Mohr circles in geophysics
Correction

October 23, 2014

We consider a fault block 5km thick which rests on a horizontal detachment as represented in figure 1. The block is subjected to gravity and a horizontal tensile stress and the friction on the bedrock. As we are in a geotechnic problem, we will use the geotechnic conventions for stresses (positive for compression and negative for tensile stress).

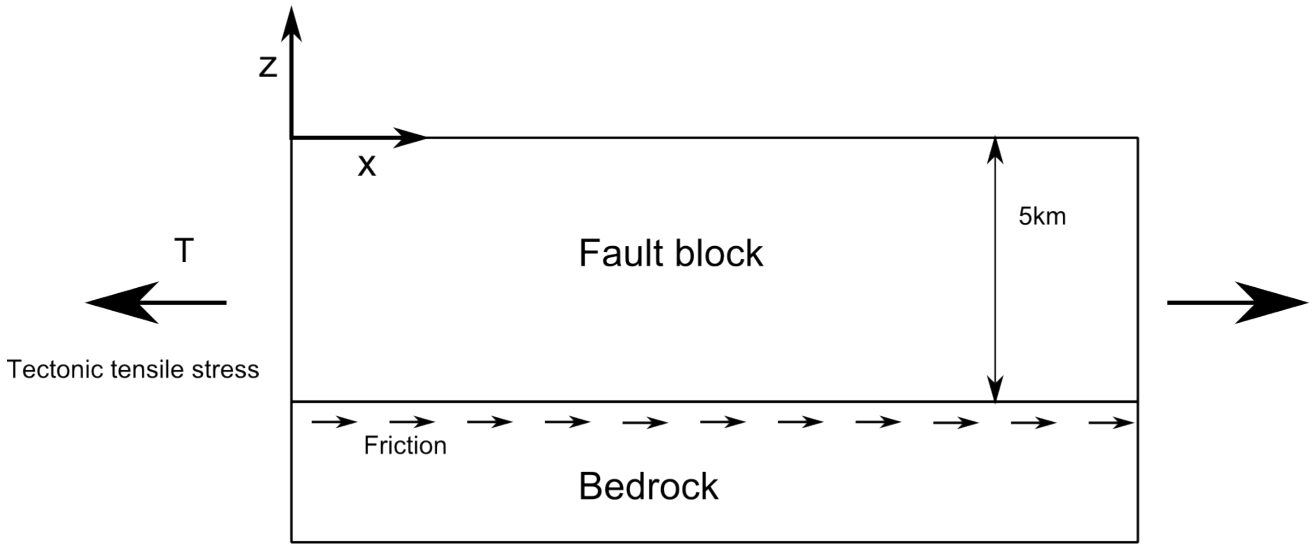


Figure 1: Model of the fault block and the different solicitations

The characteristic of the medium are listed below:

- $\rho = 2700$ kg.m^{-3}
- $g = 9.8$ m.s^{-2}
- $\nu = 0.3$
- $\mu = 0.5$
- $h = 5000$ m
- $T = 10$ MPa

1 - a Let us consider a point on the detachment surface at the depth h. Give the expression of the vertical normal stress (σ_{zz}) at that point.

Vertical normal stress = hydrostatic pressure because of the weight of the layers : $\sigma_{zz} = \rho g h$. *It is important to notice that we are in the geotechnical sign convention* b If we consider at first that $T = 0$, considering the Poisson effect, what is the horizontal stress (σ_H^ν) due to the gravity (reminder, Hooke's law in the general case is written $\varepsilon_{ij} = \frac{1}{E}\left[(1+\nu)\,\sigma_{ij} - \nu tr\left(\underline{\underline{\sigma}}\right)\delta_{ij}\right]$)?

$T = 0$ so we can consider that the medium is isotropic in directions x and y. Accordingly $\sigma_{xx} = \sigma_{yy}$ and $\epsilon_{xx} = \epsilon_{yy} = 0$. Applying Hooke's law on the x (or y axis) gives: $\sigma_H^\nu = \frac{\nu}{1-\nu}\rho g h$.

c We consider that we can write the stresses as $\sigma_{xx} = \kappa\sigma_{zz} - T$ and $\sigma_{yy} = \kappa\sigma_{zz} - \nu T$. What is the expression of κ ?

In this case we do not have $\varepsilon_{xx} = 0$ any more. We use the Hooke's law on the y axis: $\sigma_{yy} = \nu\sigma_{xx} + \nu\sigma_{zz} \Rightarrow \kappa = \frac{\nu}{1-\nu}$

d Amonton's second law of friction states that the friction between two solid bodies is proportional to the normal stress between both. Give then the magnitude of the shear stress σ_{xz}.

