Covariant quantum field theory of tachyons is not physical II

Seminar “Exact Results in Quantum Physics and Gravitation”
Faculty of Physics, University of Warsaw
November 29 2024

Krzysztof Jodłowski

Institute for Basic Science
Center for Theoretical Physics of the Universe
Particle Theory and Cosmology Group
Daejeon, S. Korea

Based on
KJ, 2406.14225 (sent to PRD)

Outline

Motivation: Recent works on tachyons are an interesting attempt of connecting tachyon non-determinism to foundations of QM. Dragan, Ekert (New J.Phys. 22 (2020) 3, 033038)

Claims:
We show that the twin space [1] Paczos et al. (Phys.Rev.D 110 (2024) 1, 015006)
does not lead to a covariant quantum scalar field with negative m^2.

We also show that the Dhar-Sudurshan Feynman propagator leads to unitarity violation due to complex poles at $p^0 = \pm im$ Dhar, Sudurshan PhysRev.174.1808

Finally, we discuss LSZ formalism for tachyons in a model-independent way.
We show that one cannot prove the LSZ asymptotic condition just by replacing plane waves with wavepackets.

Instead of history of tachyons

https://inspirehep.net/files/64b806fa56cd9a8ca5ff719d39af1f29

2 of 4

To get the first paper on tachyons by George Sudarshen, his student V.K. Deshpande, and myself published – we had to resort to some diplomatic maneuvering. At a meeting of the American Physical Society in 1961, I spotted Walter Michels, the editor of the American Journal of Physics, chatting with the chairman of the Rochester Physics Department, Robert Marshak. I managed to join their conversation and found an opportunity to ask Bob Marshak to assure professor Michels that George Sudarshan and Oleksa Bilaniuk were respectable, *bona fide* physicists. Moreover, Bob Marshak asked Michels to examine our prospective submission on "Metarelativity" thoroughly and seriously, and not throw it automatically into the waste basket, which on first sight might appear the normal thing to do. As a result, our publication appeared in record time.

Our paper rapidly became widely quoted and a number of intrepid experimentalists started looking for faster-than-light particles, particularly after Gerald Feinberg published, in 1967, an attempt at quantization of our "metaparticles", which he aptly renamed tachyons (from Greek tacis=swift).

But the acceptance was not universal. A challenge came, of all places, from the "dean" of science-fiction writers, Isaac Asimov, who published an article [12] on the subject, entitled "*Impossible, that's all*." Improbably, who came to our defense but another renowned science fiction writer, Arthur C. Clarke, with the article [13] "*Possible, that's all*."

In the end, Isaac Asimov wrote a contrite article [14] "*The Luxon Wall*", in which he admitted that he had been "left flat-footed by the advance of physics." He even called me and as a result he was invited to give a talk at Swarthmore. He was a sincere and deep-thinking person and we parted as best of friends.

On a more serious note, over 600 publication on tachyons, experimental and theoretical, appeared in the first 18 years following our 1962 "Metarelativity" paper. Please see Ref. 15 for a full listing. A number of these dealt with the resolution of the causality problem, which remained vexing for a while. In the end, it became clear that tachyons do not lead to causality violation either in the framework of special or of general relativity [16,17].

Meta Relativity #9

O.M.P. Bilaniuk, V.K. Deshpande, E.C.G. Sudarshan (Oct, 1962)

Published in: *Am.J.Phys.* 30 (1962) 718-723

DOI cite claim reference search 242 citations

quest.ph.utexas.edu/sudarshan_tachyons.html

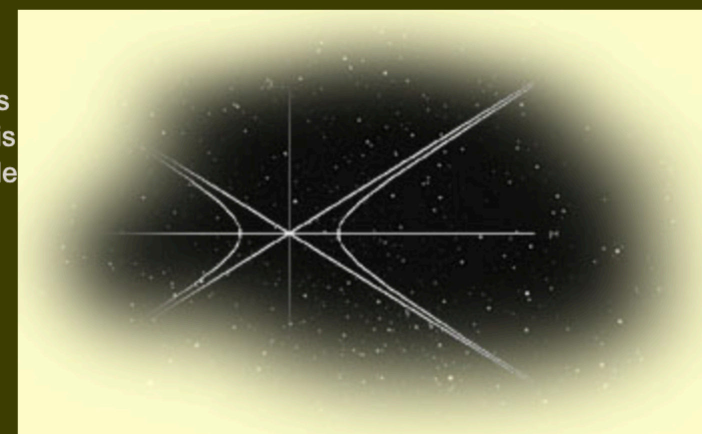
Sudarshan

SEVEN SCIENCE QUESTS

VI Theory of Tachyons

Faster than Light Propagation

It had long been assumed that the theory of relativity has established the speed of light as the highest speed allowed. If this were so, we would never be able to study the contemporary physical conditions beyond our galaxy; and in practice, beyond the solar system. It would be, therefore, desirable to have particles that travel beyond the speed of light.



In the summer of 1958 when Sudarshan was at the University of Rochester, someone asked him about what happens to energy and momentum when a particle travels faster than light. Sudarshan saw that energy and momentum could be made real by taking rest mass to be imaginary for such particles. The second difficulty of the apparent traveling "backward in time" of such a particle was solved by the interchange of the emission and absorption of the particle. Along with a graduate student, V. K. Deshpande, Sudarshan wrote a short paper and sent it to Physical Review Letters. It came back from a referee who rejected it, saying it was incorrect. Sudarshan requested a second referee who said that the results of the paper were correct, but it was all well known! A third referee stated that he had "read both the previous referee reports and he agreed with both of them"! About two years later, after Sudarshan joined the University of Rochester faculty, his colleague O.M.P. Bilaniuk offered to rewrite the paper and get it published. He did it and they published it in American Journal of Physics. It attracted a lot of attention and several letters to Physics Today.

To make a quantum theory one had to quantize a scalar field with imaginary mass. Dhar and Sudarshan completed this in the spring of 1968. (By this time Feinberg at Columbia published a paper with all Sudarshan's results, without acknowledgement to him). Feinberg's work contained essential inconsistencies, but it supplied the name "tachyon" for these particles. Arons and Sudarshan corrected the mistakes in Feinberg's work and carried out the correct quantization of tachyons.

SEVEN SCIENCE QUESTS SYMPOSIUM NOV. 6-7, 2006

- Curriculum Vitae
- **Seven Science Quests**
  - V-A: Universal Theory of Weak Interaction
  - Symmetry
  - Spin Statistics
  - Quantum Optical Coherence: Sudarshan Representation
  - Quantum Tachyons Effect
  - Theory of Tachyons
  - Quantum Mechanics of Open Systems
- Friends Committees
- Home
- Coming soon:
  - Awards
  - Testimonials
  - Articles about GS

Feinberg, Phys. Rev. 159, 1089 (1967), Sudurshan et al. (PhysRev.173.1622, PhysRev.174.1808, Am.J.Phys. 30 (1962))

Instead of history of tachyons

We look into the recent work [1] studying QFT of tachyons, related to the Dragan-Ekert superluminal observers program. Paczos et al. have already pointed out some mistakes in previous ($\sim$1960s) attempts of QFT of tachyons and proposed to extend the Hilbert space to twin space, but we find that it is classical theory.

We use QFT as a framework → our results are not directly relevant to Dragan, Ekert (New J.Phys. 22 (2020) 3, 033038) Dragan et al. (Class.Quant.Grav. 40 (2023) 2, 025013)

Covariant quantum field theory of tachyons

Jerzy Paczos,[1] Kacper Dębski,[2] Szymon Cedrowski,[2] Szymon Charzyński,[3] Krzysztof Turzyński,[2] Artur Ekert,[4,5,6] and Andrzej Dragan[2,4,*]

[1]*Department of Physics, Stockholm University, SE-106 91 Stockholm, Sweden*
[2]*Institute of Theoretical Physics, University of Warsaw, Pasteura 5, 02-093 Warsaw, Poland*
[3]*Department of Mathematical Methods in Physics, University of Warsaw, Pasteura 5, 02-093 Warsaw, Poland*
[4]*Centre for Quantum Technologies, National University of Singapore, 3 Science Drive 2, 117543 Singapore, Singapore*
[5]*Mathematical Institute, University of Oxford, Woodstock Road, Oxford OX2 6GG, United Kingdom*
[6]*Okinawa Institute of Science and Technology, Onna, Okinawa 904-0495, Japan*

(Received 11 August 2023; accepted 23 May 2024; published 9 July 2024)

Three major misconceptions concerning quantized tachyon fields, the energy spectrum unbounded from below, the frame-dependent and unstable vacuum state, and the noncovariant commutation rules, are shown to be a result of misrepresenting the Lorentz group in a too small Hilbert space. By doubling this space we establish an explicitly covariant framework that allows for the proper quantization of the tachyon fields eliminating all of these issues. Our scheme that is derived to maintain the relativistic covariance also singles out the two-state formalism developed by Aharonov *et al.* [Phys. Rev. **134**, B1410 (1964)] as a preferred interpretation of the quantum theory.

DOI: 10.1103/PhysRevD.110.015006

[1] Paczos et al. (Phys.Rev.D 110 (2024) 1, 015006)

Are tachyons just a curiosity?

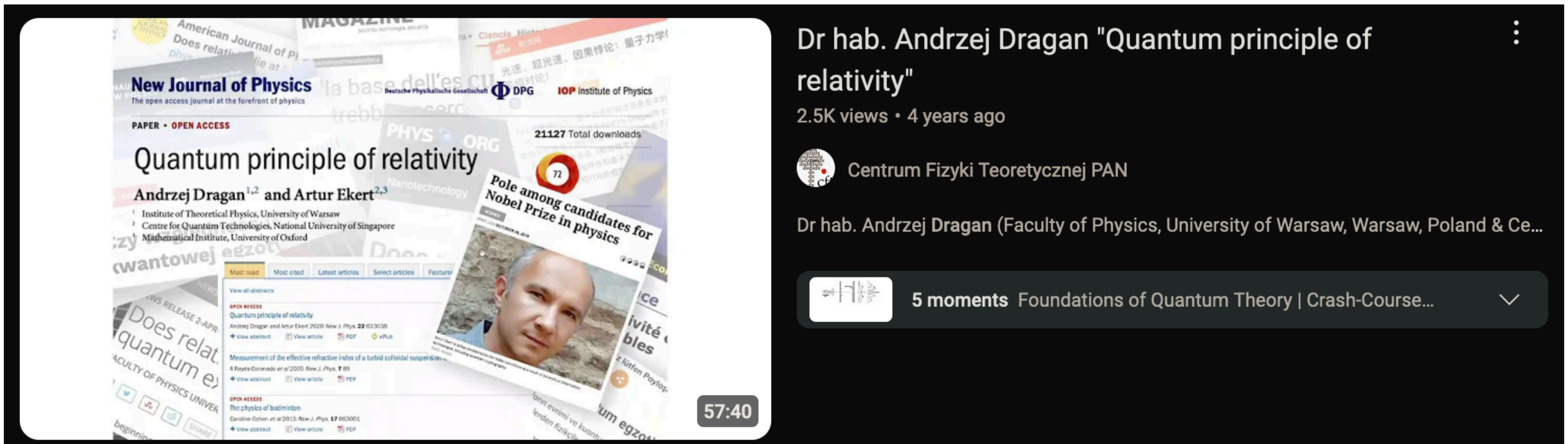
<u>Two</u> classes of transformations that preserve the speed of light

Dragan, Ekert (New J.Phys. 22 (2020) 3, 033038)

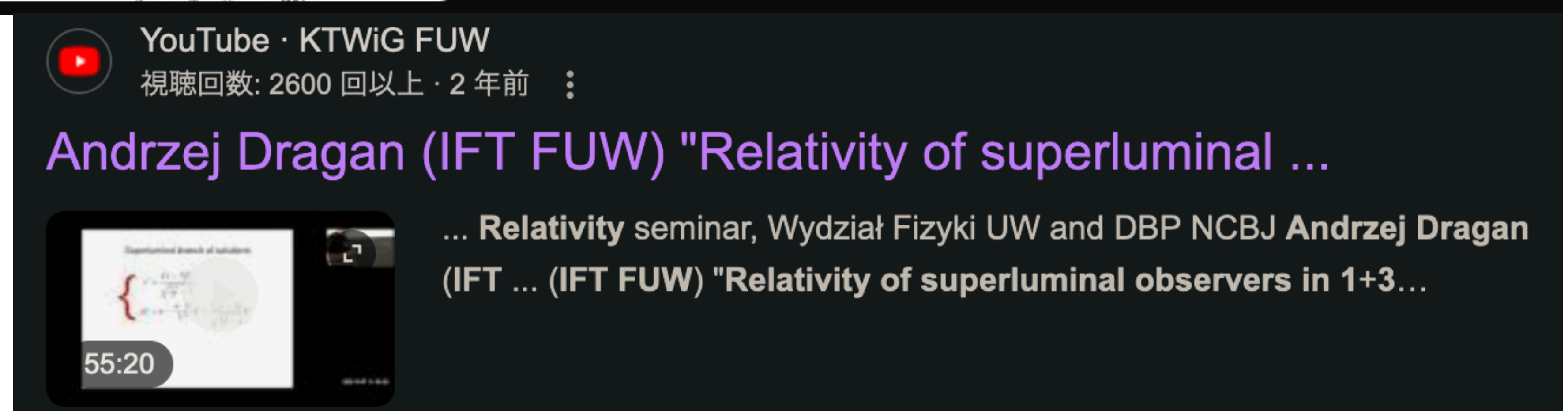
$$x' = A(V)\,x + B(V)\,t, \qquad x = A(-V)\,x' + B(-V)\,t',$$

$$x' = 0 \qquad x = Vt \qquad \frac{B(V)}{A(V)} = -V$$

$$x' = A(V)(x - Vt), \qquad t' = A(V)\left(t - \frac{A(V)A(-V) - 1}{V^2 A(V)A(-V)}\,Vx\right).$$



Please see the following seminars for <u>comprehensive</u> introduction to physics of superluminal observers.



Are tachyons just a curiosity?

Two classes of (linear) transformations that preserve the speed of light

Dragan, Ekert (New J.Phys. 22 (2020) 3, 033038)

$$x' = A(V)\,x + B(V)\,t,\qquad x = A(-V)\,x' + B(-V)\,t',$$

$$x' = 0 \qquad x = Vt \qquad \frac{B(V)}{A(V)} = -V$$

Use symmetry of flat space isotropy, homogeneity etc.

$$x' = A(V)(x - Vt),\qquad t' = A(V)\left(t - \frac{A(V)A(-V)-1}{V^2 A(V)A(-V)}Vx\right).$$

$$A(-V) = A(V) \qquad x' = \frac{x - Vt}{\sqrt{1 - V^2/c^2}},\qquad t' = \frac{t - Vx/c^2}{\sqrt{1 - V^2/c^2}}.$$

Ignatowsky 1910

$$A(-V) = -A(V) \qquad x' = \pm\frac{V}{|V|}\frac{x - Vt}{\sqrt{V^2/c^2 - 1}},\qquad t' = \pm\frac{V}{|V|}\frac{t - Vx/c^2}{\sqrt{V^2/c^2 - 1}}.$$

Parker 1969

- Full (AKA extended) principle of relativity
  - Physics: causality paradoxes Tolman 1917
  - Math: $v > c$ transformation group is possible in 1+1 spacetime
    - in 1+3 the group has to include direction-dependent time dilations

Marchildon, Antippa, Everett Phys. Rev. D 27, 1740

Are tachyons just a curiosity?

- Extended principle of relativity
  - causality paradoxes Tolman 1917
  - $v > c$ transformation group possible only in 1+1 spacetime; in 1+3 there are also direction-dependent time dilations

Marchildon, Antippa, Everett Phys. Rev. D 27, 1740

Special relativity only works with $v < c$ (?!)

Dragan, Ekert (New J.Phys. 22 (2020) 3, 033038)

- Tolman and Marchildon et al. arguments evaded because

i) causality violation has a non-deterministic character → superluminal signaling is impossible (?)

ii) switching from 1+3 to 3+1 spacetime → superluminal observers are distinguishable

$$c^2 \mathrm{d}t^2 - \mathrm{d}\boldsymbol{r} \cdot \mathrm{d}\boldsymbol{r} = \mathrm{d}x'^2 - c^2 \mathrm{d}\boldsymbol{t}' \cdot \mathrm{d}\boldsymbol{t}'.$$

Dragan et al. (Class.Quant.Grav. 40 (2023) 2, 025013)

$$x' = \frac{Vt - \frac{\boldsymbol{V}\cdot\boldsymbol{r}}{V}}{\sqrt{V^2/c^2 - 1}},$$

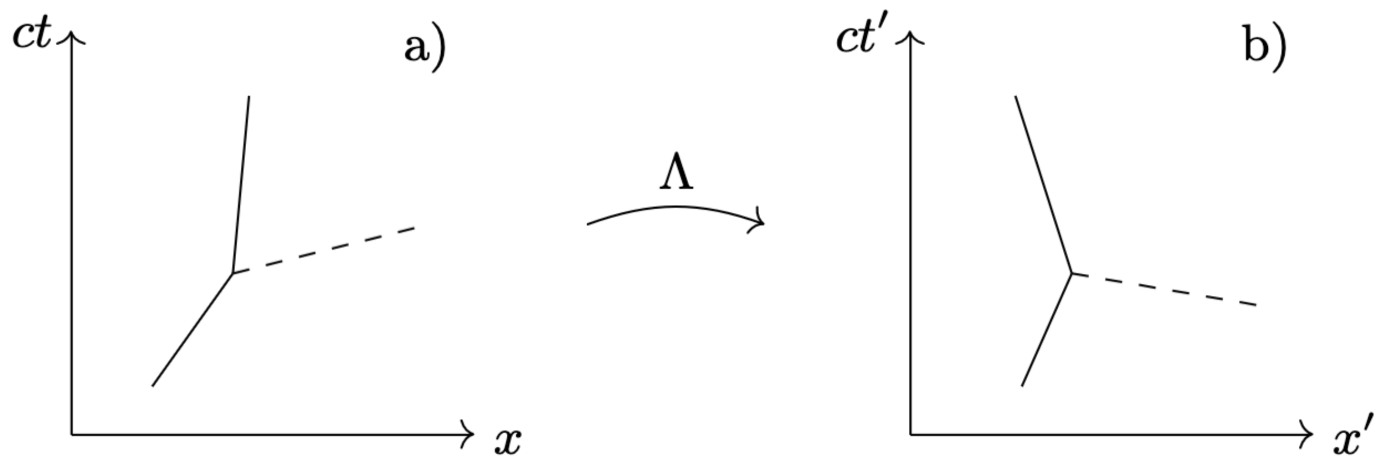
$$c\boldsymbol{t}' = \boldsymbol{r} - \frac{\boldsymbol{V}\cdot\boldsymbol{r}}{V^2}\boldsymbol{V} + \frac{\frac{\boldsymbol{V}\cdot\boldsymbol{r}}{Vc} - \frac{ct}{V}}{\sqrt{V^2/c^2 - 1}}\boldsymbol{V}.$$

$$E \equiv \frac{\sigma m c^2}{\sqrt{\frac{v^2}{c^2} - 1}},$$

$$\boldsymbol{p} \equiv \frac{\sigma m \boldsymbol{v}}{\sqrt{\frac{v^2}{c^2} - 1}},$$

$$\sigma' = \sigma \,\mathrm{sgn}\left(1 - \frac{\boldsymbol{v}\cdot\boldsymbol{V}}{c^2}\right),$$

Are tachyons just a curiosity?



Sudurshan et al. PhysRev.173.1622, Paczos et al. (Phys.Rev.D 110 (2024) 1, 015006)

- Reinterpretation principle proper

$$E \equiv \frac{\sigma m c^2}{\sqrt{\frac{v^2}{c^2} - 1}}, \qquad \boldsymbol{p} \equiv \frac{\sigma m \boldsymbol{v}}{\sqrt{\frac{v^2}{c^2} - 1}}, \qquad \sigma' = \sigma \operatorname{sgn}\left(1 - \frac{\boldsymbol{v} \cdot \boldsymbol{V}}{c^2}\right),$$

Dragan et al. (Class.Quant.Grav. 40 (2023) 2, 025013)

Dragan, Ekert (New J.Phys. 22 (2020) 3, 033038)

- **Galilean principle with superluminal observers → QM (?!)**
  - Quantum Principle of Relativity

    "*existence of a local and deterministic mode of description of any process should not depend on the choice of the inertial reference frame*"

    superluminal observers *require* wave-like description of Nature using complex numbers → (part of) QM is recovered

We use QFT as a framework → our results are not directly relevant to

Dragan, Ekert (New J.Phys. 22 (2020) 3, 033038)
Dragan et al. (Class.Quant.Grav. 40 (2023) 2, 025013)

Are tachyons just a curiosity?

Comment on ‘Quantum principle of relativity’

Horodecki New J.Phys. 25 (2023) 12, 128001

Ryszard Horodecki

International Centre for Theory of Quantum Technologies, University of Gdańsk, Jana Bażyńskiego 1A, 80-308 Gdańsk, Poland

National Quantum Information Center of Gdańsk, Wita Stwosza 53, 80-308 Gdańsk, Poland

E-mail: ryszard.horodecki@ug.edu.pl

Keywords: principle of relativity, quantum theory, special relativity

Abstract

Dragan and Ekert in the paper (2020 *New. J. Phys.* **22** 033038) presented ‘quantum principle of relativity’ (QPR) based on Galilean principle of relativity, which involves both superluminal G_S and subluminal G_s families of observers and argue that then they are considered on the same footing it ‘implies the emergence of non-deterministic dynamics, together with complex probability amplitudes and multiple trajectories.’ Here we discuss QPR in the context of Heisenberg’s classification of the fundamental physical theoretical models under the role universal constants of nature: Planck’s constant h and speed of light c. We point out that both the superluminal and subluminal branches are separable in the sense that there is no mathematical coherent formalism that connect both branches. This, in particular, implies that the QPR is incomplete.

