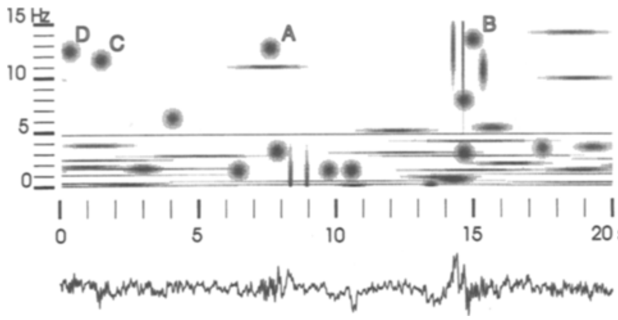

FIGURE 5. Time-frequency analysis obtained by means of MP for the signal shown below (sleep stage II, derivation C3). Spindles are marked A, B, C, and D. Coordinates as in Fig. 1.

locked loop (3,8), complex demodulation (6), autoregressive modeling (5), and matched filtering (4,5). The last method gave the most promising results. However, its limitations are due to the fact that the matched filter has fixed frequency and shape. The *a priori* choice of filter obviously biases the results. In Reference 5, five matched filters of frequencies differing by 1 Hz were applied. In MP, the number of waveforms describing data structures is practically infinite, therefore the accuracy of the method is superior to any known procedure.

In Fig. 5, four spindles are visible. Spindles A and B appearing in the signal at the same time as the other short-time structures are nevertheless clearly resolved. Low-intensity spindles C and D may be elusive to conventional methods of analysis, but their characteristics identifies them as spindles (Table 1).

Since spindles are described by only one atom each, it is possible to follow their evolution in time and space. Figure 6 shows the characteristics of one spindle at different derivations. The topographic representation of spindles, in terms of their parameters, makes possible to follow their propagation and ultimately might bring a considerable progress in understanding of the mechanisms of their generation. Well-defined characteristics of spindles offered by MP also make possible their automatic recognition based on the criteria concerning their frequency, time spread, and amplitude.

The EEG analysis by means of MP approach is by no means limited to spindles. Different data structures can be traced and their spatiotemporal characteristics followed.

TABLE 1. Parameters of spindles marked A, B, C, and D in Fig. 7.

	A	B	C	D
Position (s)	7.58	14.92	1.48	0.39
Frequency (Hz)	12.8	13.6	11.8	12.6
Amplitude (arbitrary units)	640	626	433	407
Scale s (points)	64	64	64	64

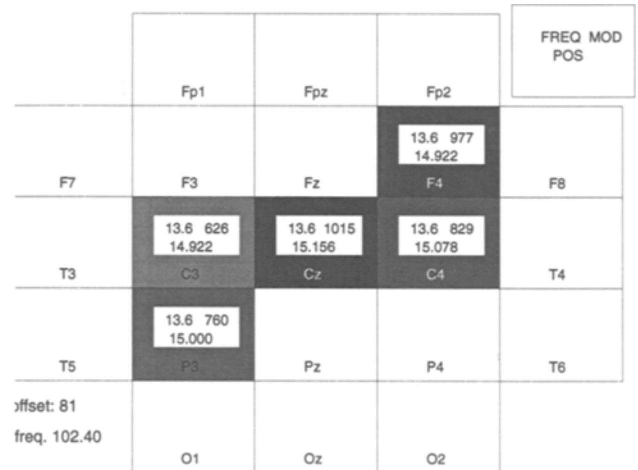

FIGURE 6. Topographical representation of the parameters of spindle B in Fig. 5 and Table 1, for other derivations. FREQ = frequency; POS = time coordinate (in seconds); MOD = intensity.

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