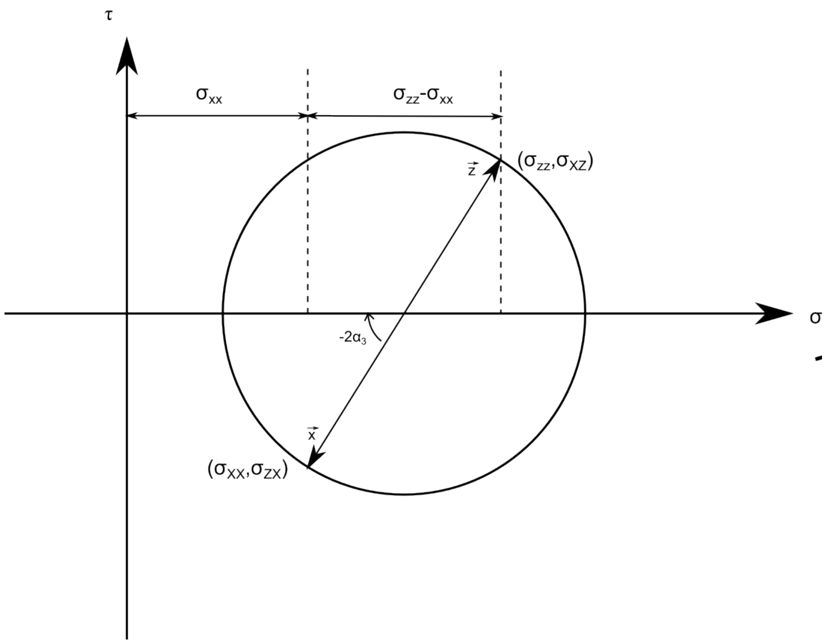
The normal stress applying on the detachment plane is σ_{zz}. The shear stress on the detachment plane is then $\sigma_{xz} = +\mu\rho g h$. (xz because its along x applying on a surface normal to z. The stress is positive because its rotating counterclockwise.

2 - Construct the Mohr circle for the two-dimensional stress that acts at this considered point. Give the radius and the coordinates of the center of the circle.

To draw the circle we use the fact that the x and z axis are orthogonal in the physical space so the are diametrically opposed in the Mohr circle. We jusst have to place the two points $(\sigma_{xx}, \sigma_{zx})$ and $(\sigma_{xx}, \sigma_{zx})$ and draw the circle which go through these two points. To obtain the two points, one just have to remember that $\underline{T} = \underline{\underline{\sigma}} \cdot \underline{n}$ with $\underline{T}$ the stress tensor and $\underline{n}$ the normal to the cutting plane.

In our case $\underline{\underline{\sigma}} = \begin{pmatrix} \sigma_{xx} & 0 & \sigma_{xz} \\ 0 & \sigma_{yy} & 0 \\ \sigma_{zx} & 0 & \sigma_{zz} \end{pmatrix}$ *and* $\underline{n} = (1, 0, 0)$ *and* $(0, 0, 1)$.

Numerical application gives: $\sigma_{xx} = 49.5$ MPa, $\sigma_{xz} = 66.2$ MPa and $\sigma_{zz} = 132.3$ MPa.
The center of the circle is at the position $\frac{\sigma_{zz} - \sigma_{xx}}{2} = 90.9$ MPa.
The radius of the circle is : $r^2 = \frac{1}{4}\left(\sigma_{zz} - \sigma_{xx}\right)^2 - \sigma_{xz}^2 \Rightarrow r = 78.1$ MPa



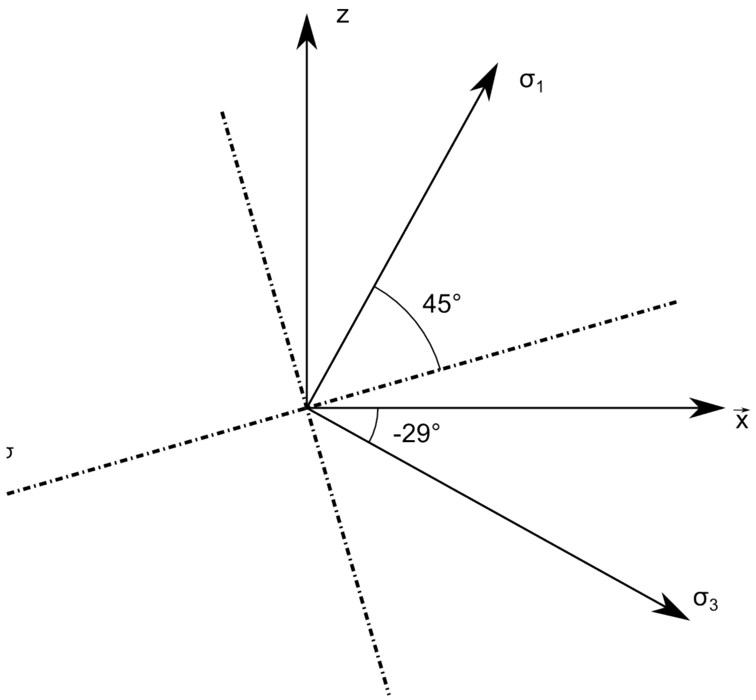
3 - What are the orientation and the magnitude of the principal stresses in the plane (x, z). Plot the axis in the physical space.