Are tachyons just a curiosity?

- Extended principle of relativity
    - causality paradoxes Tolman 1917
    - $v > c$ transformation group possible only in 1+1 spacetime; in 1+3 there are also direction-dependent time dilations

Marchildon, Antippa, Everett Phys. Rev. D 27, 1740

Special relativity only works with $v < c$ (?!)

Dragan, Ekert (New J.Phys. 22 (2020) 3, 033038)

- **Galilean principle with superluminal observers → QM (?!)**
    - comments questioning the “Quantum Principle of Relativity”

QPR is wrong Del Santo, Horvat New J.Phys. 24 (2022) 12, 128001

QPR is not needed Grudka, Wojcik New J.Phys. 24 (2022) 098001

QPR is incomplete Horodecki New J.Phys. 25 (2023) 12, 128001

It seems the QPR cannot be considered as a full-fledged physical principle.
Moreover, “Superluminal observers do not explain quantum superpositions” Grudka et al. Phys. Lett. A 487, 129127 (2023)

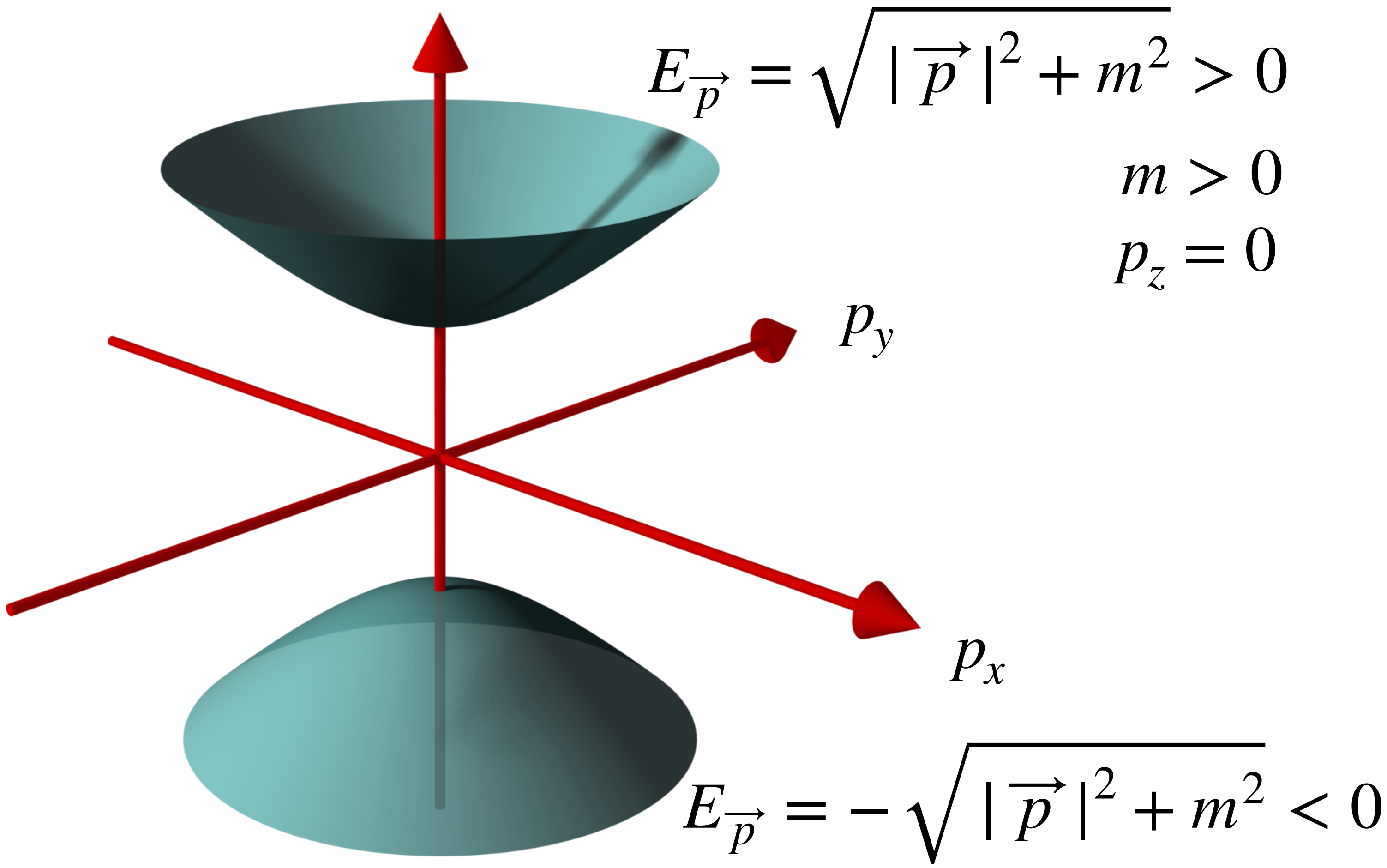
Towards tachyon QFT

- Classical Field Theory with superluminal observers
  - exchange time and space dimensions 1+3 → 3+1 (ala black hole behind the event horizon) Grudka, Wojcik New J.Phys. 24 (2022) 098001
  - field theory as a direct consequence of extended special relativity Dragan et al. (Class.Quant.Grav. 40 (2023) 2, 025013)
- QFT of tachyons - previous attempts have holes Paczos et al. (Phys.Rev.D 110 (2024) 1, 015006)
  - both bose/fermi commutation relations were tried; none leads to covariant description → twin space
  - unstable and frame-dependent vacuum (energy spectrum unbounded from below)
  - lack of microcausality - claims it does not matter since Feinberg, Phys. Rev. 159, 1089 (1967) signal sending is impossible due to "basis incompleteness"
  - Sudurshan et al. (Am.J.Phys. 30 (1962), Phys Rev. 173.1622, Phys. Rev. 174.1808)
  - Tanaka Prog. Theor. Phys. 24 (1960) 171
  - Feinberg, Phys. Rev. 159, 1089 (1967)

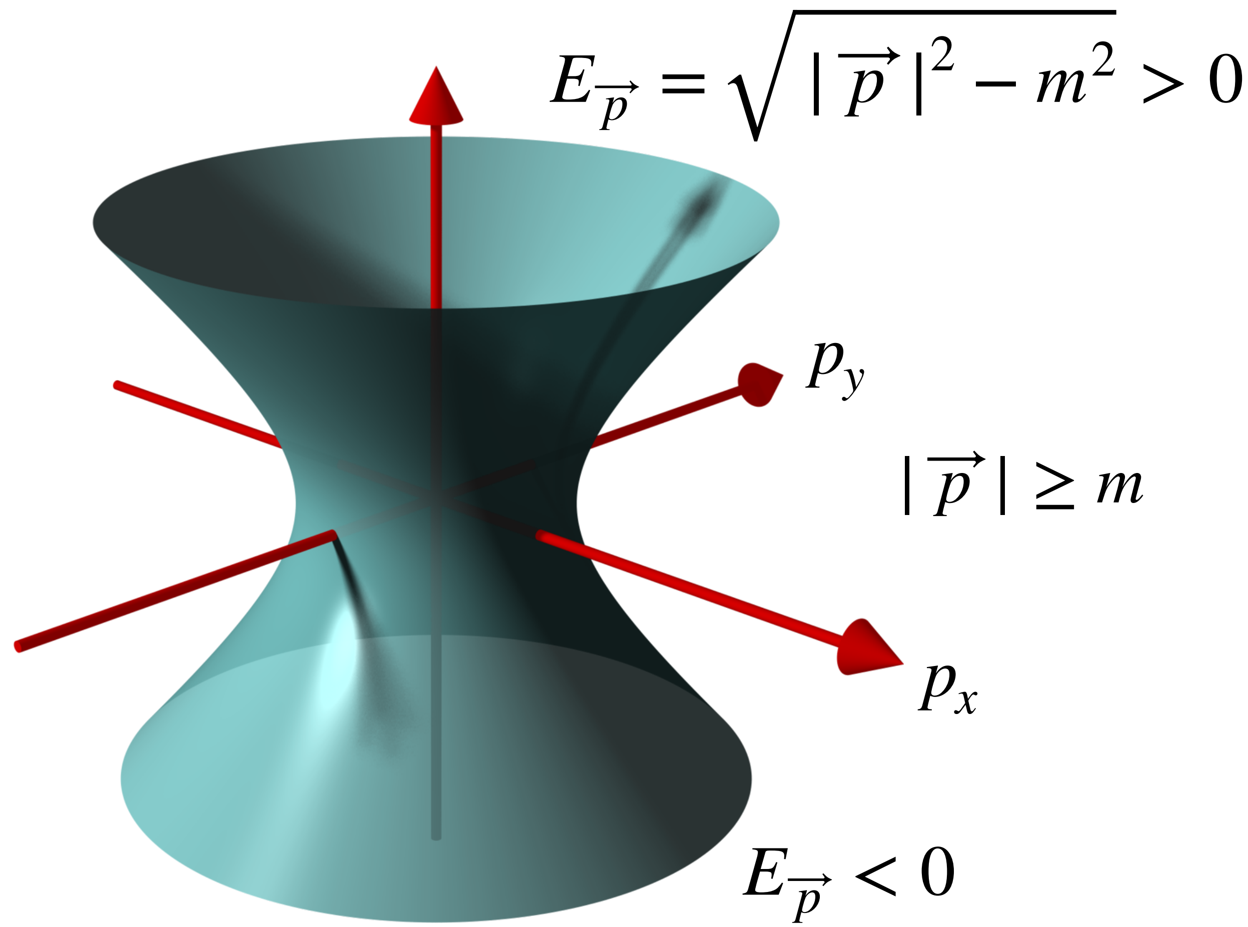
  - employ two-state formalism of Aharonov et al.
    - Fock space contains *both* past and future
    - gives covariant description and LI vacuum

      Paczos et al. (Phys.Rev.D 110 (2024) 1, 015006)

Some well-known facts: the mass shell: $p^2 \mp m^2 = 0$



a) Two (disconnected) sheet hyperboloid



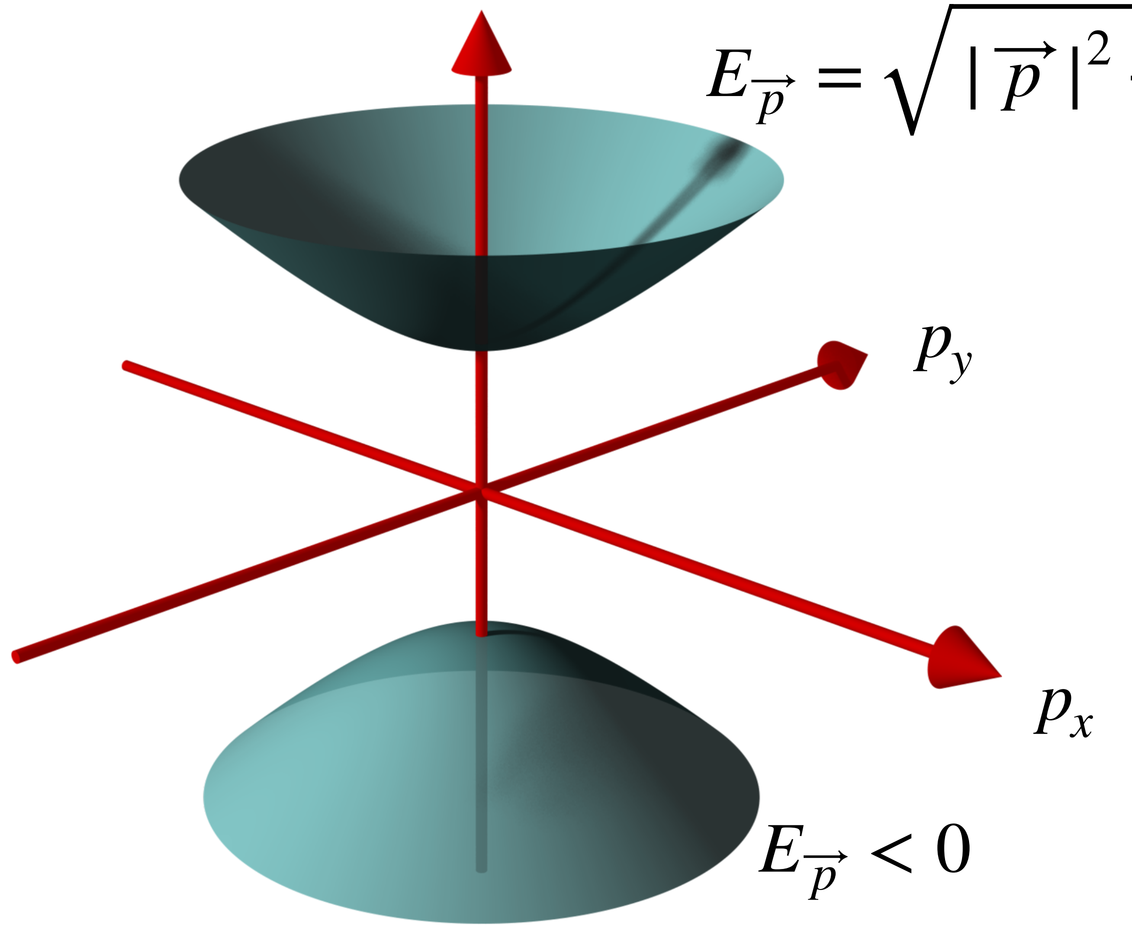
b) One sheet hyperboloid

For subliminal observers, boosts preserve each sheet, while for superluminal observers, there is only one sheet and there are boosts that change the sign of time/energy of a space-like 4-vector.

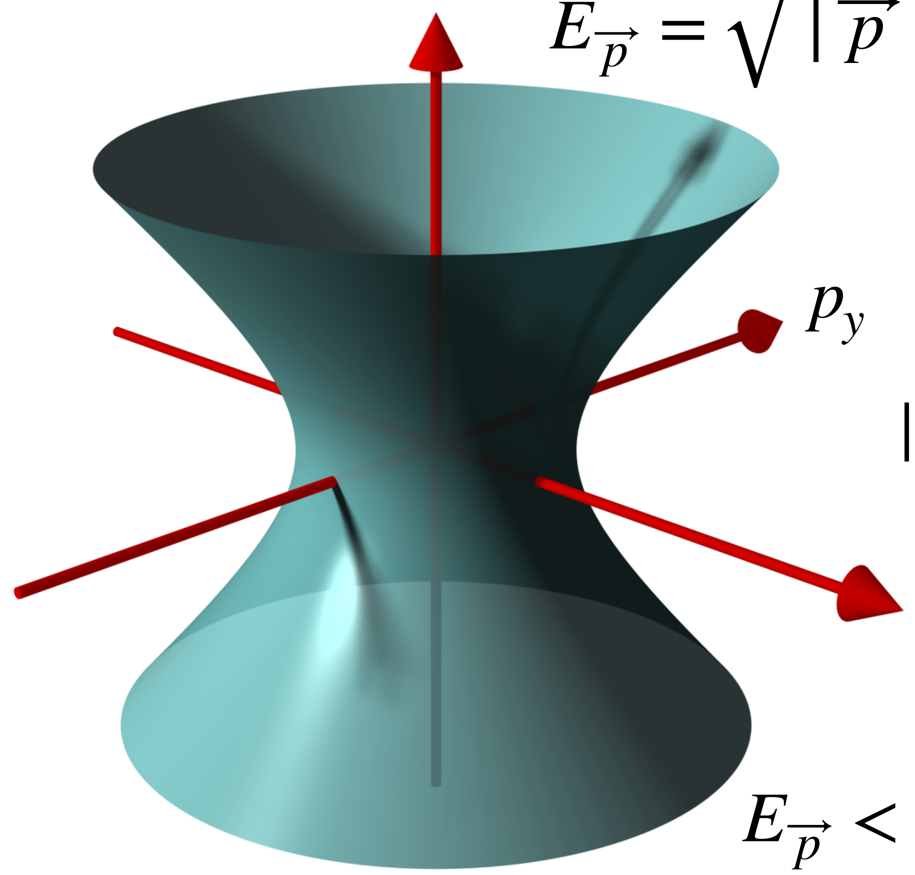
All proposals tachyon QFTs do not overcome this fact if the CCR of fields are to be satisfied.

Moreover, note that we have excluded $|\vec{p}| \leq m$ from b), since they lead to complex energies, and then $\phi \sim e^{-ikx} \sim e^{-\Gamma t}$, which is non-normalizable at $t \to -\infty$.

Mass shell: $p^2 \mp m^2 = 0$



a) Two (disconnected) sheet hyperboloid



b) One sheet hyperboloid

$$
\begin{aligned}
dp'_x &= dp_x \\
dp'_y &= dp_y \\
dp'_z &= \gamma(dp_z - \beta dE) = \gamma dp_z(1 - \beta p_z/E) \\
&= dp_z E'/E
\end{aligned}
$$

We used $dE/dp_z = p_z/E$ and $E' = \gamma(E - \beta p_z)$.

Therefore, $d^3p'/E' = d^3p/E$ is LIM for a).

$$\int_{p^0 \geq 0} \frac{\mathrm{d}^4 p}{(2\pi\hbar)^4} \Big((2\pi\hbar)\delta(p^2 - m^2c^2) \Big) f(p)$$

We use LIM to normalize single-particle states to 1: $\langle \vec{p} \,|\, \vec{q} \rangle = (2\pi)^3\, 2E_{\vec{p}}\, \delta^3(\vec{p} - \vec{q})$.

Twin space

$$\mathcal{F} \otimes \mathcal{F}^{\star} \quad \text{where} \quad \mathcal{F} \equiv \bigoplus_{n=0}^{\infty} S\left(\mathcal{H}^{\otimes n}\right)$$

[1] Paczos et al. (Phys.Rev.D 110 (2024) 1, 015006)

Twin space

5.6 Tensor Product of Fock Spaces

In this subsection we describe the so-called *exponential law for Fock spaces.*

Let $\mathcal{Z}_1$ and $\mathcal{Z}_2$ be Hilbert spaces. We introduce the identification

$$U : \Gamma^{\rm fin}_{\rm s/a}(\mathcal{Z}_1) \otimes \Gamma^{\rm fin}_{\rm s/a}(\mathcal{Z}_2) \to \Gamma^{\rm fin}_{\rm s/a}(\mathcal{Z}_1 \oplus \mathcal{Z}_2)$$

as follows. Let $\Psi_1 \in \Gamma^n_{\rm s/a}(\mathcal{Z}_1)$, $\Psi_2 \in \Gamma^m_{\rm s/a}(\mathcal{Z}_2)$. Let j_i be the imbedding of $\mathcal{Z}_i$ in $\mathcal{Z}_1 \oplus \mathcal{Z}_2$. Then

$$U(\Psi_1 \otimes \Psi_2) := \sqrt{\tfrac{(n+m)!}{n!m!}}(\Gamma(j_1)\Psi_1) \otimes_{\rm s/a} (\Gamma(j_2)\Psi_2) \ . \qquad (36)$$

Theorem 16. 1) $U(\Omega_1 \otimes \Omega_2) = \Omega$.
2) U *extends to a unitary operator* $\Gamma_{\rm s/a}(\mathcal{Z}_1) \otimes \Gamma_{\rm s/a}(\mathcal{Z}_2) \to \Gamma_{\rm s/a}(\mathcal{Z}_1 \oplus \mathcal{Z}_2)$.
3) *If* $h_i \in B(\mathcal{Z}_i)$, *then*

$$U\big(\mathrm{d}\Gamma(h_1) \otimes 1 + 1 \otimes \mathrm{d}\Gamma(h_2)\big) = \mathrm{d}\Gamma(h_1 \oplus h_2)U.$$

4) *If* $p_i \in B(\mathcal{Z}_i)$, *then*

$$U\left(\Gamma(p_1) \otimes \Gamma(p_2)\right) = \Gamma(p_1 \oplus p_2)U.$$

Dereziński Lect.Notes Phys. 695:63-143, 2006

$$\mathcal{F} \otimes \mathcal{F}^\star \text{ where } \mathcal{F} \equiv \bigoplus_{n=0}^{\infty} S\left(\mathcal{H}^{\otimes n}\right)$$

Paczos et al. (Phys.Rev.D 110 (2024) 1, 015006)

Using the identification above - the twin space is <u>explicitly</u> a proper Fock space. What about operators?

