Poromechanics of the exploitation of an oilfield, a brief introduction
Correction

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1 - We are in a 2D case. The vertical stress is the hydrostatic pressure of the upper layers: $\sigma_v = -\rho g h$ (non geotechnical convention). Hooke's law is not applicable because it considers an homogeneous material and not a porous one. The stress tensor and strain tensor are:

$$\underline{\underline{\sigma}} = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_2 \end{pmatrix} \qquad \underline{\underline{\varepsilon}} = \begin{pmatrix} \varepsilon_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Accordingly, the deviatoric stress tensor becomes :

$$\underline{\underline{s}} = \underline{\underline{\sigma}} - \sigma = \frac{1}{3} \begin{pmatrix} 2\sigma_1 - 2\sigma_2 & 0 & 0 \\ 0 & \sigma_2 - \sigma_1 & 0 \\ 0 & 0 & \sigma_2 - \sigma_1 \end{pmatrix}$$

And the deviatoric strain tensor is

$$\underline{\underline{e}} = \underline{\underline{\varepsilon}} - \frac{1}{3}\epsilon \underline{\underline{1}} = \frac{1}{3} \begin{pmatrix} 2\varepsilon_1 & 0 & 0 \\ 0 & -\varepsilon_1 & 0 \\ 0 & 0 & -\varepsilon_1 \end{pmatrix}$$

The second equation of poromechanics gives us:

$$\underline{\underline{s}} = 2G\underline{\underline{e}}$$

with G the shear modulus of the system.

$$\begin{cases} s_1 = \frac{4}{3}G\varepsilon_1 \\ s_2 = s_3 = -\frac{2}{3}G\varepsilon_1 \end{cases} \Rightarrow \sigma_1 - \sigma_2 = 2G\varepsilon_1 \Rightarrow \sigma = \sigma_1 - \frac{4}{3}G\varepsilon_1 \quad \text{with} \quad \sigma = \frac{1}{3}tr\left(\underline{\underline{\sigma}}\right)$$

Using now the first equation of poromechanics:

$$\begin{aligned} \sigma &= K\epsilon - bp \\ \sigma_1 - \frac{4}{3}G\varepsilon_1 &= K\varepsilon_1 - bp \\ \sigma_1 &= \left(K + \frac{4}{3}G\right)\varepsilon_1 - bp \end{aligned}$$

Similarly we obtain :

$$\sigma_2 = \left(K - \frac{2}{3}\right)\varepsilon_1 - bp$$

We are in an undrained situation. The saturated poromechanics gives us:

$$\begin{aligned}
\sigma &= K\epsilon - bp = K_u\epsilon \\
\varepsilon_1 &= \frac{bp}{K - K_u} \\
p &= \frac{\frac{\sigma_1}{b}}{\frac{K+4/3G}{K-K_u} - 1} \approx 14.2\text{MPa}
\end{aligned}$$

with $K_u = K + b^2 M$ the undrained modulus and $\frac{1}{M} = \frac{\phi_0}{K_O} + \frac{1}{N}$. K, K_{sk} are calculated with the relations $\frac{1}{N} = \frac{b-\phi_0}{K_{sk}}$ and $b = 1 - \frac{K}{K_{sk}}$. $G = \frac{3}{2}\frac{1-2\nu}{1+\nu}$.

2 - The state equation for oil is $\frac{d\rho}{\rho} = \frac{dp}{K_O}$. Integration this relation we obtain :

$$\rho_O = \rho_{atm}\exp\left(\frac{p - p_{atm}}{K_O}\right)$$

$$\frac{\rho - \rho_{atm}}{\rho_{atm}} \approx 1.05\%$$

On the range of pressure considered, the oil can be considered as incompressible.

3 - Hydraulic resistance of the well : $R_{well} = \frac{8\eta h}{\pi r^4} \approx 40$ Pa / m^{-3}.
Hydraulic resistance of the formation : $R_{formation} = \frac{\eta L}{A k_0} \approx 2.65.10^{12}$ Pa / m^{-3}.
With A the area of the will which is producing ($= 2\pi r d$ and $d = 80$m).

4 - We are in a drained case now. The production can be written as :

$$\frac{dm}{dt} = Q(t)\rho_O$$

Using Darcy's law for the produced flow rate (we consider that the well is producing along its whole length, we have $Q(t) = \frac{Ak_0}{\eta}\frac{p-p_{well}}{L}$.

As nothing is replacing the oil, the only production is created by the change of porosity following the decrease of pressure :

$$\begin{aligned}
dm &= -\rho_O d\varphi = -\rho\left(bd\epsilon + \frac{dp}{N}\right) \\
d\sigma &= Kd\epsilon - bdp \Rightarrow d\epsilon = \frac{b}{K}dp
\end{aligned}$$

There is no change of confining stress during the process. Only the pore pressure changes.

$$\begin{aligned}
-\rho_O\left(\frac{b^2}{K}dp + \frac{dp}{N}\right) &= \rho_O\frac{Ak_0}{\eta}\frac{p - p_{well}}{L}dt \\
-\frac{dp}{dt}\left(\frac{b^2}{K} + \frac{1}{N}\right) &= \frac{Ak_0}{\eta}\frac{p - p_{well}}{L} \\
\frac{dp}{dt} + \frac{1}{\tau}p &= \frac{p_{well}}{\tau} \\
p(t) &= p_{well} + (p_0 - p_{well})\exp(-t/\tau)
\end{aligned}$$

with $\tau = \frac{\eta L}{A k_0}\left(b^2/K + 1/N\right) \approx 373$ s. p_{well} is the pressure at the bottom of the well. We can consider at first approximation that is corresponds to the hydrostatic pressure of oil : $p_{well} = 13.2$ MPa

The change of porosity during this early stage is:

$$\varphi = \left(\frac{1}{N} + \frac{b^2}{K}\right)(p_{well} - p_0) \approx -0.015\%$$

The percentage of oil retrieved is $\% = \frac{\varphi}{\phi_0 - \varphi} = 0.075\%$.

