On the use of evolutionary methods in spaces of Euclidean signature

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Chicago, 26 February 2019

The main message:

some of the arguments and techniques developed originally and applied so far exclusively only in the Lorentzian case do also apply to Riemannian spaces

Based on some recent works:

- I. Rácz: *Is the Bianchi identity always hyperbolic?*, CQG **31** 155004 (2014)
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All the involved results are valid for arbitrary dimension: i.e. for $dim(M) = n$ (≥ 4). Nevertheless, for the sake simplicity attention will be restricted to the case of $n = 4$.

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Outline:

- **Einsteinian spaces:** (M, g_{ab})
  - First part
  - Second part
  - in both cases metrics of Euclidean signature will be involved
  - no gauge condition

    ... arbitrary choice of foliations & "evolutionary" vector field

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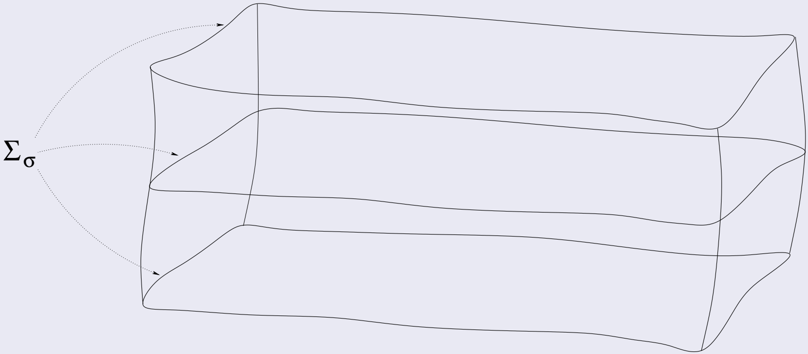

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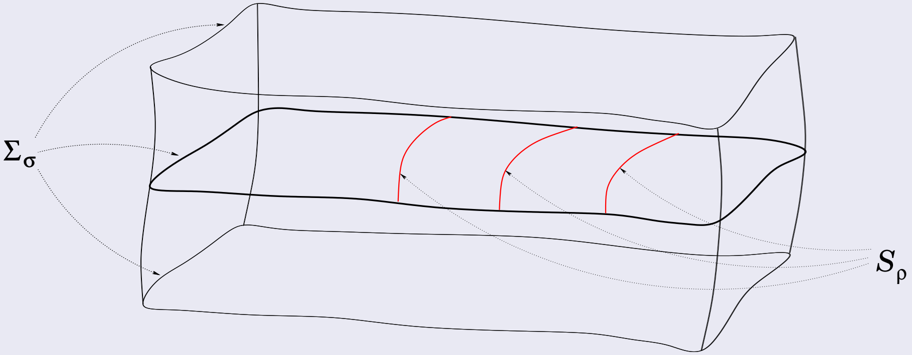

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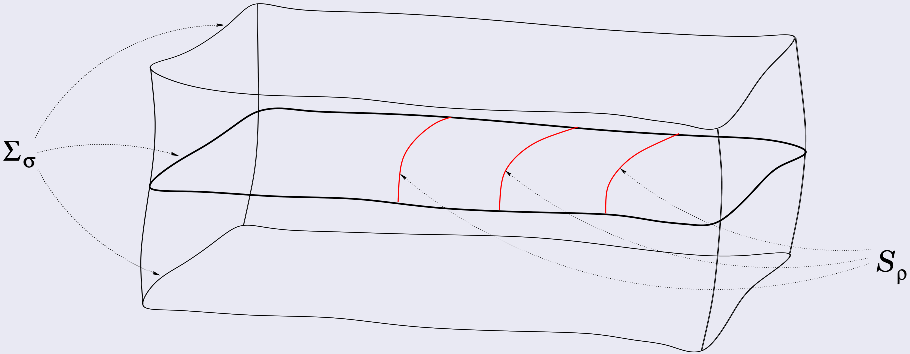

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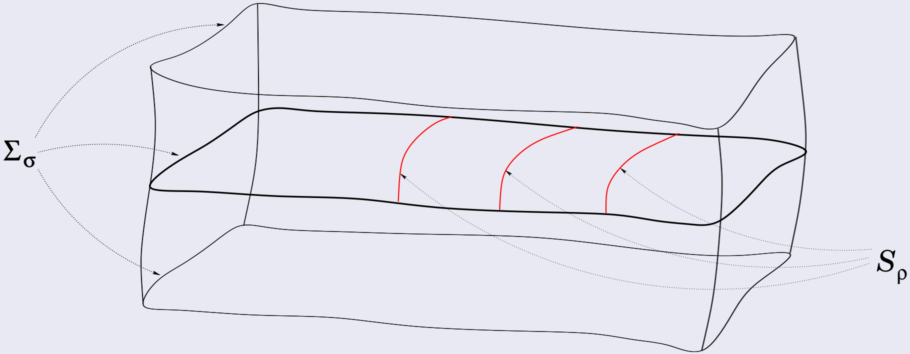