Twin space

Time-symmetric formulation of QM (*two-state formalism*) Aharanov, Bergmann, and Lebowitz 1964

Invoked to have a <u>covariant</u> tachyon quantum field $\hat{\mathbf{\Phi}}$ Paczos et al. (Phys.Rev.D 110 (2024) 1, 015006)

Let us first consider $\hat{\boldsymbol{\phi}}$ acting in the regular Fock space:

$$\hat{\phi}(t,\boldsymbol{r}) \equiv \int_{|\boldsymbol{k}|>m} \mathrm{d}^3\boldsymbol{k}\, \left(u_{\boldsymbol{k}}(t,\boldsymbol{r})\,\hat{a}_{\boldsymbol{k}} + u^*_{\boldsymbol{k}}(t,\boldsymbol{r})\,\hat{a}^\dagger_{\boldsymbol{k}}\right), \qquad [\hat{a}_{\boldsymbol{k}},\hat{a}^\dagger_{\boldsymbol{l}}] = 2\omega_k(2\pi)^3\delta^{(3)}(\boldsymbol{k}-\boldsymbol{l}).$$

$$\hat{\phi}(k) = (2\pi)\,\theta(|\vec{k}|-m)\,e^{-ikx}\,\delta(k^2+m^2)\,\left(\theta(k^0)\hat{a}_{\vec{k}} + \theta(-k^0)\hat{a}^\dagger_{-\vec{k}}\right).$$

$$\hat{\phi}(x) = \int \frac{d^3\vec{k}}{(2\pi)^3}\,\theta(|\vec{k}|-m)\,\left(u_{\vec{k}}(x)\,\hat{a}_{\vec{k}} + u^*_{\vec{k}}(x)\,\hat{a}^\dagger_{\vec{k}}\right) = \int \frac{d^3\vec{k}}{(2\pi)^3}\,\theta(|\vec{k}|-m)\,\left(\frac{e^{-i\omega_{\vec{k}}x^0+i\vec{k}\cdot\vec{x}}}{2\omega_{\vec{k}}}\hat{a}_{\vec{k}} + \frac{e^{i\omega_{\vec{k}}x^0+i\vec{k}\cdot\vec{x}}}{2\omega_{\vec{k}}}\hat{a}^\dagger_{-\vec{k}}\right)$$

$$= \int \frac{d^3\vec{k}}{(2\pi)^3}\int dk^0\,\theta(|\vec{k}|-m)\,e^{-ikx}\,\frac{\delta(k^0-\omega_{\vec{k}})+\delta(k^0+\omega_{\vec{k}})}{|2k^0|}\,\left(\theta(k^0)\hat{a}_{\vec{k}} + \theta(-k^0)\hat{a}^\dagger_{-\vec{k}}\right)$$

$$= \int \frac{d^4k}{(2\pi)^4}(2\pi)\,\theta(|\vec{k}|-m)\,e^{-ikx}\,\delta(k^2+m^2)\,\left(\theta(k^0)\hat{a}_{\vec{k}} + \theta(-k^0)\hat{a}^\dagger_{-\vec{k}}\right).$$

$$\omega_{\vec{k}} = \sqrt{|\vec{k}|^2-m^2}\,, \qquad \delta(k^2+m^2), = \frac{\delta(k^0-\omega_{\vec{k}})+\delta(k^0+\omega_{\vec{k}})}{|2k^0|}, \qquad \vec{k}\to-\vec{k}$$

Twin space

Time-symmetric formulation of QM (*two-state formalism*) Aharanov, Bergmann, and Lebowitz 1964

Invoked to have a <u>covariant</u> tachyon quantum field $\hat{\Phi}$ Paczos et al. (Phys.Rev.D 110 (2024) 1, 015006)

Let us first consider $\hat{\phi}$ acting in the regular Fock space:

$$\hat{\phi}(t,\boldsymbol{r}) \equiv \int_{|\boldsymbol{k}|>m} \mathrm{d}^3\boldsymbol{k} \left(u_{\boldsymbol{k}}(t,\boldsymbol{r})\,\hat{a}_{\boldsymbol{k}} + u^*_{\boldsymbol{k}}(t,\boldsymbol{r})\,\hat{a}^\dagger_{\boldsymbol{k}}\right) \qquad [\hat{a}_{\boldsymbol{k}}, \hat{a}^\dagger_{\boldsymbol{l}}] = 2\omega_k(2\pi)^3\delta^{(3)}(\boldsymbol{k}-\boldsymbol{l}).$$

$$\hat{\phi}(k) = (2\pi)\,\theta(|\vec{k}|-m)\,e^{-ikx}\,\delta(k^2+m^2)\,\left(\theta(k^0)\hat{a}_{\vec{k}} + \theta(-k^0)\hat{a}^\dagger_{-\vec{k}}\right).$$

Demanding $\hat{\Phi}(\Lambda^{-1}x) = U(\Lambda)^{-1}\,\hat{\Phi}(x)\,U(\Lambda)$, $\det\Lambda = 1$ and $\Lambda^0_0 > 0$, requires

$$\theta(|\vec{l}|-m)\left(\theta(l^0)\,\hat{a}_{\vec{l}} + \theta(-l^0)\,\hat{a}^\dagger_{-\vec{l}}\right) = \theta(|\vec{k}|-m)\left(U(\Lambda)\,\hat{a}_{\vec{k}}\,U(\Lambda)^{-1} + U(\Lambda)\,\hat{a}^\dagger_{-\vec{k}}\,U(\Lambda)^{-1}\right),$$

For time-like k, $\mathrm{sgn}(k^0)$ is LI, so $\hat{\phi}$ is LI: $\hat{a}_{\vec{l}} = U(\Lambda)\,\hat{a}_{\vec{k}}\,U(\Lambda)^{-1}$, and $\hat{a}^\dagger_{\vec{l}} = U(\Lambda)\,\hat{a}^\dagger_{\vec{k}}\,U(\Lambda)^{-1}$.

For space-like k, there are boosts that flip $\mathrm{sgn}(k^0)$, so $\hat{a}_{\vec{l}} = U(\Lambda)\,\hat{a}^\dagger_{-\vec{k}}\,U(\Lambda)^{-1}$ and $\hat{a}^\dagger_{\vec{l}} = U(\Lambda)\,\hat{a}_{-\vec{k}}\,U(\Lambda)^{-1}$,

which changes a into $a^\dagger$ and hence commutator changes sign.

Paczos et al. (Phys.Rev.D 110 (2024) 1, 015006) noticed that neither Feinberg, Phys. Rev. 159, 1089 (1967) nor Dhar, Sudurshan PhysRev.174.1808 solve this problem.

Twin space

QM CCR: $[x, p] = i$

$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}(a^\dagger + a)$$
$$\hat{p} = i\sqrt{\frac{\hbar m\omega}{2}}(a^\dagger - a)\ .$$

$$a = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} + \frac{i}{m\omega}\hat{p}\right)$$
$$a^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} - \frac{i}{m\omega}\hat{p}\right)$$

QM CCR: $[a, a^\dagger] = 1$

QFT CCR:

$[\Phi(0, \vec{x}), \partial_t \Phi(0, \vec{y})] = i\delta(\vec{x} - \vec{y})$

Is normally equivalent to:

$[a_{\vec{k}}, a^\dagger_{\vec{l}}] = \delta^3(\vec{k} - \vec{l})$.

But what about tachyons?

Feinberg, Phys. Rev. 159, 1089 (1967)

It is clear that these relations and the assumption that L is unitary are inconsistent with the canonical commutation relations

$$[a(k), a^\dagger(k')] = \delta^3(k-k')\,, \qquad (4.8)$$
$$[a(k), a(k')] = 0\,,$$

since a Lorentz transform changing $a(k)$ into $a^\dagger(k)$ will change the sign of the left-hand side of (4.8) without changing the right-hand side. On the other hand, if we quantize with anticommutators, no such trouble will arise. Therefore, we shall take the a, $a^\dagger$ to satisfy

$$a(k)a^\dagger(k') + a^\dagger(k')a(k) = \delta^3(k-k')\,, \qquad (4.9)$$
$$a(k)a(k') + a(k')a(k) = 0\,, \quad (k_0, k_0' \neq 0)$$

and these are consistent with (4.7) and a unitary L. Therefore the tachyons are fermions, even though they have spin-zero. Such a violation of the connection between spin and statistics is not in contradiction with the known theorems on this connection,[11] since we do not assume "microscopic causality."

Twin space

For completeness, we need to show that $|\vec{k}| > m$ is LI on-shell

$$|\vec{k'}| = \frac{|\vec{k} - E\vec{u}|}{\sqrt{1-u^2}} \geq \frac{|\vec{k}| - |E|u}{\sqrt{1-u^2}} = \frac{m(v-u)}{\sqrt{1-u^2}\sqrt{v^2-1}} \overset{?}{\geq} m$$

$$E \equiv \frac{\sigma m c^2}{\sqrt{\frac{v^2}{c^2} - 1}},$$

$$\boldsymbol{p} \equiv \frac{\sigma m \boldsymbol{v}}{\sqrt{\frac{v^2}{c^2} - 1}},$$

$$\sigma' = \sigma \operatorname{sgn}\left(1 - \frac{\boldsymbol{v} \cdot \boldsymbol{V}}{c^2}\right),$$

$$v = |\vec{v}| > 1 > u = |\vec{u}|$$

$$\vec{v} = \vec{k}/E = \vec{k}/\omega_{\vec{k}}$$

$$|\vec{k}| > m$$

is equivalent to

$$(1 - uv)^2 \geq 0.$$

Twin space

We use LIM to normalize single-particle states to 1: $\langle \overrightarrow{p} \,|\, \overrightarrow{q} \rangle = (2\pi)^3\, 2E_{\overrightarrow{p}}\, \delta^3(\overrightarrow{p} - \overrightarrow{q})$.

Equivalent to $[\hat{a}_{\vec{p}}, \hat{a}^{\dagger}_{\vec{q}}] = (2\pi)^3\, 2\omega_{\vec{p}}\, \delta^3(\vec{p} - \vec{q})$, this does not yet mean we have QM.

Paczos et al. (Phys.Rev.D 110 (2024) 1, 015006)

$\hat{a}^{\star}_{\vec{k}} \langle 0| \equiv \langle \vec{k}|$, $\quad \hat{a}^{\star\dagger}_{\vec{k}} \langle 0| \equiv 0$, If we want $\langle \vec{l}|\vec{k}\rangle_{F^\star} = (2\pi)^3\, 2\omega_k\, \delta(\vec{k} - \vec{l})$

then $[\hat{a}^{\dagger\star}_{\vec{q}}, \hat{a}^{\star}_{\vec{p}}] = (2\pi)^3\, 2\omega_{\vec{p}}\, \delta^3(\vec{p} - \vec{q})$. This is consistent with $(A \circ B)^\star = B^\star \circ A^\star$,

$$\hat{\Phi} : \mathcal{F} \otimes \mathcal{F}^\star \to \mathcal{F} \otimes \mathcal{F}^\star \qquad \hat{\Phi}(x) = \frac{1}{2}\left(\hat{\phi}(x) \otimes \hat{\mathbb{1}} + \hat{\mathbb{1}} \otimes \hat{\phi}^\star(x)\right),$$

$$\hat{\phi}^\star_1(x) \equiv \int \frac{d^3\vec{k}}{(2\pi)^3}\, \theta(|\vec{k}| - m)\, \left(u_{\vec{k}}(x)\, \hat{a}^{\star}_{\vec{k}} + u^*_{\vec{k}}(x)\, \hat{a}^{\star\dagger}_{\vec{k}}\right), \qquad \sim \boxed{\hat{\phi}^T(x)}$$

$$\hat{\phi}^\star_2(x) \equiv \int \frac{d^3\vec{k}}{(2\pi)^3}\, \theta(|\vec{k}| - m)\, \left(u^*_{\vec{k}}(x)\, \hat{a}^{\star}_{\vec{k}} + u_{\vec{k}}(x)\, \hat{a}^{\star\dagger}_{\vec{k}}\right), \qquad \sim \hat{\phi}^\dagger(x)$$

Twin space

We use LIM to normalize single-particle states to 1: $\langle \vec{p} \,|\, \vec{q} \rangle = (2\pi)^3 \, 2E_{\vec{p}} \, \delta^3(\vec{p} - \vec{q})$.

Equivalent to $[\hat{a}_{\vec{p}}, \hat{a}^{\dagger}_{\vec{q}}] = (2\pi)^3 \, 2\omega_{\vec{p}} \, \delta^3(\vec{p} - \vec{q})$, this does not yet mean we have QM.

Paczos et al. (Phys.Rev.D 110 (2024) 1, 015006)

$\hat{a}^{\star}_{\vec{k}} \, \langle 0| \equiv \langle \vec{k}| \,, \quad \hat{a}^{\star\dagger}_{\vec{k}} \, \langle 0| \equiv 0 \,,$ If we want $\langle \vec{l}|\vec{k}\rangle_{F^\star} = (2\pi)^3 \, 2\omega_k \, \delta(\vec{k} - \vec{l})$

then $[\hat{a}^{\dagger\star}_{\vec{q}}, \hat{a}^{\star}_{\vec{p}}] = (2\pi)^3 \, 2\omega_{\vec{p}} \, \delta^3(\vec{p} - \vec{q})$. This is consistent with $(A \circ B)^\star = B^\star \circ A^\star$,

$$\hat{\Phi} : \mathcal{F} \otimes \mathcal{F}^\star \to \mathcal{F} \otimes \mathcal{F}^\star \qquad \hat{\Phi}(x) = \frac{1}{2}\Big(\hat{\phi}(x) \otimes \hat{\mathbb{1}} + \hat{\mathbb{1}} \otimes \hat{\phi}^\star(x)\Big) \,,$$

$$\hat{\phi}^\star_1(x) \equiv \int \frac{d^3\vec{k}}{(2\pi)^3} \, \theta(|\vec{k}| - m) \, \Big(u_{\vec{k}}(x) \, \hat{a}^{\star}_{\vec{k}} + u^*_{\vec{k}}(x) \, \hat{a}^{\star\dagger}_{\vec{k}}\Big) \,, \qquad \sim \boxed{\hat{\phi}^T(x)}$$

This choice leads to covariant, non-quantum field.

$$\hat{\phi}^\star_2(x) \equiv \int \frac{d^3\vec{k}}{(2\pi)^3} \, \theta(|\vec{k}| - m) \, \Big(u^*_{\vec{k}}(x) \, \hat{a}^{\star}_{\vec{k}} + u_{\vec{k}}(x) \, \hat{a}^{\star\dagger}_{\vec{k}}\Big) \,, \qquad \sim \hat{\phi}^\dagger(x)$$

The other choice leads to quantum, non-covariant field, e.g., micro-causality violation.

Twin space

$$u_{\vec{k}}(x) = \frac{e^{-i\omega_{\vec{k}}t + i\vec{k}\cdot\vec{x}}}{(2\pi)^2\, 2\omega_{\vec{k}}}\,,$$

$$\hat{\phi}_1^\star(x) \equiv \int \frac{d^3\vec{k}}{(2\pi)^3}\,\theta(|\vec{k}|-m)\,\left(u_{\vec{k}}(x)\,\hat{a}^\star_{\vec{k}} + u^*_{\vec{k}}(x)\,\hat{a}^{\star\dagger}_{\vec{k}}\right)\,, \quad \hat{\phi}(t,\boldsymbol{r}) \equiv \int_{|\boldsymbol{k}|>m} \mathrm{d}^3\boldsymbol{k}\,\left(u_{\boldsymbol{k}}(t,\boldsymbol{r})\,\hat{a}_{\boldsymbol{k}} + u^*_{\boldsymbol{k}}(t,\boldsymbol{r})\,\hat{a}^\dagger_{\boldsymbol{k}}\right)$$

$$\hat{\Phi}(x) = \int \frac{d^3\vec{k}}{(2\pi)^3}\,\theta(|\vec{k}|-m)\,\left(u_{\vec{k}}(x)\,\hat{c}_{\vec{k}} + u^*_{\vec{k}}(x)\,\hat{c}^\dagger_{\vec{k}}\right) = \hat{\Phi}^\dagger(x)\,, \qquad \hat{c}_{\vec{k}} \equiv \hat{a}_{\vec{k}} \otimes \hat{1} + \hat{1} \otimes \hat{a}^\star_{\vec{k}}$$

Covariance guaranteed since $U(\Lambda)\left(\hat{a}_{\boldsymbol{k}} \otimes \hat{\mathbb{1}} + \hat{\mathbb{1}} \otimes \hat{a}^\star_{\boldsymbol{k}}\right) U(\Lambda)^{-1} = \left(\hat{a}_{\boldsymbol{l}'} \otimes \hat{\mathbb{1}} + \hat{\mathbb{1}} \otimes \hat{a}^\star_{\boldsymbol{l}'}\right)^\dagger$,

Indeed, $[c_{\vec{k}}, c^\dagger_{\vec{l}}] = 0$, and vanishing CCRs are preserved by boosts. However, the CCR are not satisfied since $[\hat{\Phi}(0,\vec{x}), \partial_t\hat{\Phi}(0,\vec{y})|_{t=0}] = 0 \neq i\,\delta(\overrightarrow{x} - \overrightarrow{y})$.

Moreover, $[\Phi(x), \Phi(y)] = 0$ for any x, y.
Indeed, $\langle 0\,|\,\phi^\star(x)\phi^\star(y)\,|\,0\rangle_{F^\star} = (\langle 0\,|\,\phi(y)\phi(x)\,|\,0\rangle_F)^\star = \langle 0\,|\,\phi(y)\phi(x)\,|\,0\rangle_F$. Therefore, the terms with $\phi^\star$ have opposite sign to ϕ and $[\Phi(x), \Phi(y)] = 0$.

$$\langle 0\,|\,\phi(x)\phi(y)\,|\,0\rangle = \int_{|\vec{k}|>m} \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}\frac{1}{2\omega_{\mathbf{p}}}e^{-ip(x-y)}$$

$$\langle 0\,|\,\overset{\star}{\phi}(x)\overset{\star}{\phi}(y)\,|\,0\rangle_\star = \int_{|\vec{k}|>m} \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}\frac{1}{2\omega_{\mathbf{p}}}e^{+ip(x-y)}$$

$$[\hat{\Phi}(x), \hat{\Phi}(y)] = \frac{1}{2}\left([\hat{\phi}(x), \hat{\phi}(y)] + [\hat{\phi}^\star(x), \hat{\phi}^\star(y)]\right) =$$

$$= \frac{1}{2}\left([\hat{\phi}(x), \hat{\phi}(y)] - [\hat{\phi}(x), \hat{\phi}(y)]^\star\right) = \frac{1}{2}(C - C^\star) =$$

$$= \frac{1}{2}(C - C) = 0\,.$$

$$\boxed{(A \circ B)^\star = B^\star \circ A^\star\,,}$$

Twin space

$$u_{\vec{k}}(x) = \frac{e^{-i\omega_{\vec{k}}t + i\vec{k}\cdot\vec{x}}}{(2\pi)^2\, 2\omega_{\vec{k}}}\,,$$

$$\hat{\phi}_1^\star(x) \equiv \int \frac{d^3\vec{k}}{(2\pi)^3}\,\theta(|\vec{k}|-m)\,\left(u_{\vec{k}}(x)\,\hat{a}^\star_{\vec{k}} + u^*_{\vec{k}}(x)\,\hat{a}^{\star\dagger}_{\vec{k}}\right)\,, \qquad \hat{\phi}(t,\boldsymbol{r}) \equiv \int_{|\boldsymbol{k}|>m} \mathrm{d}^3\boldsymbol{k}\,\left(u_{\boldsymbol{k}}(t,\boldsymbol{r})\,\hat{a}_{\boldsymbol{k}} + u^*_{\boldsymbol{k}}(t,\boldsymbol{r})\,\hat{a}^\dagger_{\boldsymbol{k}}\right)$$

$$\hat{\Phi}(x) = \int \frac{d^3\vec{k}}{(2\pi)^3}\,\theta(|\vec{k}|-m)\,\left(u_{\vec{k}}(x)\,\hat{c}_{\vec{k}} + u^*_{\vec{k}}(x)\,\hat{c}^\dagger_{\vec{k}}\right) = \hat{\Phi}^\dagger(x)\,, \qquad \hat{c}_{\vec{k}} \equiv \hat{a}_{\vec{k}} \otimes \hat{1} + \hat{1} \otimes \hat{a}^\star_{\vec{k}}$$

Covariance guaranteed since $U(\Lambda)\left(\hat{a}_{\boldsymbol{k}} \otimes \hat{\mathbb{1}} + \hat{\mathbb{1}} \otimes \hat{a}^\star_{\boldsymbol{k}}\right)U(\Lambda)^{-1} = \left(\hat{a}_{\boldsymbol{l}'} \otimes \hat{\mathbb{1}} + \hat{\mathbb{1}} \otimes \hat{a}^\star_{\boldsymbol{l}'}\right)^\dagger,$

Indeed, $[c_{\vec{k}}, c^\dagger_{\vec{l}}] = 0$, and vanishing CCRs are preserved by boosts. However, the CCR are not satisfied since $[\hat{\Phi}(0,\vec{x}), \partial_t\hat{\Phi}(0,\vec{y})|_{t=0}] = 0 \neq i\,\delta(\vec{x} - \vec{y})$.

Moreover, $[\Phi(x), \Phi(y)] = 0$ for any x, y.
Indeed, $\langle 0\,|\,\phi^\star(x)\phi^\star(y)\,|\,0\rangle_{F^\star} = (\langle 0\,|\,\phi(y)\phi(x)\,|\,0\rangle_F)^\star = \langle 0\,|\,\phi(y)\phi(x)\,|\,0\rangle_F$. Therefore, the terms with $\phi^\star$ have opposite sign to ϕ and $[\Phi(x), \Phi(y)] = 0$.

Take derivative wrt. $t = y_0$. Then $[\Phi(x), \partial_t\Phi(y)] = 0$.

There is no quantum dynamics since everything commutes, which is indeed LI.