With the pumping system, $\varphi \approx -0.199\%$ and the yield becomes 1.005%.

5 - First, let us consider the expansion of the gas cap. We can model the gas with the perfect gas law. Considering the initial and the final state, we obtain :

$$\begin{aligned}
\Delta\phi_{g1} &= \frac{\Delta V_g}{V_{tot}} \\
\Delta\phi_{g1} &= \frac{n_g RT}{V_{tot}}\left(\frac{1}{p_{final}} - \frac{1}{p_0}\right) \\
\Delta\phi_{g1} &= \phi S_{rg}\left(\frac{p_0}{p_{final}} - 1\right)
\end{aligned}$$

with $V_{tot} = \frac{V_g^0}{\phi S_{rg}}$.

Let us now consider the degasing of oil. Henry's law gives us $x_0 = p_0/K_H$ and $x_{final} = p_{final}/K_H$. The additional quantity of gas obtained is then:

$$\Delta n_g = \frac{x_0}{1 - x_0} n_O - \frac{x_f}{1 - x_f} n_O$$

using the definition of the mole fraction. Expressing now the oil quantities with the density and the molar mass, we obtain :

$$\begin{aligned}
\Delta\phi_{g2} &= \frac{RT}{p_{final}} \phi S_O^0 \frac{\rho_O}{M_O}\left(\frac{x_0}{1 - x_0} - \frac{x_f}{1 - x_f}\right) \\
\Delta\phi_{g2} &= \frac{RT}{p_{final}} \phi S_O^0 \frac{\rho_O}{M_O}\left(\frac{p_0}{K_H - p_0} - \frac{p_{final}}{K_H - p_{final}}\right)
\end{aligned}$$

The percentage of recovered oil is then :

$$\frac{\Delta\phi_{g1}}{\phi_O} = \begin{cases} 0.42\% & \text{without pumping} \\ 744\% & \text{with pumping} \end{cases}$$

$$\frac{\Delta\phi_{g2}}{\phi_O} = \begin{cases} 1.19\% & \text{without pumping} \\ 1945\% & \text{with pumping} \end{cases}$$

The recovery of oil with pumping system is larger than 100 %. However, in oil fields, it is well known that the primary recovery is rarely larger than 30 %. The explanation of this difference can come from : use of the perfect gas equation which overestimates by a large margin the quantity of gas, the assumption that there is no gas production and the neglection of capillary pressure.

6 - Considering that both pressure of gas and fluid are the same, and using the equations of question 1, we have :

$$\Delta\varepsilon = \frac{b\,(p_0 - p_{final})}{K + 4/3G}$$

The thickness of the aquifer is 100m, this leads to :

$$\Delta h = \begin{cases} 9.3\text{cm} & \text{without pumping} \\ 28\text{cm} & \text{with pumping} \end{cases}$$

7 - Considering that the field is oil-wet, water will have a intermediate wettability.

Capillary number is defined as $Ca = \frac{v_w \eta_w}{\gamma_{o/w}} = \frac{Q}{A}\frac{\eta_w}{\gamma_{w/o}} \approx 5.3.10^{-5}$. The viscosity ratio gives $M = 0.1$. The regime will then be capillary fingering.

There are 3 main parameters which the operator can change in order to recover more oil.

- polymer flooding : addition of polymers in the injected water can increase the viscosity of the water and thus allow to go toward a more homogeneous displacement
- temperature control : temperature acts on both viscosities. As the influence of the temperature on the viscosity of both fluids is different, playing with the temperature allows to change the viscosity ratio. Usually, an increase of temperature is the best solution. To do this increase of temperature, one can inject hot water or even steam in the field.
- interfacial tension control : adding to the water some surfactants allows to decrease the interfacial tension between water and oil and then increasing the capillary number. This has an effect similar to the polymer flooding.

8 - Injecting a miscible fluid in the field allows to avoid the problem of multiphase transport. Carbon dioxide will mix with the oil, decrease its viscosity and increase the pressure inside the field while keeping it as a single phase flow. As a result, theoretically one can achieve a 100 % recovery. However, at some point, the oil produced is too concentrated with CO_2 and the technique is not economically viable.This being said, the need of climate mitigation techniques like the underground storage of CO_2 combines interestingly with this CO_2 flooding and is currently being used in numerous oilfield, among them the famous Weyburn in Alberta (Canada).

The techniques described in questions 7 and 8 are called EOR (enhanced oil recovery). It is important to remember that these techniques are not systematically used and the economic profitability is the key parameter in this case. Oilfield are regularly closed because of a lack of profitability. However, if the price of oil goes up, some fields become profitable again and are reopened and reexploited.