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- **Einsteinian spaces:** (M, g_{ab})
  - M : 4-dimensional, smooth, paracompact, connected, orientable manifold
  - g_{ab}: smooth Lorentzian$_{(-,+,+,+)}$ or Riemannian$_{(+,+,+,+)}$ metric
- **Einstein's equations:**

$$\boxed{G_{ab} - \mathscr{G}_{ab} = 0} \quad \text{with source term:} \quad \boxed{\nabla^a \mathscr{G}_{ab} = 0}$$

  - in a more familiar setup: **Einstein's equations** with cosmological constant Λ

$$\boxed{[R_{ab} - \tfrac{1}{2}\, g_{ab}\, R] + \Lambda\, g_{ab} = 8\pi\, T_{ab}}$$

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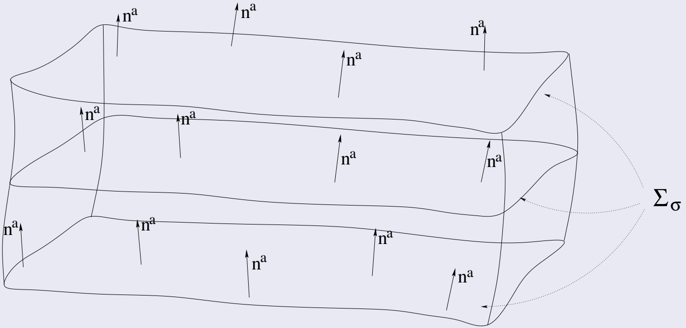
PART I:

The primary splitting

- **Assume:** M is foliated by a one-parameter family of homologous hypersurfaces, i.e. $M \simeq \mathbb{R} \times \Sigma$, for some three-dimensional manifold Σ.
  - known to **hold for globally hyperbolic spacetimes** (Lorentzian case)
  - **equivalent to** the existence of a smooth function $\sigma : M \to \mathbb{R}$ with non-vanishing gradient $\partial_a \sigma$ such that the $\sigma = const$ level surfaces $\Sigma_\sigma = \{\sigma\} \times \Sigma$ comprise the one-parameter foliation of M.
  - $$n_a \sim \partial_a \sigma \ \ldots \ \& \ldots \ g^{ab} \ \longrightarrow \ n^a = g^{ab} n_b$$



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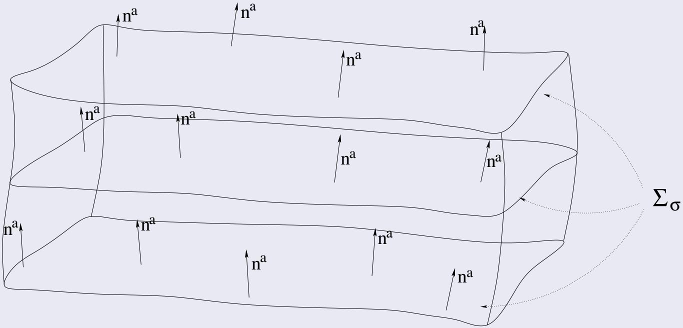

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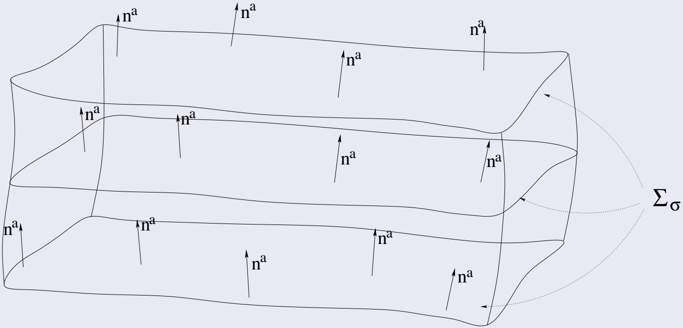