Twin space

$$u_{\vec{k}}(x) = \frac{e^{-i\omega_{\vec{k}} t + i\vec{k}\cdot\vec{x}}}{(2\pi)^2\, 2\omega_{\vec{k}}}\,,$$

$$\hat{\phi}^\star_1(x) \equiv \int \frac{d^3\vec{k}}{(2\pi)^3}\,\theta(|\vec{k}|-m)\,\left(u_{\vec{k}}(x)\,\hat{a}^\star_{\vec{k}} + u^*_{\vec{k}}(x)\,\hat{a}^{\star\dagger}_{\vec{k}}\right)\,, \quad \hat{\phi}(t,\boldsymbol{r}) \equiv \int_{|\boldsymbol{k}|>m} \mathrm{d}^3\boldsymbol{k}\,\left(u_{\boldsymbol{k}}(t,\boldsymbol{r})\,\hat{a}_{\boldsymbol{k}} + u^*_{\boldsymbol{k}}(t,\boldsymbol{r})\,\hat{a}^\dagger_{\boldsymbol{k}}\right)$$

$$\hat{\Phi}(x) = \int \frac{d^3\vec{k}}{(2\pi)^3}\,\theta(|\vec{k}|-m)\,\left(u_{\vec{k}}(x)\,\hat{c}_{\vec{k}} + u^*_{\vec{k}}(x)\,\hat{c}^\dagger_{\vec{k}}\right) = \hat{\Phi}^\dagger(x)\,, \qquad \hat{c}_{\vec{k}} \equiv \hat{a}_{\vec{k}} \otimes \hat{1} + \hat{1} \otimes \hat{a}^\star_{\vec{k}}$$

Covariance guaranteed since $U(\Lambda)\,(\hat{a}_{\boldsymbol{k}} \otimes \hat{\mathbb{1}} + \hat{\mathbb{1}} \otimes \hat{a}^\star_{\boldsymbol{k}})\,U(\Lambda)^{-1} = (\hat{a}_{\boldsymbol{l}'} \otimes \hat{\mathbb{1}} + \hat{\mathbb{1}} \otimes \hat{a}^\star_{\boldsymbol{l}'})^\dagger\,,$

Indeed, $[c_{\vec{k}}, c^\dagger_{\vec{l}}] = 0$, and vanishing CCRs are preserved by boosts. However, the CCR are not satisfied since $[\hat{\Phi}(0,\vec{x}), \partial_t\hat{\Phi}(0,\vec{y})|_{t=0}] = 0 \neq i\,\delta(\vec{x} - \vec{y})$.

$$[\hat{\phi}(0,\vec{x}), \partial_t\hat{\phi}(0,\vec{y})|_{t=0}] = i\delta(\vec{x}-\vec{y})$$
$$+\,i\,\frac{\sin\left(m|\vec{x}-\vec{y}|\right) - m|\vec{x}-\vec{y}|\cos\left(m|\vec{x}-\vec{y}|\right)}{2\pi^2|\vec{x}-\vec{y}|^3}\,,$$

is also c-number so the same applies

$$\pi(\vec{x}) = \frac{1}{2}\int_{|\vec{k}|>m} \frac{d^3\vec{k}}{(2\pi)^3}\left(-i\,e^{i\vec{k}\cdot\vec{x}}\,\hat{a}_{\vec{k}} + \underline{i\,e^{-i\vec{k}\cdot\vec{x}}\,\hat{a}^\dagger_{\vec{k}}}\right)$$

$$\pi^\star(\vec{x}) = \frac{1}{2}\int_{|\vec{k}|>m} \frac{d^3\vec{k}}{(2\pi)^3}\left(\underline{-i\,e^{i\vec{k}\cdot\vec{x}}\,\hat{a}^\star_{\vec{k}}} + i\,e^{-i\vec{k}\cdot\vec{x}}\,\hat{a}^{\dagger\star}_{\vec{k}}\right)$$

The point is, these have an opposite sign so $[\phi^\star(\vec{x}), \pi^\star(\vec{y})] = -\,[\phi(\vec{x}), \pi(\vec{y})]$.

Note that $\delta(x-y) = \delta(-x+y)$.

What if $\phi^\star \sim \phi^\dagger$

$$\langle 0 \,|\, \phi^\star(x)\phi^\star(y) \,|\, 0 \rangle_{F^\star} = (\langle 0 \,|\, \phi(y)\phi(x) \,|\, 0 \rangle_F)^\star = \langle 0 \,|\, \phi(y)\phi(x) \,|\, 0 \rangle_F^* = \langle 0 \,|\, \phi(x)\phi(y) \,|\, 0 \rangle_F$$

$$\langle 0 \,|\, \phi(x)\phi(y) \,|\, 0 \rangle = \int_{|\vec{k}|>m} \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3} \frac{1}{2\omega_{\mathbf{p}}} e^{-ip(x-y)}$$

$$\langle 0 \,|\, \phi^\star(x)\phi^\star(y) \,|\, 0 \rangle_\star = \int_{|\vec{k}|>m} \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3} \frac{1}{2\omega_{\mathbf{p}}} e^{+ip(x-y)}$$

Same dynamics as in regular Fock space for $\hat{\phi}$.

$$\Delta(x-y) = [\hat{\Phi}(x), \hat{\Phi}(y)]_{\mathcal{F}\otimes\mathcal{F}^\star} = [\hat{\phi}(x), \hat{\phi}(y)]_{\mathcal{F}} = \int_{|\vec{k}|>m} \frac{d^3\vec{k}}{(2\pi)^3\, 2\omega_{\vec{k}}} \left(e^{-ik(x-y)} - e^{ik(x-y)}\right)$$

$$= \int_{|\vec{k}|>m} \frac{d^3\vec{k}}{(2\pi)^3\, 2\omega_{\vec{k}}} \left(e^{-i\omega_{\vec{k}}(x^0-y^0)} - e^{i\omega_{\vec{k}}(x^0-y^0)}\right) e^{i\vec{k}\cdot(\vec{x}-\vec{y})} = \int_{|\vec{k}|>m} \frac{d^4k}{(2\pi)^4}\, \delta(k^2+m^2)\, e^{-ik(x-y)} \left(\theta(k^0) - \theta(-k^0)\right) ,$$

The $\mathbf{\Phi}$ field inherits all the problems of Dhar and Sudarshan 1968

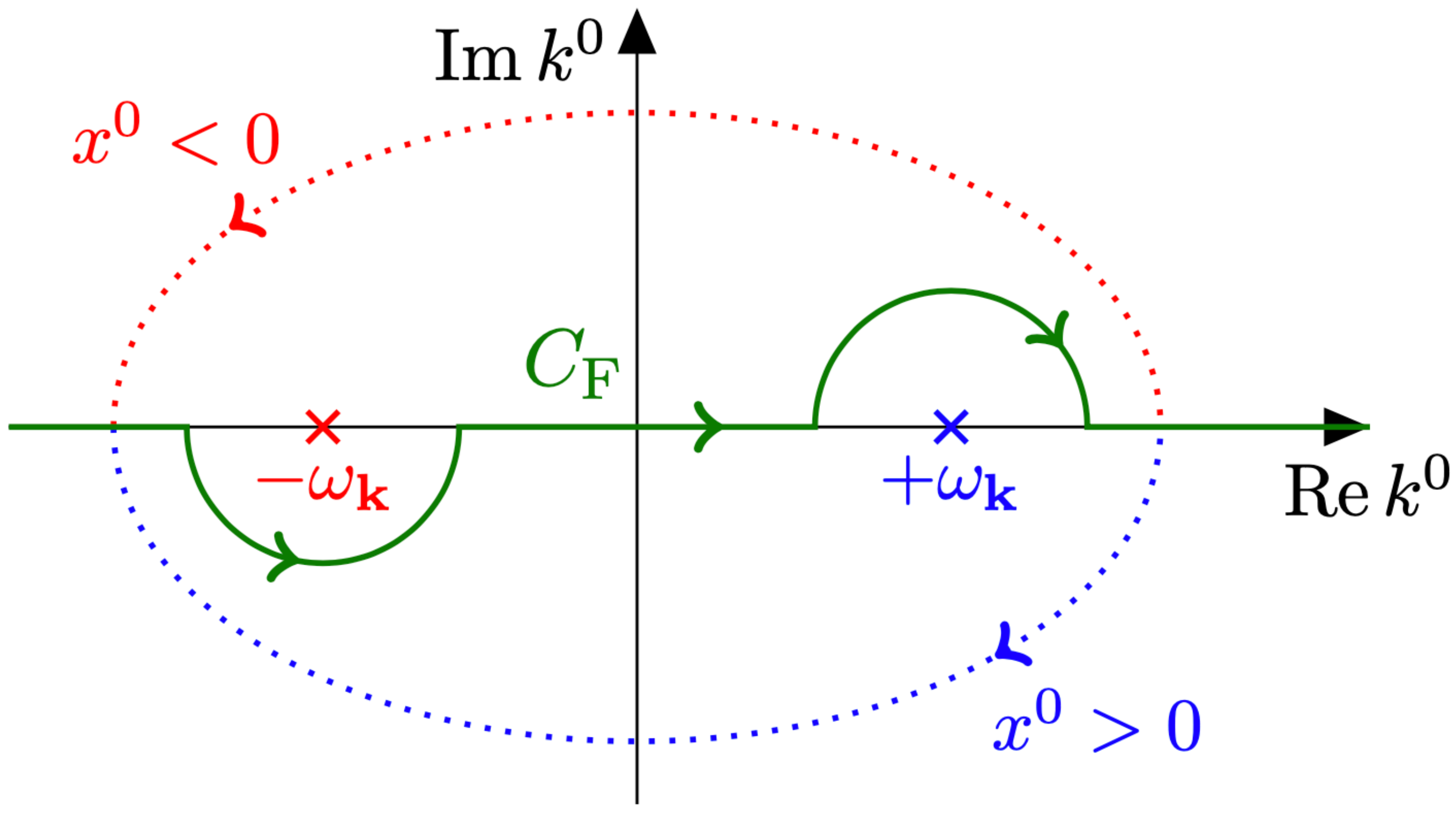
Commutator and any TOCF are not LI (in particular, two-point TOCF - FP).

Micro-causality is violated → LI scattering theory is impossible, causality violation.

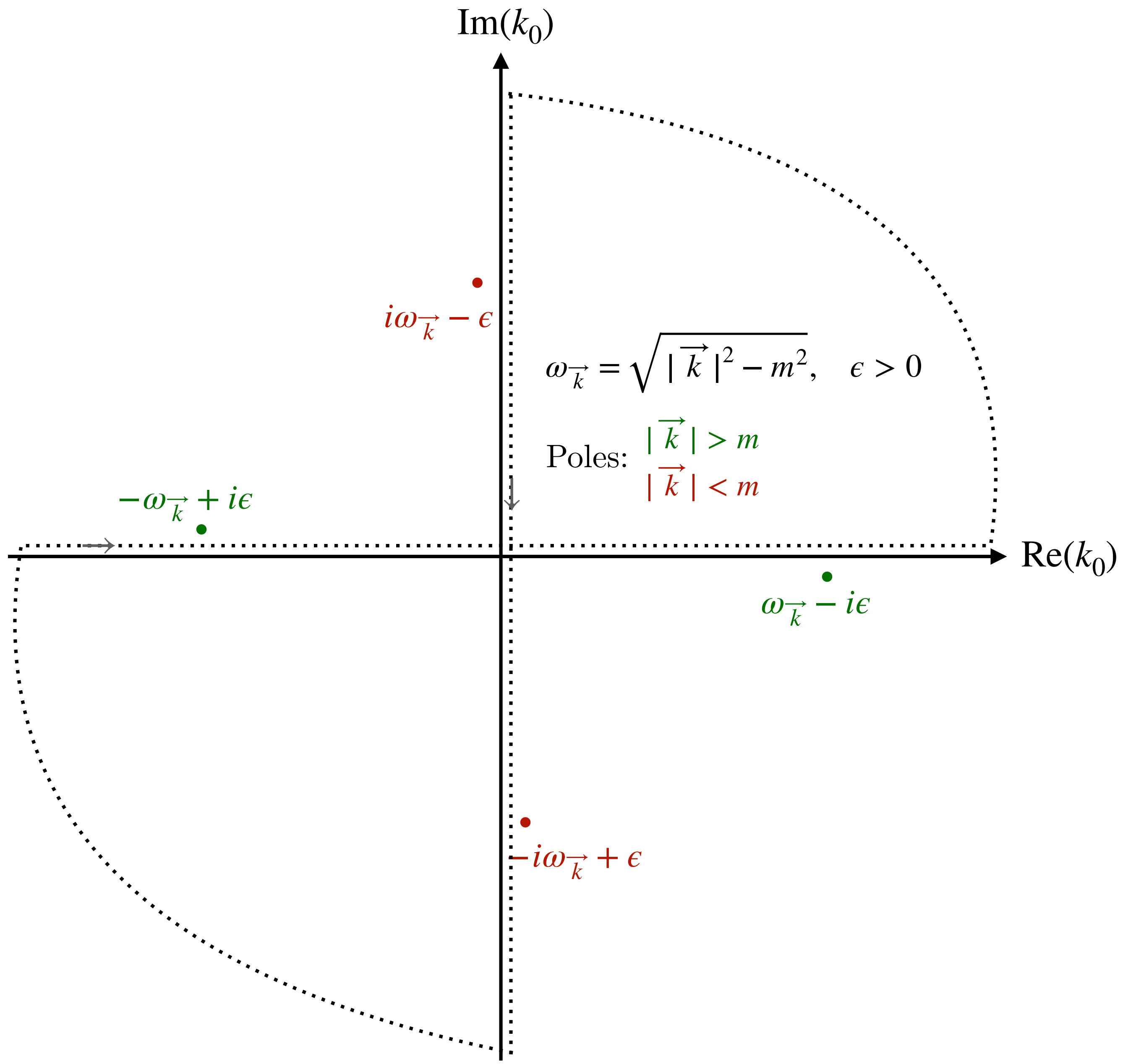
Dhar-Sudarshan Feynman propagator

$$\int \frac{d^4k}{(2\pi)^4} \frac{i\, e^{-ik(x-y)}}{k^2+m^2+i\epsilon}.$$

Dhar and Sudarshan 1968



This is not a two-point TOCF of Φ (or ϕ), since the $k < m$ modes were put back in.
Note that FP propagates *positive energy* into the future and negative energy states into the past.



Red/blue contour on the left also includes imaginary pole!

Dhar-Sudarshan Feynman propagator

$$\int \frac{d^4k}{(2\pi)^4}\,\frac{i\,e^{-ik(x-y)}}{k^2+m^2+i\epsilon}.$$

Dhar and Sudarshan 1968

The complete interaction is

$$-\tfrac{1}{2}g^2\int d^4x\int d^4y\,\bar{\psi}(x)\psi(x)G(x-y)\bar{\psi}(y)\psi(y). \quad (3.5)$$

The factor $\frac{1}{2}$ is added to compensate for the double counting. Choosing Δ_F as the Green's function according to (2.8) we get for the effective interaction

$$-\tfrac{1}{2}g^2\int d^4x\int d^4y\int d^4k\,\bar{\psi}(x)\psi(x)e^{ik\cdot x}\frac{1}{k^2+m^2+i\epsilon} \times e^{-ik\cdot y}\bar{\psi}(y)\psi(y). \quad (3.6)$$

It is interesting to note that (3.6) involves the invariant Green's function rather than the noninvariant contraction function. This means that the tachyon-fermion Yukawa interaction is not simply expressible as a trilinear interaction in the *interaction picture*, though the original interaction in the Heisenberg picture is trilinear.

placing (4.2) by

$$W_1(x)=g\bar{\psi}(x)\psi(x)\chi_{\text{in}}(x)+\tfrac{1}{2}g^2\bar{\psi}(x)\psi(x) \times\int d^4y\,D_1(x-y;\eta)\bar{\psi}(y)\psi(y), \quad (4.4)$$

where

$$D_1(x-y;\eta)=D(x-y;\eta)-\tfrac{1}{2}i\Delta^{(1)}(x-y). \quad (4.5)$$

The reduction of the S matrix now proceeds as in the usual theory: We rewrite (4.1) in a normal-ordered expansion for the asymptotic field $\chi_{in}(x)$. The coefficients of appropriate normal-ordered operators yield the various transition amplitudes. The results so calculated of course contain the standard divergences and would have to be subjected to a suitable renormalization before physically meaningful results can be extracted. There is essentially no difference between the renormalization of this theory and one in which the tachyon field is replaced by an ordinary scalar meson field; we shall content ourselves, therefore, with a derivation of the unrenormalized covariant perturbation expansion.

Dhar-Sudarshan Feynman propagator

$$\int \frac{d^4k}{(2\pi)^4} \frac{i\, e^{-ik(x-y)}}{k^2+m^2+i\epsilon}.$$

Dhar and Sudarshan 1968

Previously, I interpreted orange sentences as statements about DS FP, since virtual tachyons are unavoidable in QFT.

However, the proper expression for FP in Paczos et al. (Phys.Rev.D 110 (2024) 1, 015006) is actually not given.

where $\hat{\mathbb{H}}_{0+}$ denotes the operator corresponding to the free Hamiltonian $\hat{H}_0$. With this, we can write the S-matrix element, $S_{\alpha\beta} = \langle \beta_{\rm out}|\alpha_{\rm in}\rangle$, in the following way:

$$S_{\alpha\beta} = \lim_{T\to\infty} \mathrm{Tr}(e^{-i\hat{\mathbb{H}}_+ T} e^{i\hat{\mathbb{H}}_{0+} T} |\alpha_0\rangle \otimes \langle\beta_0|). \tag{20}$$

The above formula allows us to compute the S-matrix elements within the twin space formalism along the same lines as in standard QFT.

It should be emphasized that the operators $\hat{\mathbb{H}}_\pm$ are not derived from the field operator $\hat{\boldsymbol{\Phi}}$. Instead, to obtain them one constructs the single Fock space operator $\hat{H}$ from the operator $\hat{\phi}$ as if it were a Hamiltonian on $\mathcal{F}$, and extends it to the twin space $\mathcal{F}\otimes\mathcal{F}^\star$ like in (18). This approach results in the covariant expression (20) providing a straightforward reference to the standard formulation of QFT.

Using Eq. (20) we can compute the S-matrix elements in a perturbative way. It is important to note, however, that the contraction function

$$\langle 0|T\hat{\phi}(x)\hat{\phi}(y)|0\rangle = \int_{|\boldsymbol{k}|>m} \frac{d^4k}{(2\pi)^4} \frac{ie^{-ik(x-y)}}{k^2+m^2+i\epsilon} \tag{21}$$

is not relativistically invariant because of the restriction $|\boldsymbol{k}|>m$. As proposed by Dhar and Sudarshan [23] the contraction function can be extended by dropping the condition $|\boldsymbol{k}|>m$, which would correspond to including the virtual tachyons with $|\boldsymbol{k}|<m$ into considerations. The propagator obtained in this way is relativistically invariant, as desired. This is similar to the situation in quantum electrodynamics, where (in the Coulomb gauge) we need to include in the propagator the nonphysical longitudinal and scalar photons. Just like the longitudinal and scalar photons, the tachyons with $|\boldsymbol{k}|<m$ are allowed to appear as virtual particles, but are excluded from the space of asymptotic states.

of subluminal particles also shift to $k' = \Lambda k$ and $l' = \Lambda l$. In this boosted frame, we find that the transformed matrix element takes the form

$$-ig(2\pi)^4\delta^{(4)}(k'-l'+p') \equiv -ig(2\pi)^4\delta^{(4)}(\Lambda(k-l-p)), \tag{23}$$

which demonstrates the covariance of the scattering process.

The renormalization procedure for this theory can be carried out at one loop in a standard way, since the UV divergences in diagrams involving tachyons are the same as for scalars with positive mass squared. At the technical level, one needs to subtract singularities from on-shell particles on intermediate lines to avoid double counting and to restrict integration over momenta for tachyons which does not affect logarithmic divergences.

Similar reasoning can be carried out for other types of covariant interactions. Even if some tachyons are boosted from the initial to the final states, this change is compensated by the minus sign of the boosted momentum. As a result, the conditions of momentum conservation at each vertex transform covariantly between all inertial frames.

V. DISCUSSION AND CONCLUSIONS

We showed how to covariantly quantize a tachyonic field while maintaining the positive-energy spectrum and preserving a stable, Lorentz-invariant vacuum state. Unlike Feinberg [21], Arons, Sudarshan, and Dhar [22,23], Schwartz [90], as well as others, but similar to Schwartz [41] we proposed to solve this problem by extending the Hilbert space to $\mathcal{F}\otimes\mathcal{F}^\star$. We developed an explicitly covariant framework that keeps the commutation relations the same in all reference frames, and it ensures the dynamical stability and relativistic invariance of the vacuum state. We also applied our framework to account for interactions with other fields.

Paczos et al. (Phys.Rev.D 110 (2024) 1, 015006)

Dhar-Sudarshan Feynman propagator

$$G_F^{ij}(x-y) = \langle 0 | T A^i(x) A^j(y) | 0 \rangle$$

The Feynman propagator of a tachyon violates unitarity

$$\int \frac{d^4k}{(2\pi)^4} \frac{i\,e^{-ik(x-y)}}{k^2+m^2+i\epsilon}.$$

$$= \theta(x^0-y^0) \int \frac{d^3k}{(2\pi)^3 2\omega_k} \sum_{\lambda=\pm} \epsilon^{i\lambda}(k)\epsilon^{j\lambda *}(k)\, e^{-ik(x-y)}$$

$$+\theta(y^0-x^0) \int \frac{d^3k}{(2\pi)^3 2\omega_k} \sum_{\lambda=\pm} \epsilon^{i\lambda}(k)\epsilon^{j\lambda *}(k)\, e^{+ik(x-y)}$$

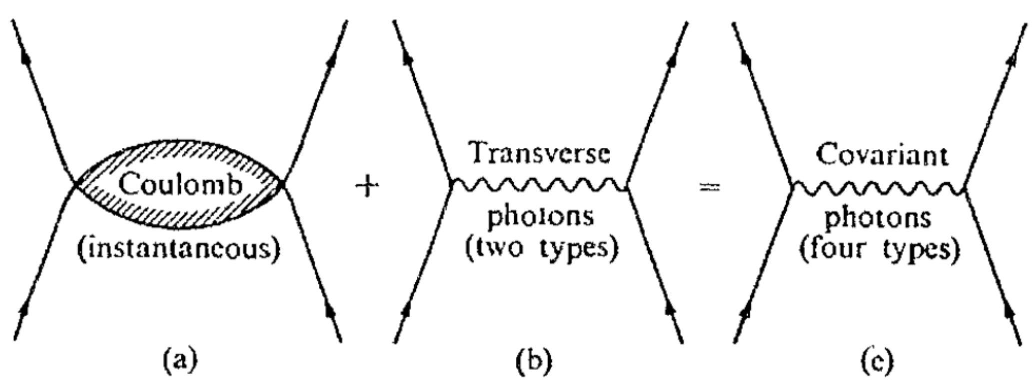
$$= \int \frac{d^4k}{(2\pi)^4} \frac{i\,e^{-ik(x-y)}}{k^2+i\epsilon} \left(\delta^{ij} - \frac{k^i k^j}{|\vec{k}|^2} \right)$$

Photon propagator in Coulomb gauge is not Lorentz invariant (preferred reference frame where $\partial_t A_0 = 0$).