The equation of the Mohr circle is :

$$\begin{cases} \sigma = \frac{\sigma_1+\sigma_3}{2} + \frac{\sigma_1-\sigma_3}{2}\cos(-2\alpha) \\ \tau = \frac{\sigma_1-\sigma_3}{2}\sin(-2\alpha) \end{cases}$$

The values of the principal stresses are then: $\begin{cases} \sigma_1 = \sigma_n + r = 169.0\ MPa \\ \sigma_3 = \sigma_n - r = 12.8\ MPa \end{cases}$

The orientation of the principal stress direction are obtained geometrically: $\sin(-2\alpha_3) = \frac{\sigma_{zx}}{r}$ *from* $\underline{x}$ *to* $\underline{\sigma}_3$. $\Rightarrow \alpha_3 = -29°$

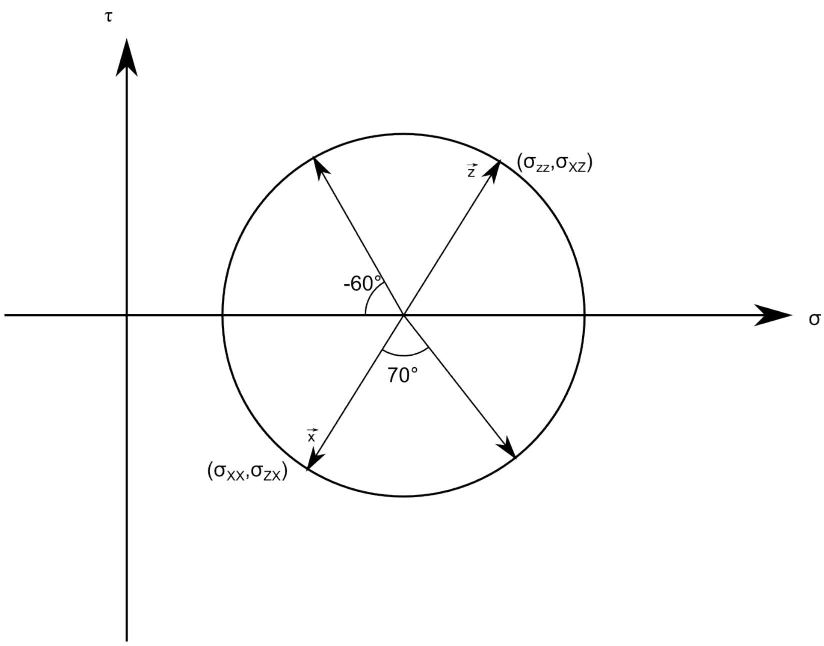


4 - What are the orientation and the values of the maximal shear stress.

The maximum shear stress is equal to the radius of the circle : $\tau_{max} = r = 78.1\ MPa$. *They are oriented as 45° from the principal axis. There are then 2 planes whose normales are oriented at 45 ° respectively from* $\underline{\sigma_1}$ *and* $\underline{\sigma_1}$

5 - What are the values of the normal and shear components on the following planes:

- perpendicular to (x, z), its normal at an angle of 35° from $\underline{x}$.
- perpendicular to (x, z), its normal at an angle of $-30°$ from the principal axis $\underline{e_3}$.



We use the equation of the circle:

$$\begin{cases} \sigma = 90.9 + 78.1\cos(-52) = 138.9\ MPa \\ \tau = 78.1\sin(-52) = 61.5\ MPa \end{cases}$$

And for the other orientation:

$$\begin{cases} \sigma = 90.9 + 78.1\cos(120) = 51.85\ MPa \\ \tau = 78.1\sin(120) = 67.63\ MPa \end{cases}$$