Dhar and Sudarshan 1968:

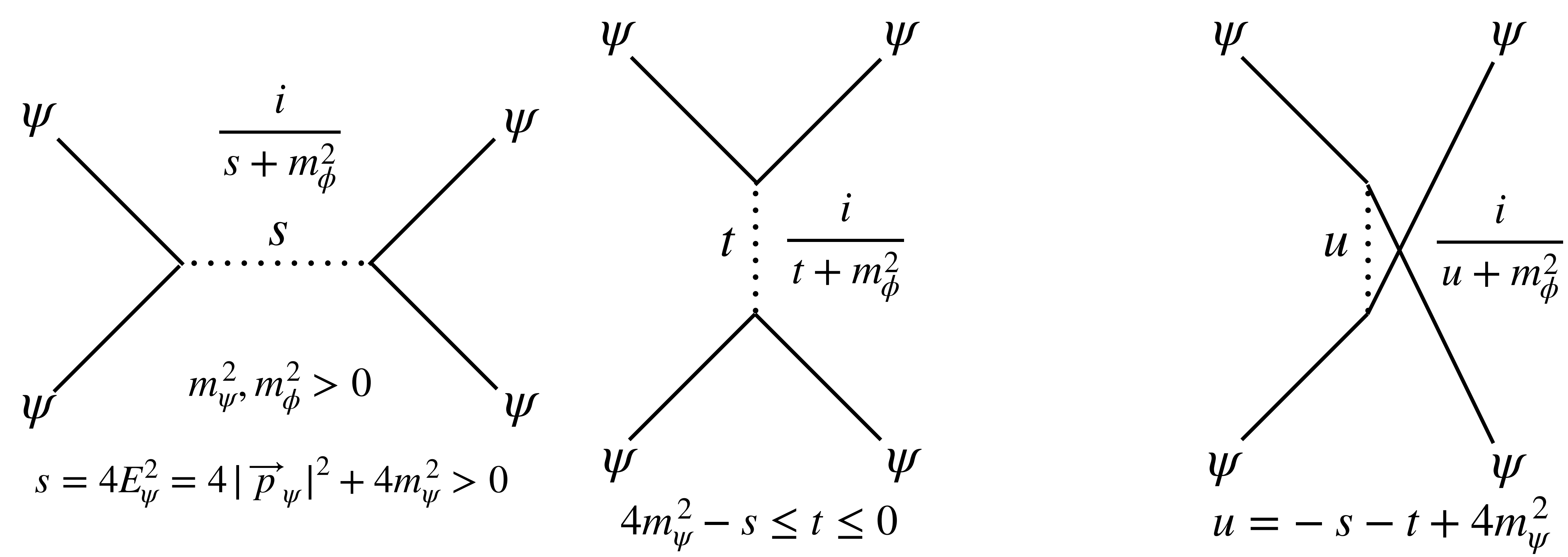
"The situation here is very similar to quantum electrodynamics in the radiation (Coulomb) gauge. In this case, for each value of the momentum there are only two types of photons which are both transverse. The contraction function of two such Maxwell field operators is not covariant. Hence we have to add the direct Coulomb interaction between the electric charge densities with the coupling strength e. The net result of all this is that the perturbation series can be developed as if the contraction function were covariant and as if there were longitudinal and scalar photons."

Conservation of the QED current causes decoupling of the unphysical modes.
But tachyons have no gauge invariance!



Sakurai "Advanced QM" **Fig. 4–20.** Exchange of "covariant photons."

Tree-level unitarity violation



Consider $2 \to 2$ elastic scattering of subliminal states mediated by a virtual tachyon. Kinematics is the same as usual and the allowed momentum transfer $t_{min} \le q^2 = t = (p_1 - p_3)^2 \le t_{max} = 0$. Simple calculation for $m_1 = m_2 = m_3 = m_4 = m_\psi$ gives $t_{min} = 4m_\psi^2 - s$. Since $s = 4E^2 = 4p^2 + 4m_\psi^2$ in the CMF, the pole in the tachyon propagator $(t + m_\psi^2 = 0)$ in the t-channel is hit for $t = t_{min}$, when $p^2 = m_\phi^2/4$. You can also find p, which will satisfy this condition for <u>any</u> $t = c\, t_{min}$, $0 < c \le 1$. It is $p^2 = m_\psi^2/(4c)$ There is also divergence in the u-channel for any $t = 4m_\psi^2 + m_\phi^2 - s$, e.g., for $u = u(t_{max})$.

These are <u>purely kinematical divergences in the physical region: $\theta \ne 0, \pi$</u>.

Tree-level unitarity violation

Already noted in
Mrówczyński 1983

The divergence that arise at $S < 0$ are linear and come from L^2_{+-} and L^2_{-+} terms. For finding L^N with $N > 2$, the recurrence formula (2) is introduced

$$L^{N+1}(S, \pm m_1^2, \ldots\ldots, \pm m_N^2, \pm m_{N+1}^2) =$$

$$= \int d^4 p_{N+1} \delta(p_{N+1}^2 \pm m_{N+1}^2) L^N (S', \pm m_1^2, \ldots\ldots, \pm m_N^2), \qquad (2)$$

where $P^\mu P_\mu = S, (P^\mu - P^\mu_{N+1})^2 = S'$.

The divergences found for L^2 make that L^N for $N > 2$ is divergent for any S. This unexpected result shows that there arise not only the interpretation difficulties quoted previously when we try to build the formalism of tachyons which is a simple extension of methods for particles slower than light. The above divergences are of a completely different nature than those in QED, for example, since they come from pure kinematics but do not depend on interaction phenomena. They arise for tachyons and bradyons as well, because such divergences are related to the way the negative energies are taken into account.

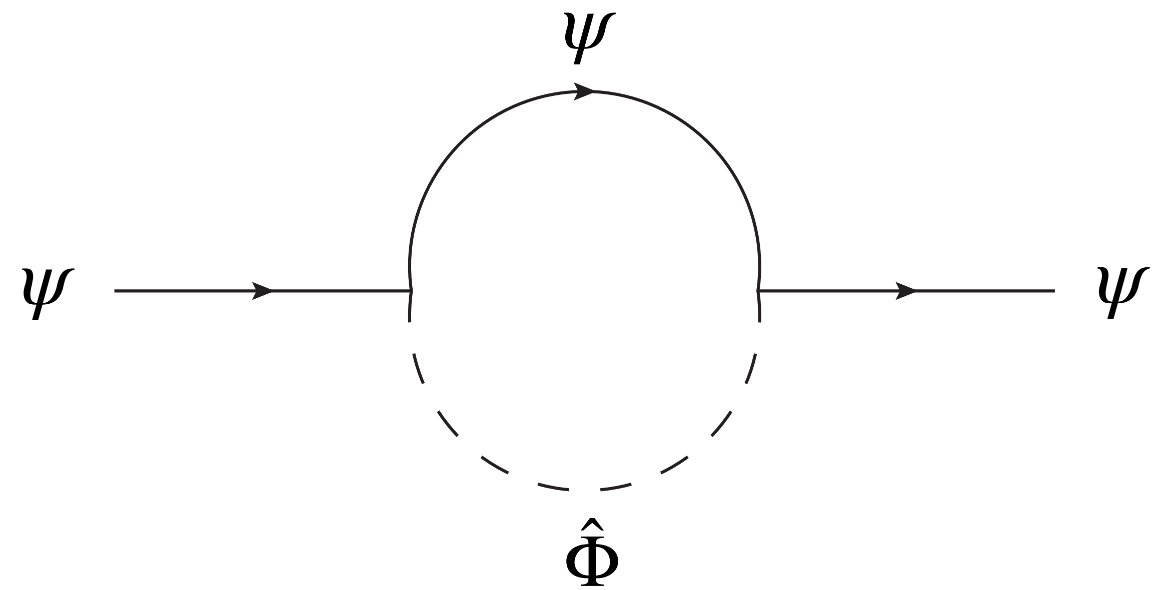
Mrøwczyński St. E2-83-299
The Phase-Space of Tachyons

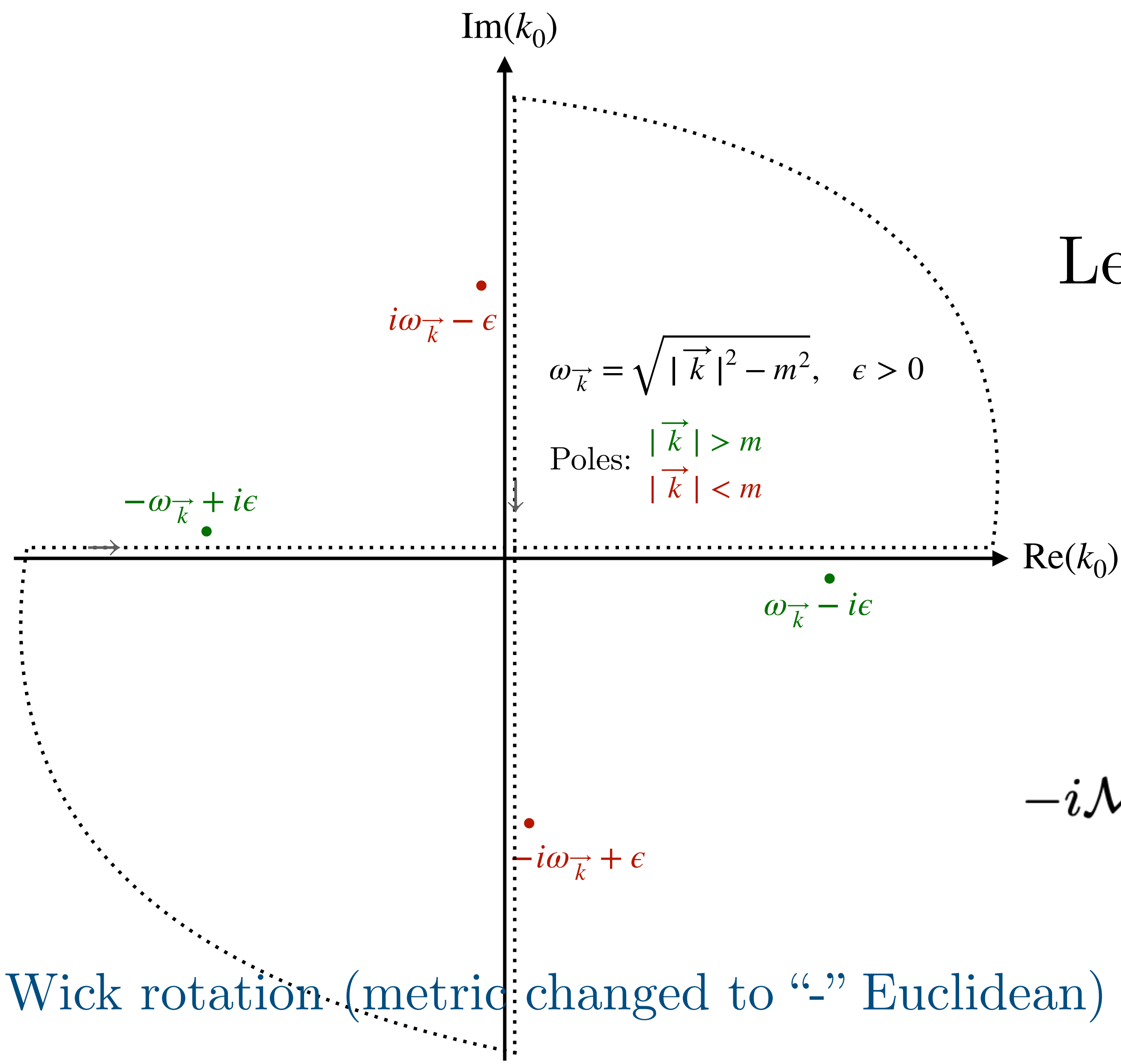
The phase-space of quantized systems that contain tachyons has been investigated. Interpretation difficulties and unexpected divergences are found when we consider the volume of Lorentz invariant phase-space. These problems can be overcome however at the expense of Lorentz invariancy.

1-loops with virtual tachyons

b) $$\int \frac{d^4k}{(2\pi)^4} \frac{i\, e^{-ik(x-y)}}{k^2+m^2+i\epsilon}.$$

Dhar and Sudarshan 1968





Let us compute virtual tachyon $\hat{\Phi}$ contribution to the mass renormalization of a scalar ψ.

$$\mathcal{L} = \begin{cases} g\Lambda\, \phi\, |\psi|^2 & \psi \text{ is a boson,} \\ g\, \phi\, \bar{\psi}\psi & \psi \text{ is a fermion,} \end{cases}$$

$$-i\mathcal{M} = (g\Lambda)^2 \mu^{2\epsilon} \int \frac{d^Dk}{(2\pi)^D} \frac{1}{(k^2-m_0^2+i\epsilon)((p+k)^2-m_1^2+i\epsilon)}$$

Wick rotation (metric changed to "-" Euclidean) is the same as in standard QFT since we avoid all the poles.

1-loops with virtual tachyons

$$
\begin{aligned}
-i\mathcal{M} &= (g\Lambda)^2\mu^{2\epsilon}\int\frac{d^Dk}{(2\pi)^D}\frac{1}{(k^2-m_0^2+i\epsilon)((p+k)^2-m_1^2+i\epsilon)} = (g\Lambda)^2\mu^{2\epsilon}\int_0^1 dx\int\frac{d^Dl}{(2\pi)^D}\frac{1}{(l^2-\Delta)^2}\\
&= \frac{i(g\Lambda)^2}{16\pi^2}\mu^{2\epsilon}\,(4\pi)^\epsilon\,\Gamma(\epsilon)\int_0^1 dx\frac{1}{\Delta^\epsilon} = \frac{i(g\Lambda)^2}{16\pi^2}\left[\left(\frac{1}{\epsilon}-\gamma_E+\ln\left(4\pi\mu^2\right)\right)-\int_0^1 dx\log\left(-x(1-x)p^2+(1-x)m_0^2+xm_1^2-i\epsilon\right)\right]\\
&= \frac{i(g\Lambda)^2}{16\pi^2}\left[\mathrm{UV}-I(p^2)\right] = \frac{i(g\Lambda)^2}{16\pi^2}\left[\mathrm{UV}-\int_0^1 dx\Big(\log\left(p^2-i\epsilon\right)+\log\left((x-x_+)(x-x_-)\right)\Big)\right]\\
&= \frac{i(g\Lambda)^2}{16\pi^2}\left[\mathrm{UV}-\Big(\log\left(p^2-i\epsilon\right)+(1-x_+)\log\left(1-x_+\right)+(1-x_-)\log\left(1-x_-\right)-2+x_+\log(-x_+)+x_-\log(-x_-)\Big)\right],
\end{aligned}
\tag{4}
$$

where $D=4-2\epsilon<4$, $l=k+px$, $\Delta(x)=-x(1-x)p^2+(1-x)m_0^2+xm_1^2-i\epsilon$, and $\Delta(x_\pm)=0$ for

$$x_\pm=\frac{1}{2}\left(1+\frac{m_0^2-m_1^2}{p^2}\right)\pm\sqrt{\left(1+\frac{m_0^2-m_1^2}{p^2}\right)^2-4\frac{m_0^2-i\epsilon}{p^2}}.$$ We used

$$\Gamma(\epsilon)\simeq\frac{1}{\epsilon}-\gamma_E$$

$$A^\epsilon\simeq 1+\epsilon\log(A)$$

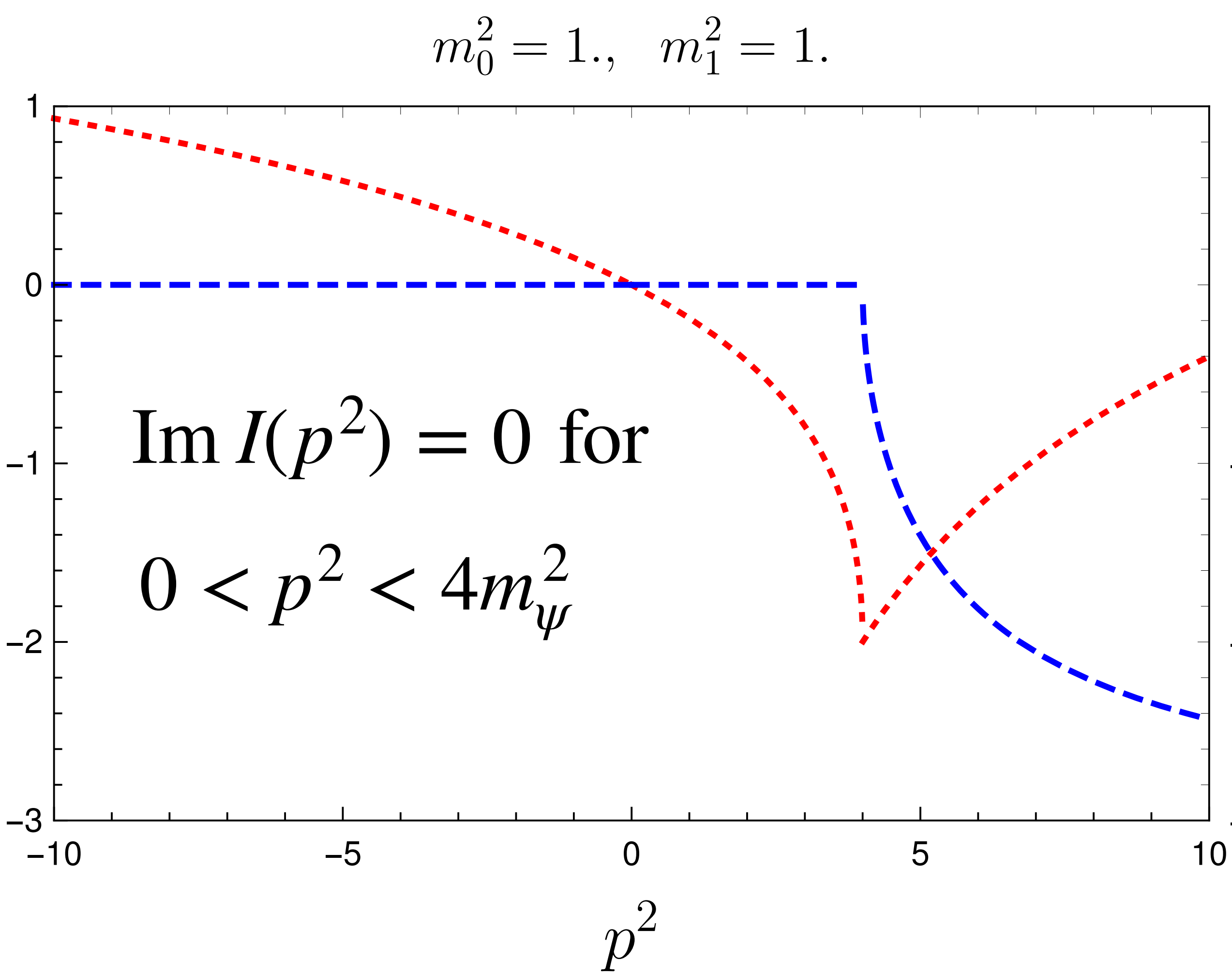
Actually, we only need the imaginary part, which is easier to calculate:

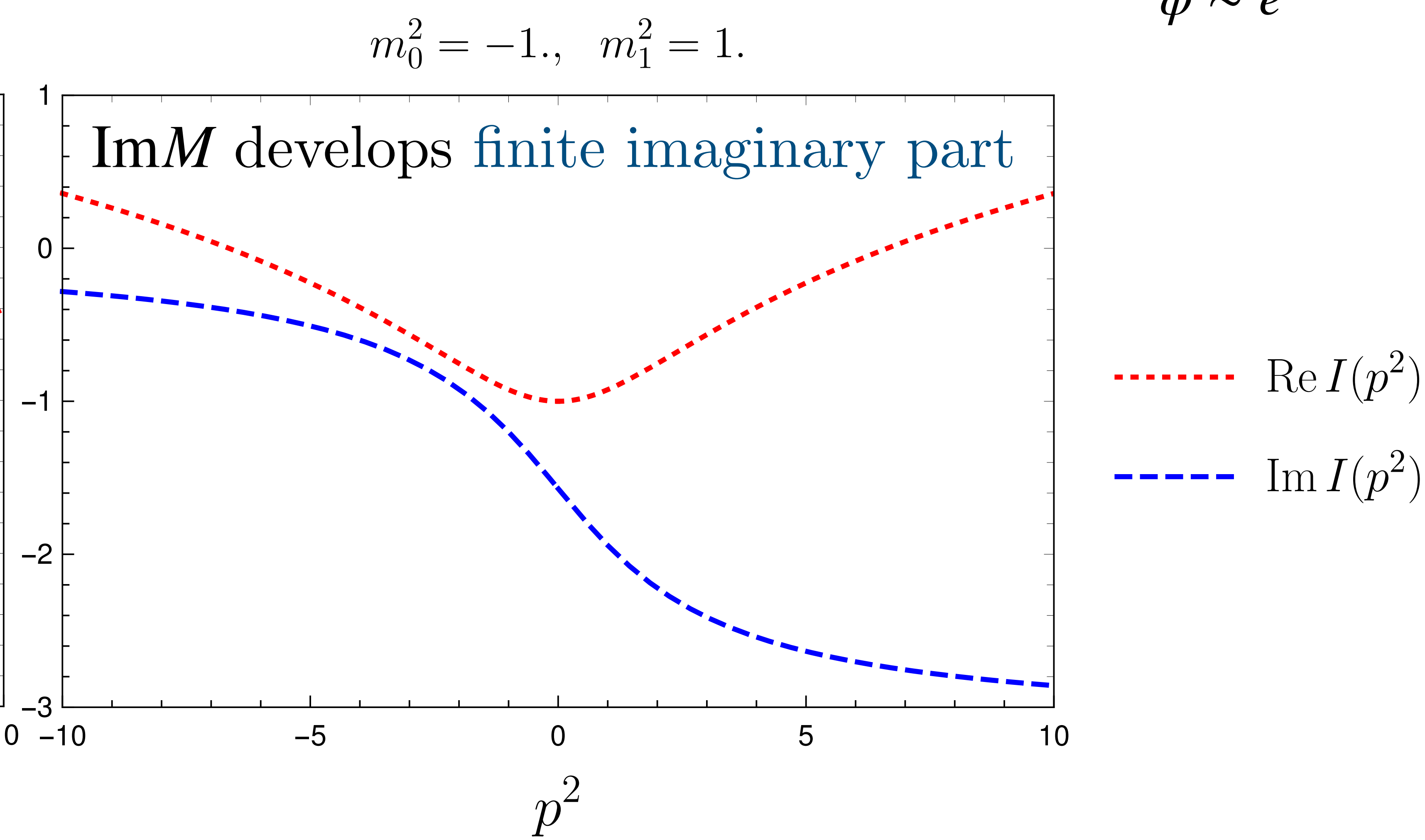
$$Im(M)=-\pi\int_0^1 dx\,\theta\left[x(1-x)p^2-(1-x)m_0^2-xm_1^2\right]$$

Unitarity violation

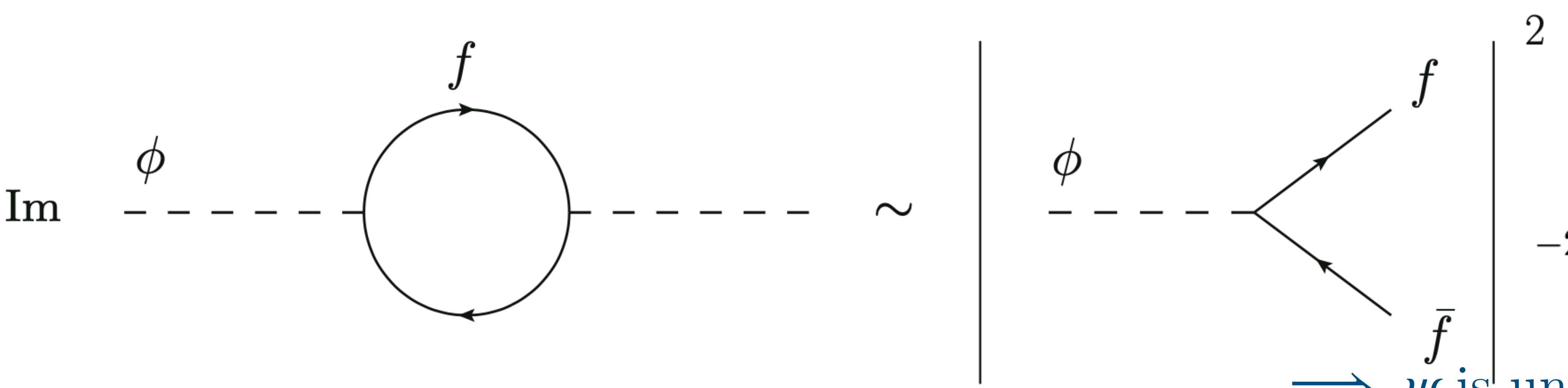
Propagator for *stable* $\frac{-i}{k^2 - m^2 + i\epsilon}$ and *unstable* $\frac{-i}{k^2 - m^2 + im\Gamma}$ particles → pole at $k^0 = \sqrt{m^2 - im\Gamma} \simeq m - i\Gamma/2$

$$\phi \sim e^{-\Gamma t/2}$$





$$\mathcal{S}^\dagger \mathcal{S} = (\mathcal{I} + i\mathcal{M}^\dagger)(\mathcal{I} - i\mathcal{M}) = \mathcal{I} - i\mathcal{M} + i\mathcal{M}^\dagger + \mathcal{M}^\dagger \mathcal{M} = \mathcal{I}.$$



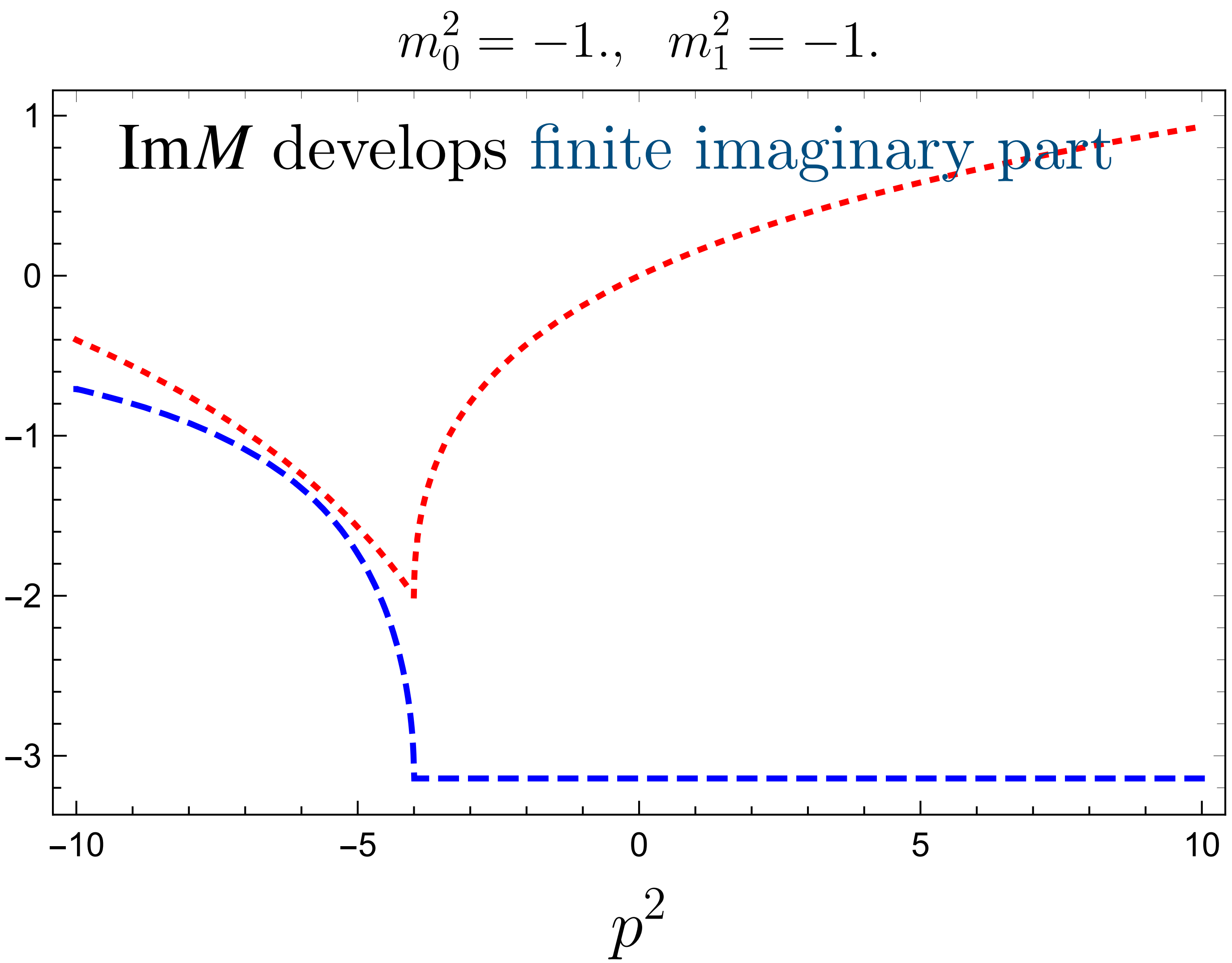
Optical theorem

$$i(\mathcal{M} - \mathcal{M}^\dagger) = \mathcal{M}^\dagger \mathcal{M}.$$

$$-2Im(\overline{\sum}\mathcal{M}_{i\to i}) = 2M_a \sum_n \Gamma(a \to n) = 2M_a\, \Gamma(a \to all)$$

$\Longrightarrow$ ψ is unstable due to the tachyon in the loop

Unitarity violation - self-interactions



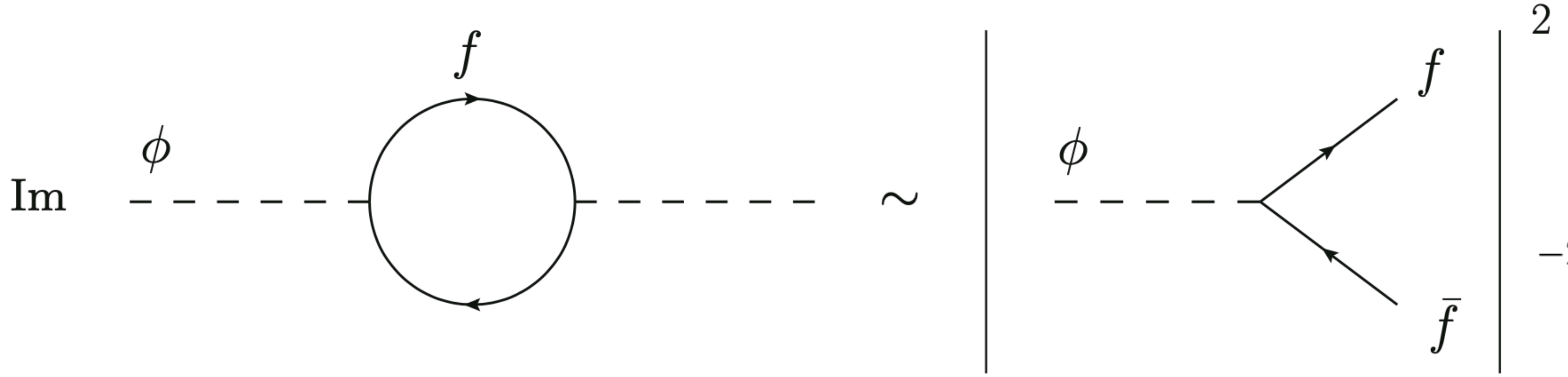
$$Im(M) = -\pi\left(\left(\sqrt{\frac{p^2+4}{p^2}}-1\right)(\theta(p^2+4)-1)+\theta(p^2+4)\right)$$

Re $I(p^2)$

Im $I(p^2)$

No threshold for $s = p^2$ for the LHS.
→ Tachyon self-interactions (cubic term) make it unstable

$$\mathcal{S}^\dagger\mathcal{S} = (\mathcal{I}+i\mathcal{M}^\dagger)(\mathcal{I}-i\mathcal{M}) = \mathcal{I}-i\mathcal{M}+i\mathcal{M}^\dagger+\mathcal{M}^\dagger\mathcal{M} = \mathcal{I}.$$

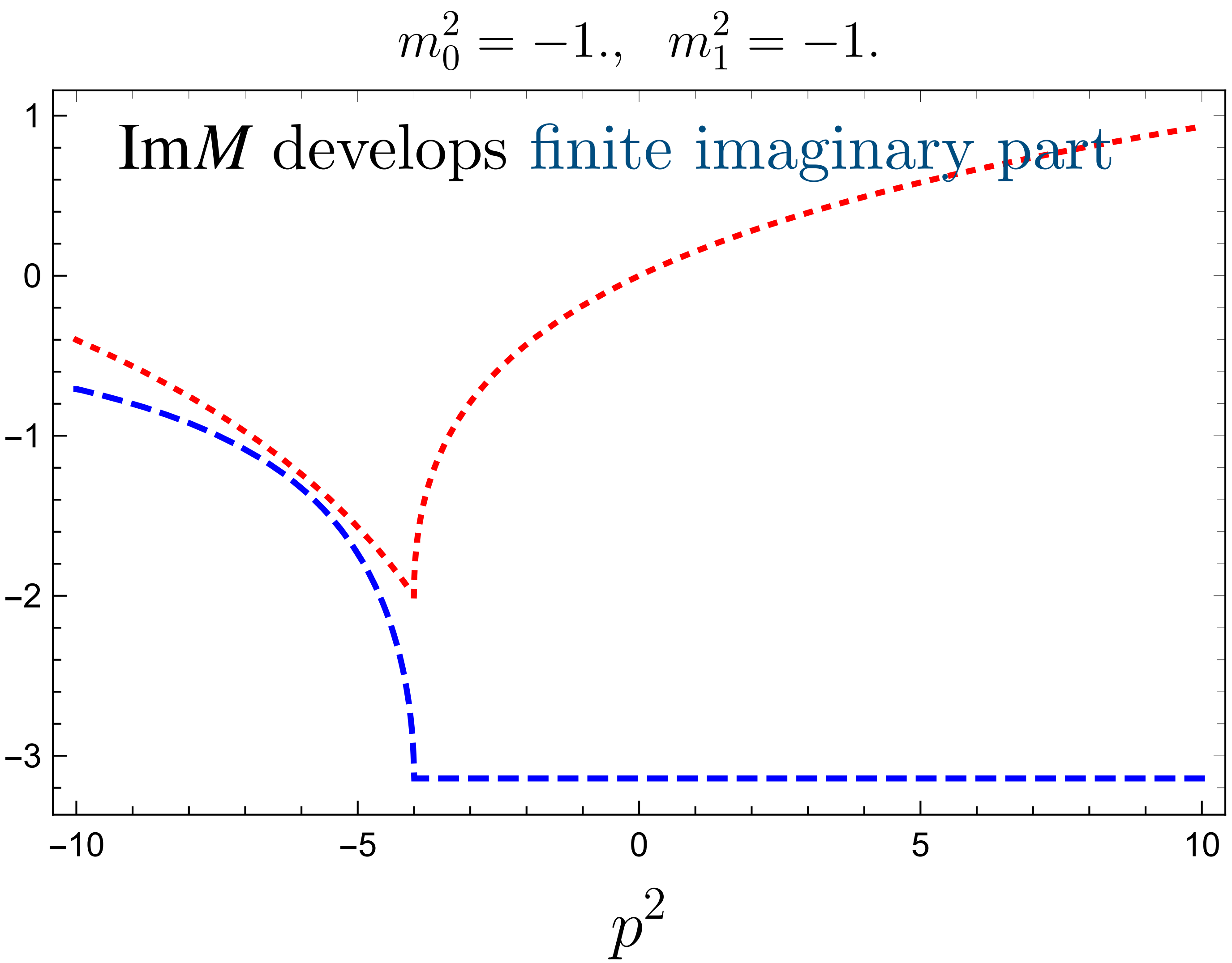


Optical theorem

$$i(\mathcal{M}-\mathcal{M}^\dagger) = \mathcal{M}^\dagger\mathcal{M}.$$

$$-2Im(\overline{\sum}\mathcal{M}_{i\to i}) = 2M_a\sum_n\Gamma(a\to n) = 2M_a\,\Gamma(a\to all)$$

Unitarity violation - self-interactions & scattering



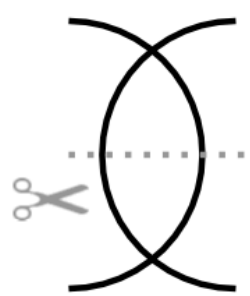
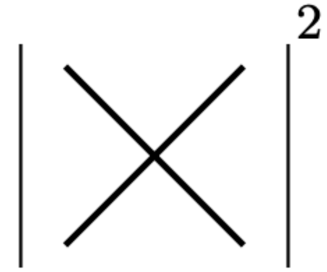
$$Im(M) = -\pi\left(\left(\sqrt{\frac{p^2+4}{p^2}}-1\right)(\theta(p^2+4)-1)+\theta(p^2+4)\right)$$

No threshold for $s = p^2$ for the LHS.
But then the RHS of optical theorem says that you can produce tachyons with imaginary energies in $2 \to 2$ scattering!

$$\mathcal{S}^\dagger\mathcal{S} = (\mathcal{I}+i\mathcal{M}^\dagger)(\mathcal{I}-i\mathcal{M}) = \mathcal{I} - i\mathcal{M} + i\mathcal{M}^\dagger + \mathcal{M}^\dagger\mathcal{M} = \mathcal{I}.$$

Optical theorem

$$i(\mathcal{M}-\mathcal{M}^\dagger) = \mathcal{M}^\dagger\mathcal{M}.$$

2 Im  $= \int d\Pi$  $= \int d\Pi\, |\mathcal{M}_4|^2$

$$-2\,\mathrm{Im}(\mathcal{M}_{i\to i}) = 2\lambda^{1/2}(s, m_a^2, m_b^2)\sum_n \sigma(a+b\to n)$$
$$= 2\lambda^{1/2}(s, m_a^2, m_b^2)\sigma(a+b\to all),$$

Tachyons and the SM?

$$\Gamma = \frac{g^2}{16\pi|\boldsymbol{k}|}\left(1 - \frac{m^2}{4|\boldsymbol{k}|^2}\right) \qquad (12$$

$$-2Im(\overline{\sum}\mathcal{M}_{i\to i}) = 2M_a \sum_n \Gamma(a \to n) = 2M_a\,\Gamma(a \to all)$$

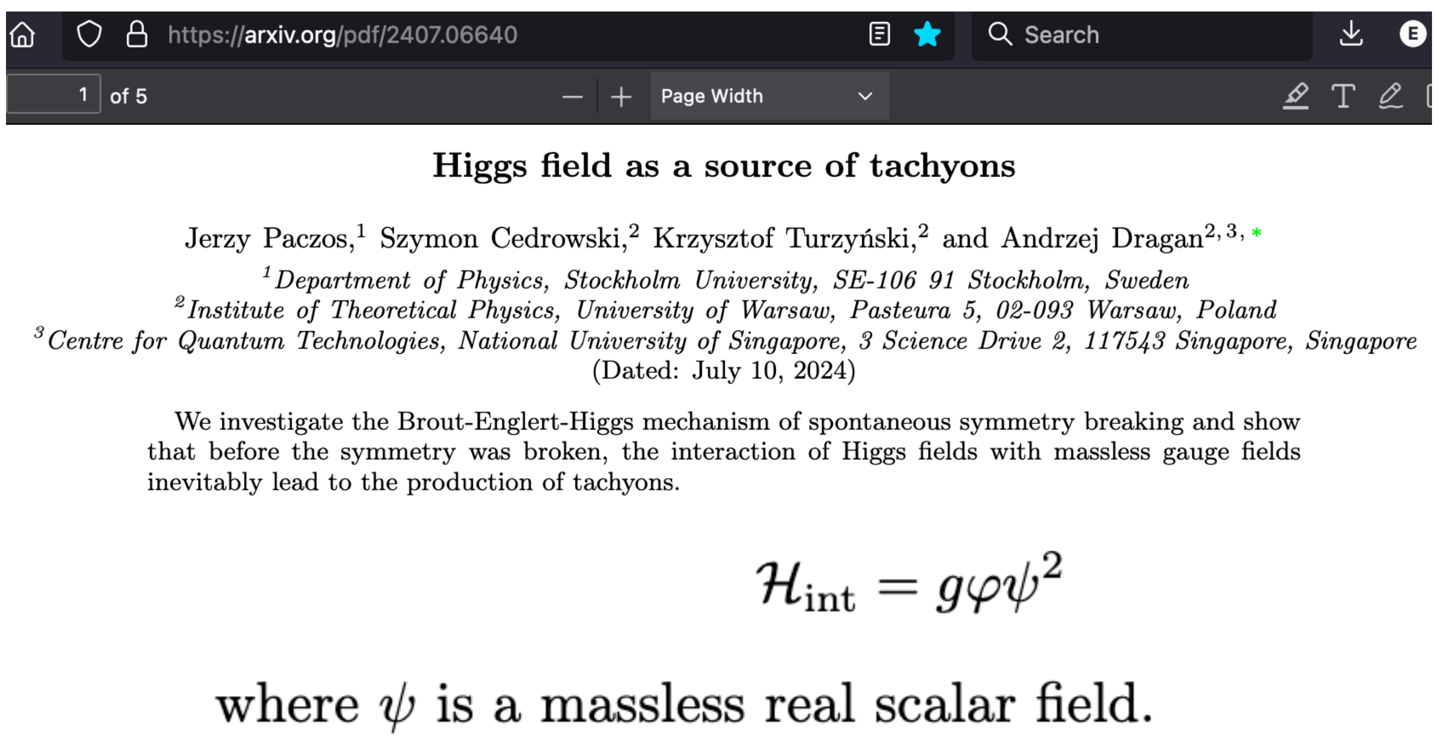
Paczos et al. 2407.06640

for $0 < m/|\boldsymbol{k}| < 2$, and vanishes otherwise. Therefore, the emission of a tachyon with mass m is possible only if the initial massless particle has energy larger than $m/2$.

Is optical theorem satisfied? - definitely not for the DS Feynman propagator.

Specifying FP is necessary to check unitarity.

$$Im(M) = \frac{\pi}{2p_2}\left[\left(-\sqrt{\frac{4}{p^4}+1}p^2 + p^2 - 2\right)\theta(p^2) + \theta(-p^2)\left(p^2(-\theta(p^2)) + \sqrt{\frac{4}{p^4}+1}p^2 + p^2 - 2\right) - 2p^2 + 4\right]$$


https://arxiv.org/pdf/2407.06640
Search
1 of 5
Page Width

Higgs field as a source of tachyons

Jerzy Paczos,[1] Szymon Cedrowski,[2] Krzysztof Turzyński,[2] and Andrzej Dragan[2,3,*]

[1] Department of Physics, Stockholm University, SE-106 91 Stockholm, Sweden
[2] Institute of Theoretical Physics, University of Warsaw, Pasteura 5, 02-093 Warsaw, Poland
[3] Centre for Quantum Technologies, National University of Singapore, 3 Science Drive 2, 117543 Singapore, Singapore

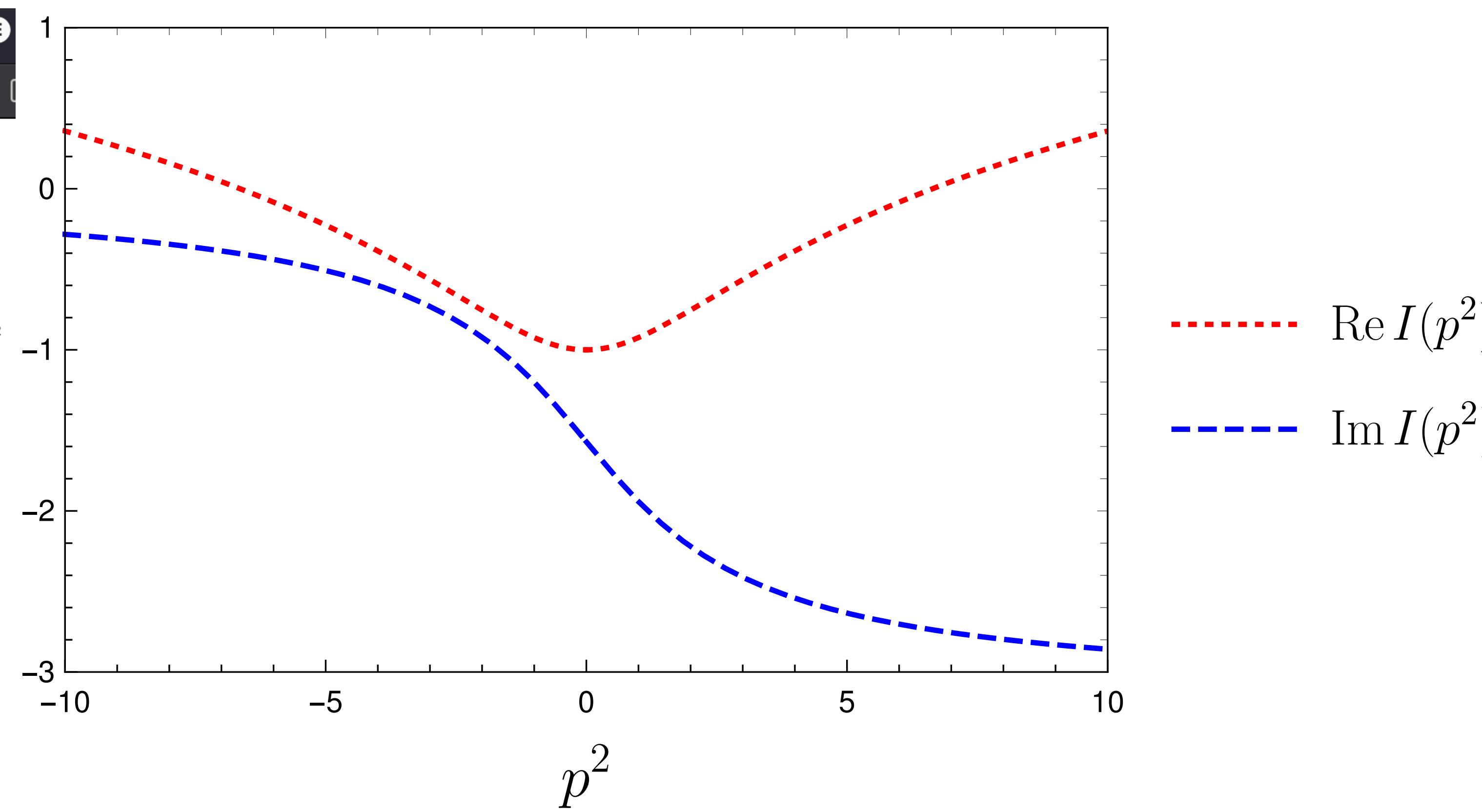
(Dated: July 10, 2024)

We investigate the Brout-Englert-Higgs mechanism of spontaneous symmetry breaking and show that before the symmetry was broken, the interaction of Higgs fields with massless gauge fields inevitably lead to the production of tachyons.

$$\mathcal{H}_{\text{int}} = g\varphi\psi^2$$

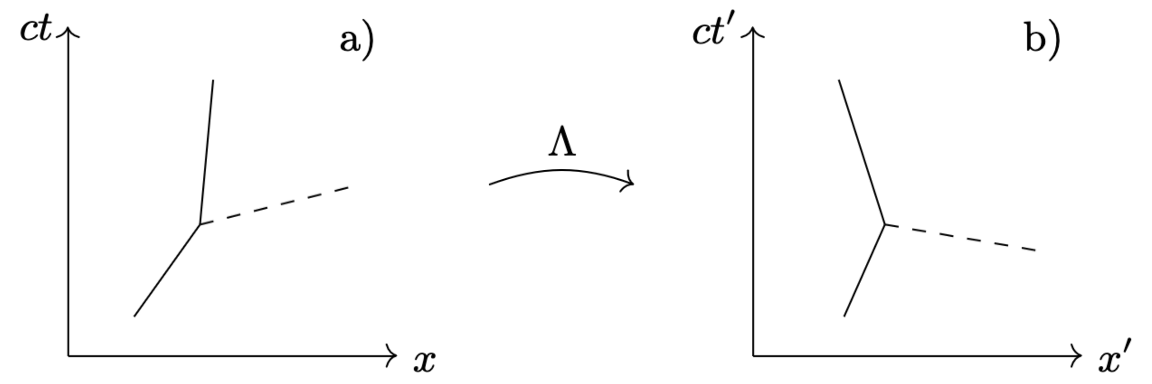
where ψ is a massless real scalar field.

$m_0^2 = -1., \quad m_1^2 = 1.$



Reinterpretation principle

- iii) Is the reaction rate for the process of tachyon emission/absorption shown in Fig. 1 of Ref. [1] well-defined?



Bilaniuk, Deshpande, Sudarshan Am. J. Phys. 30, 718 (1962)

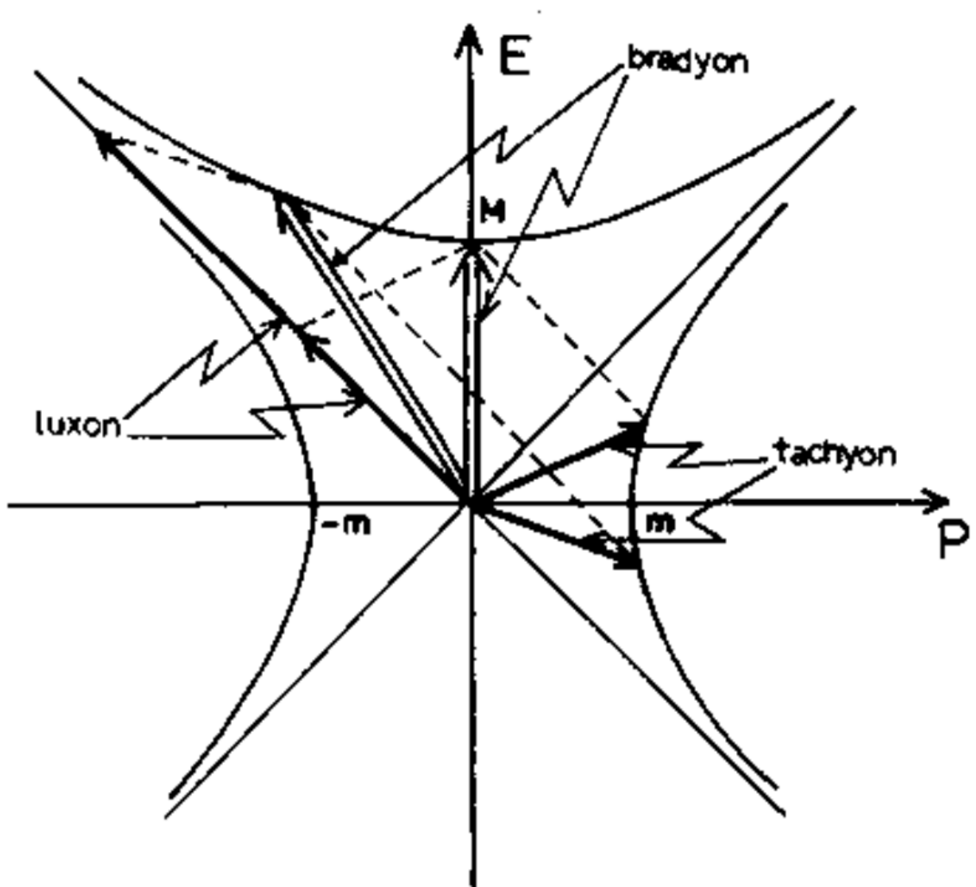
- *Reinterpretation principle*: tachyon with $E < 0$ is antitachyon with $E > 0$
    - $1 \to 2$ decay involving final state tachyon with $E < 0$ is actually $2 \to 1$ inverse-decay involving antitachyon with $E > 0$ in the initial state
    - $1 \to 2$ decay with scalar Yukawa: the S matrix element is Lorentz covariant

$$iM \times \delta^{(4)}(P_i - P_f) = -ig(2\pi)^4\delta^{(4)}(k - l - p) \to -ig(2\pi)^4\delta^{(4)}(k' - l' + p') \equiv -ig(2\pi)^4\delta^{(4)}(\Lambda(k - l - p)),$$

This S matrix element lacks **phase space** of the **final** states to be an <u>observable</u>.
Moreover, there is no Lorentz covariant operator, which transforms Γ to σ → <u>RP is not covariant</u>.

Restriction to only the positive energy states changes decay to scattering in non-covariant way. To restore it, one could average over the initial tachyon states as well - but this is a non-determinism, which is different than in ordinary QM, where we can prepare the initial states.

Reinterpretation principle



Already noted in Mrówczyński 1983

Fig.4. The decay of bradyon with mass M into luson and tachyon with mass m in the rest frame of bradyon (CM of products) and in the other moving frame. We see that the energy of tachyon in the second frame is negative.

positive in one frame and negative in another one. So in one frame such a configuration is taken into consideration while in the other one it is not.

This S matrix element lacks **phase space** of the **final** states to be an observable.
Moreover, there is no Lorentz covariant operator, which transforms Γ to σ → RP is not covariant.

Restriction to only the positive energy states changes decay to scattering in non-covariant way. To restore it, one could average over the initial tachyon states as well - but this is a non-determinism, which is different than in ordinary QM, where we can prepare the initial states.

Scattering in QFT

Weinberg QFT 1

In/out states via interaction picture

$|\psi_{in}\rangle$ $\qquad$ $\boldsymbol{X}$ $\qquad$ $|\psi_{out}\rangle$

Free evolution $\qquad$ Scattering $\qquad$ Free evolution

Hamiltonian is approximately free at early and late times.
Connect an interacting field to the free one by unitary transformation (like in QM).

$$|\psi\rangle = \Omega_- |\psi_{in}\rangle = \Omega_+ |\psi_{out}\rangle$$

Moller operators

$$\Omega_\pm = \lim_{t\to\pm\infty} e^{iHt} e^{-iH_0 t}$$

$S = \Omega_+^\dagger \Omega_-$ S matrix

States evolve with H_0, operators - with $V = H - H_0$
Strong limit (operator norm).

Haag's theorem Haag (1955), Hall, Whightman (1957)
for infinite number of generators, the canonical commutation relations do not have a unique (up to isomorphism) irreducible unitary representation.

→ Moller operators are not unitary

Two ways out: 1) Does Haag theorem apply to regularized theories? 2) Don't split H, construct exact eigenstates using interpolating field $\hat{\phi}$, which has some overlap with 1PS (Haag-Ruelle, LSZ).

Computing the S matrix

- The S matrix properties
  - Lorentz invariance
  - unitarity
  - cluster decomposition property

- i) via Dyson series from QM

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{U}(t,t_0) = H(t)\,\mathcal{U}(t,t_0)$$

$$\mathcal{U}(t,t_0)|_{t\to t_0} = \mathbb{1} \qquad (1)$$

$$\mathcal{U}(t,t_0) = \mathrm{T}\left\{\exp\Big(-\mathrm{i}\int_{t_0}^{t}\mathrm{d}t' H(t')\Big)\right\} = \sum_{n=0}^{\infty}\frac{(-\mathrm{i})^n}{n!}\int_{t_0}^{t}\mathrm{d}t_1 \ldots \int_{t_0}^{t}\mathrm{d}t_n\,\mathrm{T}\left\{H(t_1)\cdots H(t_n)\right\}$$

Time ordering is not-well defined for space-like 4-vectors unless fields commute

Drop it? Then DS does not solve (1)

Microcausality

For *subliminal* particles, S matrix is LI since $[\phi(x), \phi(y)] = 0$ for $(x-y)^2 < 0$ and the same holds after replacing $\phi \to \phi^\dagger$, $\phi \to \partial_\mu \phi \implies$ any local operator $O(x)$ preserves causality. In particular, $[H(x), H(y)] = 0$ and the S matrix defined by the Dyson series is LI.

$$U(\Lambda)\,H(x_1)\ldots H(x_n)\,U(\Lambda)^{-1} = H(\Lambda(x_1))\ldots H(\Lambda(x_n)) = TH(\Lambda(x_1))\ldots H(\Lambda(x_n))$$

WLOG we can assume the first term is already TO. The first equation assumes H is covariant, while the second uses microcausality. This establishes each term in DS is LI (note d^4x is LI).

Dyson series is not LI for tachyons which violate micro causality.

Feinberg, Phys. Rev. 159, 1089 (1967)
Dhar and Sudarshan 1968

Computing the S matrix

- via LSZ formalism

  Assume there is a vacuum state $|0\rangle$ and an interacting tachyon field $\phi'(x)$ that satisfies the following conditions:

  1. $\langle 0|\phi'(x)|0\rangle = 0$
  2. $\langle k|\phi'(x)|0\rangle = e^{ikx}\,\theta(|\vec{k}| - m)$
  3. For any <u>normalizable</u> states $|\alpha\rangle, |\beta\rangle$: $\lim_{t\to\pm\infty} \langle\alpha|\phi'(x)|\beta\rangle = \sqrt{Z}\,\langle\alpha|\phi_{\text{free}}(x)|\beta\rangle$

  LSZ asymptotic condition - in/out states

Spectral representation for interacting field shows that it creates not only one-particle states, but also a continuum of multi-particle states.

$$\int dx\, e^{ip(x-y)}\langle\Omega|T\phi(x)\phi(y)|\Omega\rangle = \frac{iZ}{p^2 - m^2 + i\epsilon} + \int_{m^2_{\text{bound}}}^{\infty} đ(M^2)\,\rho(M^2)\frac{i}{p^2 - M^2 + i\epsilon}.$$

This means the field $\phi'(x)$ "interpolates" between in/out states.
Also note that (3) is limit in the weak sense (contrary to Moller operators).
In particular, for nontrivial interactions $Z \neq 1$ and
$\lim_{t\to\pm\infty} \langle\alpha|[\phi'(x), \phi'(y)]|\beta\rangle \neq \langle\alpha|[\phi_{\text{free}}(x), \phi_{\text{free}}(y)]|\beta\rangle$.

Computing the S matrix

- via LSZ formalism

Assume there is a vacuum state $|0\rangle$ and an interacting tachyon field $\phi'(x)$ that satisfies the following conditions:

1. $\langle 0|\phi'(x)|0\rangle = 0$
2. $\langle k|\phi'(x)|0\rangle = e^{ikx}\,\theta(|\vec{k}| - m)$ LSZ asymptotic condition - in/out states
3. For any normalizable states $|\alpha\rangle, |\beta\rangle$: $\lim_{t\to\pm\infty} \langle \alpha|\phi'(x)|\beta\rangle = \sqrt{Z}\,\langle \alpha|\phi_{\text{free}}(x)|\beta\rangle$

Then, one can compute the S matrix elements via the LSZ reduction formula

$$\prod_k \int dy_k\, e^{ip_k y_k} \prod_\ell \int dx_\ell\, e^{-iq_l x_l} \langle\Omega|T\phi(y_1)\ldots\phi(x_1)\ldots|\Omega\rangle$$

$$= \prod_k \frac{i\sqrt{Z}}{p_k^2 - m^2} \prod_\ell \frac{i\sqrt{Z}}{q_\ell^2 - m^2} \langle p_1,\ldots|S|q_1,\ldots\rangle\Big|_{\text{connected}}$$

i) *Time-ordered correlator functions* have a corresponding pole structure, with a Feynman propagator factor giving poles for on-shell particles.

ii) Field $\phi_{\text{free}}(x)$ always creates or destroy a particle near a point, while $\underline{\phi'(x)}$ creates <u>both</u> single-particle and multiparticle states. Due to (3), we can project out the single-particle states by looking at the residues of the appropriate poles.

LSZ-like S matrix

Plane waves are non-localized, non-regular solution of KG. Consider wave packet $\tilde{f}(p) \equiv F(p)$ with compact support.

$$\hat{\phi}(t, \vec{x}) = \int_{|\vec{k}|>m} \frac{d^3\vec{k}}{(2\pi)^3\, 2\omega_{\vec{k}}} \tilde{f}(k)\, e^{-i\omega_{\vec{k}} t + i\vec{k}\cdot\vec{x}},$$

For any states $|\alpha\rangle, |\beta\rangle$: $\lim_{t\to\pm\infty} \langle\alpha|\phi'(x)|\beta\rangle = \sqrt{Z}\,\langle\alpha|\phi_{\text{free}}(x)|\beta\rangle$

This holds if the following limit vanishes

(overlap between the vacuum, interpolating field, and multi particle states asymptotically vanishes - localization)

$$\lim_{t\to\infty} \int \frac{d^3\overrightarrow{k}}{(2\pi)^3} \langle\psi|n\rangle \frac{F(\vec{P}_n)(\omega_{\vec{P}_n} + P_n^0)}{2\omega_{\vec{P}_n}} \langle n|\phi'(0)|0\rangle e^{-i(\omega_{\vec{P}_n} - P_n^0)t}$$

Coleman, QFT lectures

Note that $1/\omega_{\overrightarrow{k}} = 1/\sqrt{|\overrightarrow{k}|^2 - m^2}$ is not analytical, not bounded, not integrable.

$\tilde{f}/\sqrt{\omega_{\vec{k}}} \in L_2 \nRightarrow \tilde{f}/\omega_{\vec{k}} \in L_1$ and $\tilde{f} \in L_1 \nRightarrow \tilde{f}/\omega_{\vec{k}} \in L_1$

We need the latter to use the Riemann-Lebesgue lemma.

Standard proof of LSZ asymptotic condition does not work.

$\rightarrow$ just assume it, then the LSZ-like formalism may be possible.

Conclusions

- Although tachyons are well-motivated, their quantum description is problematic.
    - the CCRs are not satisfied if covariance is satisfied or quantum tachyon field is not covariant; in both cases, off-shell two-point TOCF is not LI
    - Sudarshan-Dhar FP violates unitarity
    - LSZ condition cannot be proved by replacing plane waves with wavepackets

- The Sudarshan's *reinterpretation principle* is either non-covariant or it means that one cannot prepare the initial tachyon states - different non-determinism than in QM.

- Our results confirm that tachyons in Minkowski space are instabilities, instead of particle-like objects that can be *physical* superluminal observers.

> If tachyons exist they are probably neutral. If they do exist we ought to find them. If we do not find them we ought to be able to find out why they could not exist. So far we have found no reason why they could not exist.
>
> Dhar and Sudarshan 1968

Dziękuję!

Thank you!

감사합니다

Are tachyons just a curiosity?

Dragon, Ekert “Reply to the comment on "Quantum principle of relativity"” 2309.00020

Horodecki’s main concern questions whether extended special relativity alone can be used to deduce quantum theory in its entirety, or if the proposed "quantum principle of relativity" remains incomplete in this regard. Our short answer is that we don’t know yet. While we’ve

Moreover, fields are the fundamental dof., while QM is formally a QFT in 0+1 dimensions. Comprehensive analysis of the latter (1+3) for tachyons has not been done.

Dragan and Ekert admitted that *completeness of QPR* has not been established so far, and that the reasoning leading to probability amplitudes from relativity only employed subluminal observers.

Finally, Horodecki raises a crucial and well-founded critique: does our proposal yield any measurable and potentially observable effects? The straightforward answer is: if tachyons existed, then this would undoubtedly be the case. It is worth noting that we have recently demonstrated that a covariant quantum field theory of tachyons with a positive-definite spectrum and a stable, invariant vacuum can be constructed [7]. In this study, as well as in our previous work [6], we emphasize that the Higgs mechanism incorporates tachyonic fields. As a result, our ongoing research opens the way to study fully quantized theory of spontaneous symmetry breaking. This leaves us with a hope that the answer to the last question posed by Horodecki is affirmative.

→ At present, QPR cannot be considered as a full-fledged physical principle.

→ The potential connection to the Higgs mechanism also motivates *our* work (permanently closing on tachyon QFT).

Are tachyons just a curiosity?

Horodecki references Heisenberg's classification of possible universes based on the values of physical constants $\frac{1}{c}$ and $\hbar$. Specifically, he considers the scenario where $\frac{1}{c} = 0$ and $\hbar \neq 0$, representing a quantum but not relativistic model of reality, arguing that the universe could be quantum, without being relativistic. However, taking a low-energy limit of Dirac's theory and arriving at the approximate Pauli equation does not mean that the resulting theory is truly non-relativistic. In our work we have argued that the reason we have to consider probabilistic description involving superpositions is due to relativity. At this stage it is secondary, whether the dynamical equation is strict, or only approximate. It is best illustrated by the fact that the "non-relativistic" Pauli theory still involves spin with the gyromagnetic factor $g = 2$ which is truly relativistic. It is also in principle possible to imagine a universe, in which the speed of light is infinite. However this does not invalidate our claims, that quantum effects are a consequence of relativity, either. To show that, let us draw an analogy. Elliptical

Dragon, Ekert "Reply to the comment on "Quantum principle of relativity"" 2309.0002

Pauli equation is *Galilean invariant* and *predicts* $g = 2$.

Lévy-Leblond '67 "Nonrelativistic particles and wave equations"

Commun. math. Phys. 6, 286—311 (1967)

Nonrelativistic Particles and Wave Equations

JEAN-MARC LÉVY-LEBLOND*
Laboratoire de Physique Théorique, Faculté des Sciences de Nice

Received April 1, 1967

Abstract. This paper is devoted to a detailed study of nonrelativistic particles and their properties, as described by Galilei invariant wave equations, in order to obtain a precise distinction between the specifically relativistic properties of elementary quantum mechanical systems and those which are also shared by nonrelativistic systems. After having emphasized that spin, for instance, is not such a specifically relativistic effect, we construct wave equations for nonrelativistic particles with any spin. Our derivation is based upon the theory of representations of the Galilei group, which define nonrelativistic particles. We particularly study the spin 1/2 case where we introduce a four-component wave equation, the nonrelativistic analogue of the Dirac equation. It leads to the conclusion that the spin magnetic moment, with its Landé factor $g = 2$, is not a relativistic property. More generally, nonrelativistic particles seem to possess intrinsic moments with the same values as their relativistic counterparts, but are found to possess no higher electromagnetic multipole moments. Studying "galilean electromagnetism" (i.e. the theory of spin 1 massless particles), we show that only the displacement current is responsible for the breakdown of galilean invariance in Maxwell equations, and we make some comments about such a "nonrelativistic electromagnetism". Comparing the connection between wave equations and the invariance group in both the relativistic and the nonrelativistic case, we are finally led to some vexing questions about the very concept of wave equations.

Are tachyons local objects?

$$\hat{\Phi}(x) = \int_{\left|\overrightarrow{k}\right|>m} \frac{d^3\overrightarrow{k}}{(2\pi)^3\, 2\omega_{\overrightarrow{k}}} \left(e^{-ik(x-y)}a_k + e^{ik(x-y)}a_k^\dagger\right)$$

Can one send information superluminally using tachyons?

Feinberg claimed it is not possible, since the plane-wave basis does not contain the $|\overrightarrow{k}| \leq m$ modes, hence, it is incomplete. We give a rigorous proof instead.

Because of the restriction of the wave numbers given by (3.3), the set of functions $\phi_{+,k}{}^*(\mathbf{x}, t=0) = \phi_{-,k}(\mathbf{x}, t=0)$ does not form a complete set. Instead of the usual

The incompleteness of the allowed set of solutions has several consequences.

1. Tachyons cannot be localized in space, i.e., a superposition of solutions of the form

$$\psi(x) = \int \phi_{+,k}(x) f(k) d^3k\,, \quad (|k| \geq \mu)\,,$$

which could be a tachyon wave function, cannot be made into $\delta^3(x)$. In fact, such a superposition cannot be made to vanish outside a sphere of finite radius, but

Feinberg, Phys. Rev. 159, 1089 (1967)

[22–24]. Such theory involves field operators $\hat{\phi}$ that do not commute outside of the light cones [22], but it is not immediately clear whether this leads to the possibility of superluminal signaling. This is because the mode decomposition is incomplete and it is not possible, even in principle, to construct compactly supported wavepackets that could be used for such "signaling".

Paczos et al. (Phys.Rev.D 110 (2024) 1, 015006)

Are tachyons local objects?

$$\hat{\Phi}(x) = \int_{\left|\vec{k}\right|>m} \frac{d^3\vec{k}}{(2\pi)^3\, 2\omega_{\vec{k}}} \left(e^{-ik(x-y)} a_k + e^{ik(x-y)} a_k^\dagger \right)$$

Actually, this is a statement about KG solutions in position space - and not about tachyons - since the **Paley-Wiener theorem** <u>also</u> applies to a **subliminal** scalar. Indeed, the FT of any regular solution to the KG satisfies

2.1.1 Special solutions and Green's functions

Every function ζ that solves the (homogeneous) Klein-Gordon equation

$$(-\Box + m^2)\zeta(x) = 0 \tag{2.4}$$

can be written as

$$\begin{aligned} \zeta(x) &= \int \mathrm{e}^{\mathrm{i}kx} g(k) \delta(k^2 + m^2) \frac{\mathrm{d}k}{(2\pi)^3} \\ &= \sum_{\pm} \int \frac{\mathrm{d}\vec{k}}{(2\pi)^3 2\sqrt{\vec{k}^2 + m^2}} g\Big(\pm\sqrt{\vec{k}^2 + m^2}, \vec{k}\Big) \mathrm{e}^{\mp\mathrm{i}x^0\sqrt{\vec{k}^2+m^2}+\mathrm{i}\vec{x}\vec{k}}, \end{aligned}$$

where g is a function on the two-sheeted hyperboloid $k^2 + m^2 = 0$. A special role is played by the following *3 special solutions* of the homogeneous Klein-Gordon equation.

Dereziński Lect.Notes Phys. 695:63-143, 2006

For both subluminal/superluminal scalar, solution to the KG field cannot be analytical since it has a singularity at complex/real momentum $\vec{k}$, st. $\vec{k} \cdot \vec{k} = \mp m^2$. For superluminal scalar, we excluded this pole, but then different problem remains: analytical function cannot vanish on a compact set.

Let's focus on four-dimensional Minkowski space-time, $\mathbb{M}$. In Cartesian coordinates $x^a = (x^0, x^1, x^2, x^3)$, the metric is simply $\eta_{ab} = \mathrm{diag}(1, -1, -1, -1)$. Complexified Minkowski space, $\mathbb{M}_{\mathbb{C}}$, is then just $\mathbb{C}^4$, equipped with the metric η_{ab}. The line element

$$\mathrm{d}s^2 = \eta_{ab}\,\mathrm{d}x^a\,\mathrm{d}x^b = (\mathrm{d}x^0)^2 - (\mathrm{d}x^1)^2 - (\mathrm{d}x^2)^2 - (\mathrm{d}x^3)^2\,, \tag{1.1}$$

looks the same as in real Minkowski space, with the exception that the coordinates are now allowed to take *complex* values.

Reinterpretation principle

- The Lorentz invariant phase space for N *final* state particles with $E > 0$ is

$$dQ_N = \delta^{(4)}(P_i - P_f) \times \frac{1}{(2\pi)^{3N-4}} \prod_{i=1}^{N} \frac{d^3 p_i}{2\sqrt{p_i^2 + m_i^2}}$$

- $1 \to 2$ decay: decay width (which is defined in the rest frame and is not LI)

$$\Gamma(a \to 1 + 2 + \cdots + N) = \frac{1}{2\, m_a} \frac{1}{(2 j_a + 1)} \sum_{\lambda_a, \lambda_1, \ldots, \lambda_N} \int d Q_N \left| \mathcal{M}(a \to 1 + 2 + \cdots + N) \right|^2$$

- $2 \to 1$ inverse-decay (which is LI)

$$\sigma(a + b \to 1 + \cdots + N) = \frac{1}{2\, \lambda^{1/2}(s, m_a^2, m_b^2)} \times \frac{1}{(2 j_a + 1)(2 j_b + 1)} \sum_{\lambda_a, \lambda_b, \lambda_1, \ldots, \lambda_N} \int d Q_N \left| \mathcal{M}(a + b \to 1 + \cdots + N) \right|^2$$

- To make $1 \to 2$ decay rate LI, we need to average over the initial state too - additional integral over the phase space of the initial particle
  - Non-example: thermal relic freeze-out from plasma in the early Universe due to decays and inverse decays

$$\frac{dn_\chi}{dt} + 3Hn_\chi$$

$1/\Gamma$ behaves as time-component of a 4-vector

$$= g_A \int \frac{d^3 p_A}{(2\pi)^3 2E_A} f_A \int \int \frac{d^3 p_1}{(2\pi)^3 2E_1} \frac{d^3 p_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(p_A - p_1 - p_2) |\mathcal{M}|^2$$

σ is scalar since it gives the number of interactions

$$= g_A \int \frac{d^3 p_A}{(2\pi)^3} f_A \frac{M_A}{E_A} \Gamma_A = g_A n_A \langle \Gamma_A \rangle = S \frac{dY}{dt} = -HTs \frac{dY}{dT} = Hxs \frac{dY}{dx}$$

Can't prepare the initial state? This is a different non-determinism than in QM.

Tachyons and the SM?

$$\mathcal{H}_{\text{int}} = g\varphi\psi^2$$

Paczos et al. 2407.06640

where ψ is a massless real scalar field.

$$\Gamma = \frac{g^2}{16\pi|\boldsymbol{k}|}\left(1 - \frac{m^2}{4|\boldsymbol{k}|^2}\right) \tag{12}$$

for $0 < m/|\boldsymbol{k}| < 2$, and vanishes otherwise. Therefore, the emission of a tachyon with mass m is possible only if the initial massless particle has energy larger than $m/2$.

Decay rate

$$\hat{\varphi}(t,\boldsymbol{r}) = \int_{|\boldsymbol{k}|\geq m} \mathrm{d}^3\boldsymbol{k}\, \left(u_{\boldsymbol{k}}(t,\boldsymbol{r})\,\hat{a}_{\boldsymbol{k}} + u^*_{\boldsymbol{k}}(t,\boldsymbol{r})\,\hat{a}^\dagger_{\boldsymbol{k}}\right), \tag{5}$$

$$[\hat{a}_{\boldsymbol{k}}, \hat{a}^\dagger_{\boldsymbol{l}}] = 2\omega_k(2\pi)^3\delta^{(3)}(\boldsymbol{k}-\boldsymbol{l}). \tag{6}$$

$$u_{\boldsymbol{k}}(t,\boldsymbol{r}) = \frac{1}{(2\pi)^3 2\omega_k} e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\Omega_k t)}, \tag{7}$$

$$\hat{\psi}(x) = \int \mathrm{d}^3\boldsymbol{k}\, \left(v_{\boldsymbol{k}}(x)\hat{b}_{\boldsymbol{k}} + v^*_{\boldsymbol{k}}(x)\hat{b}^\dagger_{\boldsymbol{k}}\right). \tag{8}$$

$$[\hat{b}_{\boldsymbol{k}}, \hat{b}^\dagger_{\boldsymbol{l}}] = \delta(\boldsymbol{k}-\boldsymbol{l}) \tag{9}$$

$$v_{\boldsymbol{k}}(x) = \frac{1}{\sqrt{(2\pi)^3 2|\boldsymbol{k}|}} \mathrm{e}^{-i(|\boldsymbol{k}|t-\boldsymbol{k}\cdot\boldsymbol{x})}, \tag{10}$$

We analyze the emission process $\psi \to \psi\varphi$, where a massless particle emits a tachyon. The corresponding (appropriately normalized) initial and final states are given by

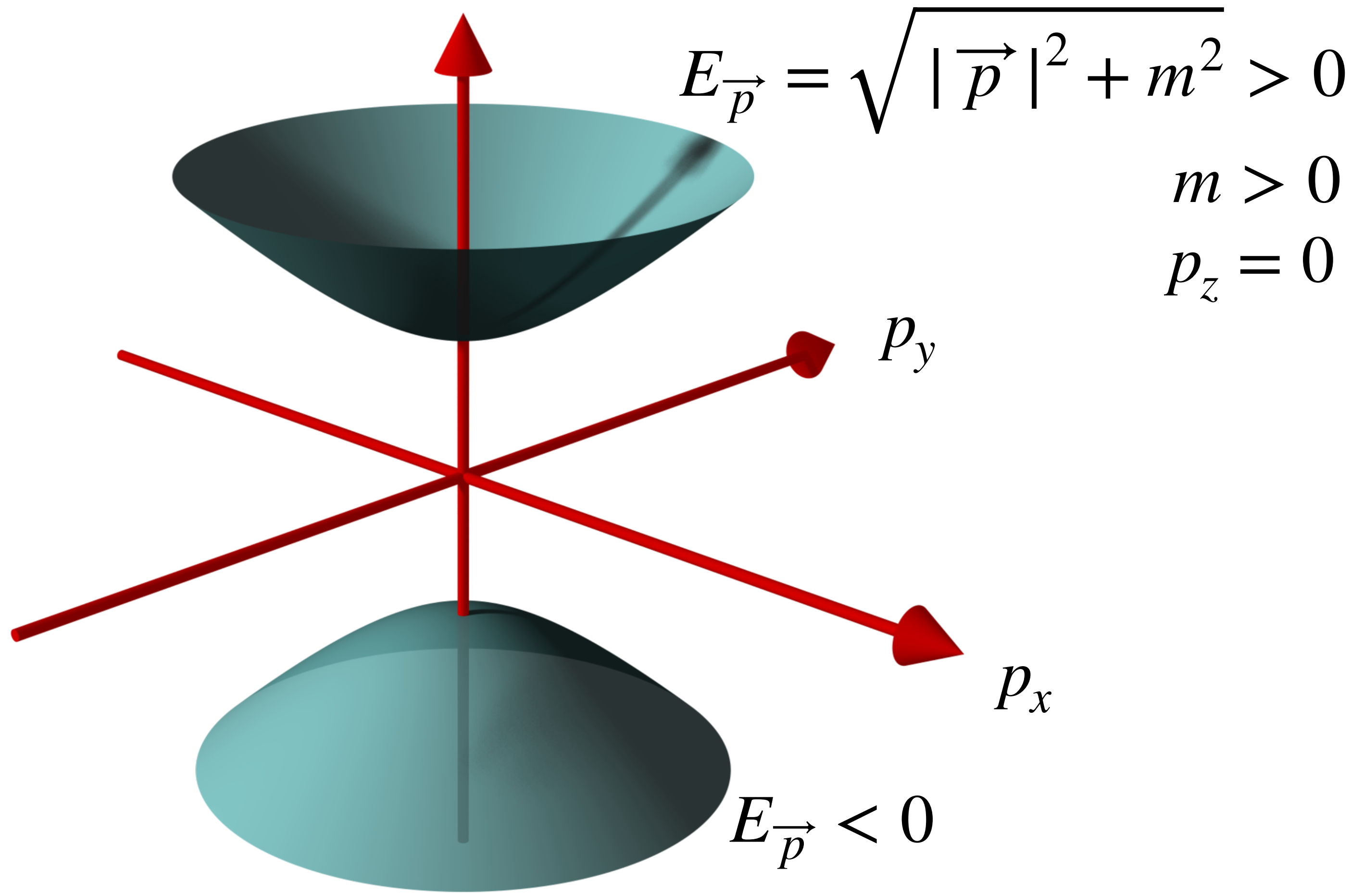
$$\begin{aligned} |i\rangle &= \sqrt{(2\pi)^3 2|\boldsymbol{k}|}\hat{b}^\dagger_{\boldsymbol{k}}\,|0\rangle\,, \\ |f\rangle &= \sqrt{(2\pi)^3 2\Omega_{\boldsymbol{l}}}\hat{a}^\dagger_{\boldsymbol{l}}\sqrt{(2\pi)^3 2|\boldsymbol{p}|}\hat{b}^\dagger_{\boldsymbol{p}}\,|0\rangle\,. \end{aligned} \tag{13}$$

$|f\rangle = \sqrt{(2\pi)^3 2\Omega_{\boldsymbol{l}}}\hat{a}^\dagger_{\boldsymbol{l}}\sqrt{(2\pi)^3 2|\boldsymbol{p}|}\hat{b}^\dagger_{\boldsymbol{p}}\,|0\rangle\,.$ $[\hat{a}_{\boldsymbol{l}}, \hat{\varphi}(x)] = u^*_{\boldsymbol{l}}(x),$ are incorrect but the final expression in Eq. 16 is correct

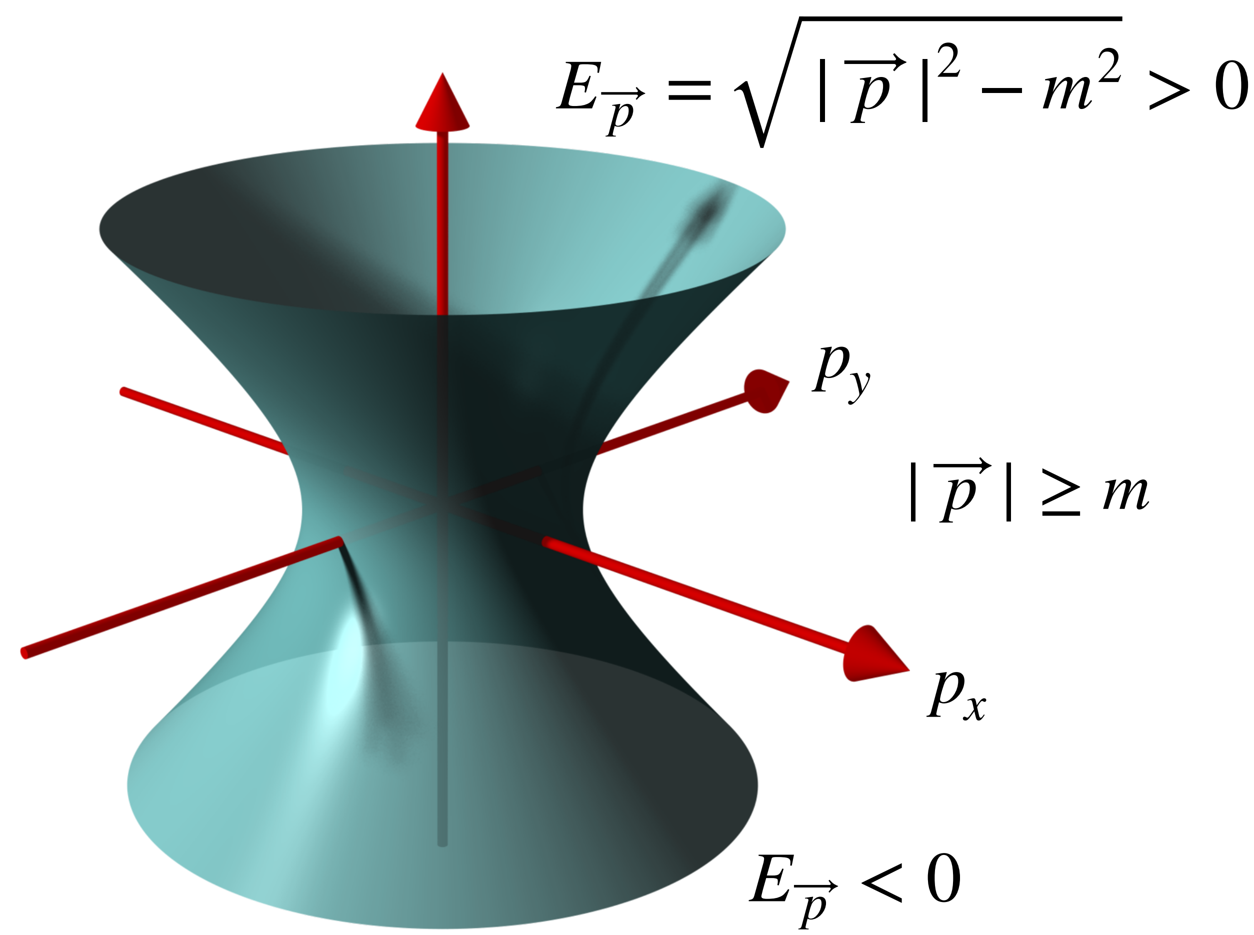
$$S_{fi} \approx -ig\sqrt{(2\pi)^9 8\Omega_{\boldsymbol{l}}|\boldsymbol{k}||\boldsymbol{p}|}\int \mathrm{d}^4x v^*_{\boldsymbol{p}}(x)u^*_{\boldsymbol{l}}(x)v_{\boldsymbol{k}}(x) = -ig(2\pi)^4\delta^{(4)}(p+l-k) \equiv (2\pi)^4\delta^{(4)}(k+l-p)(-i\mathcal{A}_{fi}), \tag{16}$$

Is optical theorem satisfied? $\Gamma = \frac{g^2}{16\pi|\boldsymbol{k}|}\left(1-\frac{m^2}{4|\boldsymbol{k}|^2}\right)\Theta\left(\frac{m}{|\boldsymbol{k}|}\right)\Theta\left(2-\frac{m}{|\boldsymbol{k}|}\right)$

Mass shell: $p^2 \mp m^2 = 0$



a) Two (disconnected) sheet hyperboloid



b) One sheet hyperboloid

For subliminal observers, boosts preserve each sheet, while for superluminal observers, there is only one sheet.

L preserves the scalar product:

$$\eta_{\mu\nu} L^{\mu}_{\ \lambda} L^{\nu}_{\ \lambda'} = \eta_{\lambda\lambda'}$$

$$(L^0_{\ 0})^2 = 1 + \sum_i (L^i_{\ 0})^2$$

$$= 1 + \sum_i (L^0_{\ i})^2$$

We show that $(Lx)^0 \geq 0$ for a boost L and a time-like x.

Sufficient to show that $|L^0_{\ i}\, x^i| \leq L^0_{\ 0}\, x^0$, since $(Lx)^0 = L^0_{\ 0}\, x^0 + L^0_{\ i}\, x^i$.

This holds since $(L^0_{\ 0} x^0)^2 \geq \sum_i (L^0_{\ i})^2 \times \sum_i (x^i)^2 \geq (L^0_{\ i}\, x^i)^2$.

We used $x^2 = (x^0)^2 - x^i x_i > 0$, and CS inequality.