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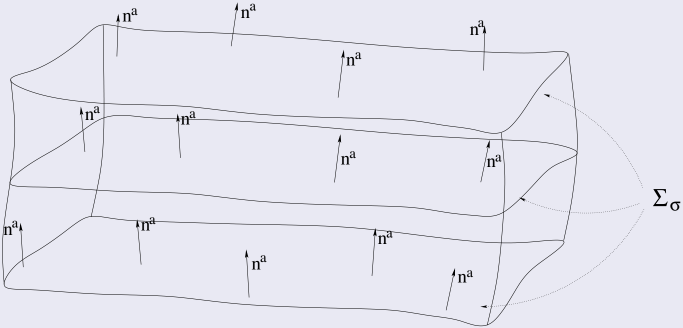

Projections:

The projection operator:

- n^a the 'unit norm' vector field that is normal to the Σ_σ level surfaces

$$n^a n_a = \epsilon$$

  - the sign is not fixed: ϵ takes the value -1 or $+1$ for Lorentzian or Riemannian metric g_{ab}, respectively.

- **the projection operator**

$$h^a{}_b = \delta^a{}_b - \epsilon\, n^a n_b$$

  to the level surfaces of $\sigma : M \to \mathbb{R}$.

- **the induced metric** on the $\sigma = const$ level surfaces

$$h_{ab} = h^e{}_a h^f{}_b\, g_{ef}$$

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- the integral curves of σ^a meet the $\sigma = const$ level surfaces precisely once
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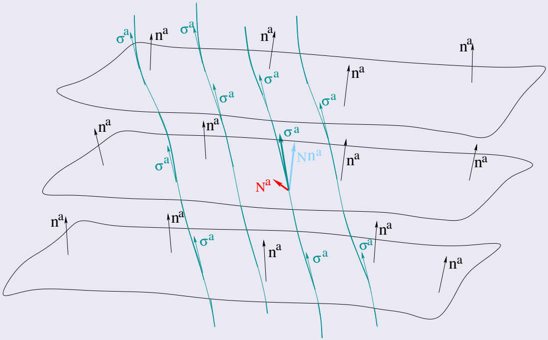

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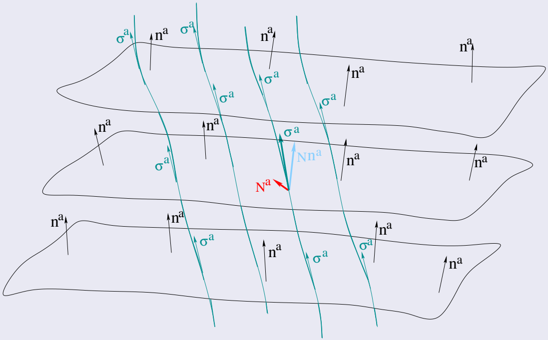

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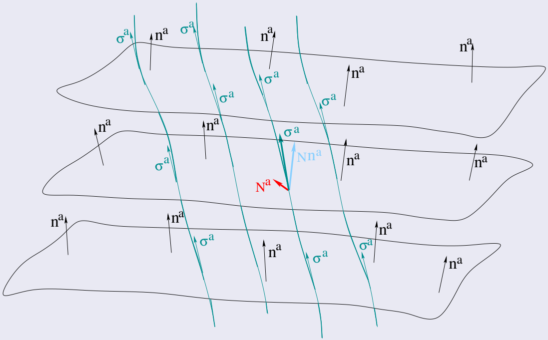

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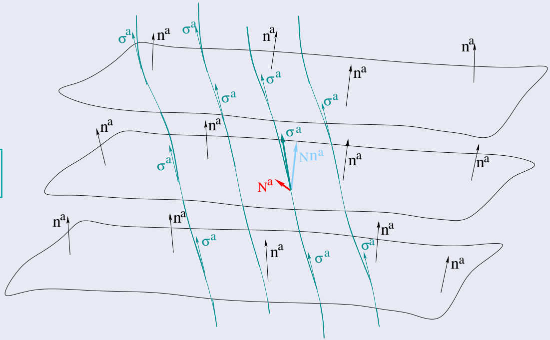

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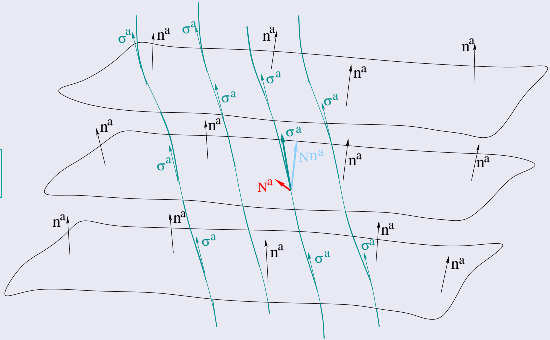

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Decompositions of various fields:

Any symmetric tensor field P_{ab} can be decomposed

in terms of n^a and fields living on the $\sigma = const$ level surfaces as

$$P_{ab} = \boldsymbol{\pi}\, n_a n_b + [n_a\, \mathbf{p}_b + n_b\, \mathbf{p}_a] + \mathbf{P}_{ab}$$

It is also rewarding to inspect the decomposition of the contraction $\nabla^a P_{ab}$:

$$\epsilon\,(\nabla^a P_{ae})\,n^e = \mathscr{L}_n \boldsymbol{\pi} + D^e \mathbf{p}_e + [\boldsymbol{\pi}\,(K^e{}_e) - \epsilon\,\mathbf{P}_{ef} K^{ef} - 2\,\epsilon\,\dot{n}^e \mathbf{p}_e]$$

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$$K_{ab} = h^e{}_a \nabla_e n_b = \tfrac{1}{2}\, \mathscr{L}_n h_{ab}$$

$$\dot{n}_a := n^e \nabla_e n_a = -\epsilon\, D_a \ln N$$

back

Decompositions of various fields:

Examples:

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- the "source term"

$$\mathscr{G}_{ab} = n_a n_b\, \mathfrak{e} + [n_a\, \mathfrak{p}_b + n_b\, \mathfrak{p}_a] + \mathfrak{S}_{ab}$$

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Div

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The main result of the first part:

Theorem

Let (M, g_{ab}) *be an Einsteinian space as specified and assume that the metric* h_{ab} *induced on the* $\sigma = const$ *level surfaces is Riemannian. Then,* **regardless whether** g_{ab} **is of Lorentzian or Euclidean signature**, *any solution to the reduced equations* $E^{(\mathcal{EVOL})}_{ab} = 0$ *is also a solution to the full set of field equations* $G_{ab} - \mathscr{G}_{ab} = 0$ *provided that the constraint expressions* $E^{(\mathcal{H})}$ *and* $E^{(\mathcal{M})}_a$ *vanish on one of the* $\sigma = const$ *level surfaces.*

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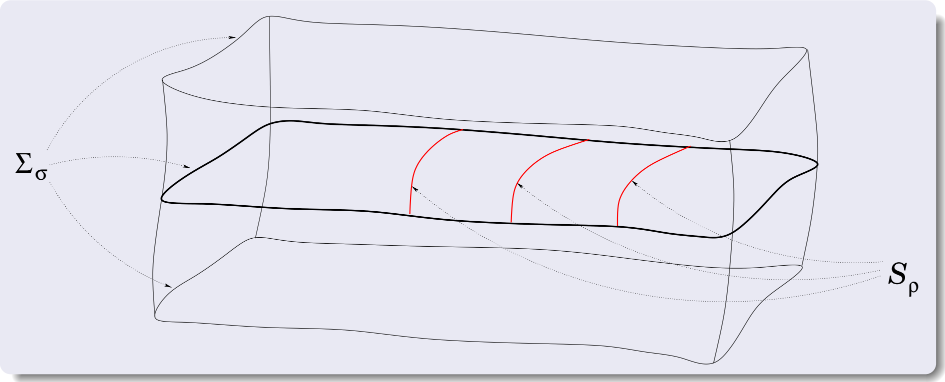
PART II:



The explicit form of the constraints:

The constraint expressions are projections of $E_{ab} = G_{ab} - \mathscr{G}_{ab}$:

$$E^{(\mathcal{H})} = n^e n^f E_{ef} = \tfrac{1}{2}\{-\epsilon\,{}^{(3)}\!R + (K^e{}_e)^2 - K_{ef}K^{ef} - 2\,\mathfrak{e}\} = 0$$

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Consider the underdetermined equation on $\Sigma \approx \mathbb{R}^2$ with some coordinates (χ, ξ)

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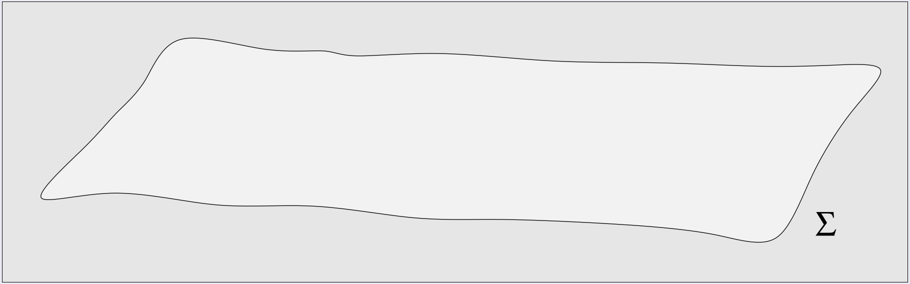

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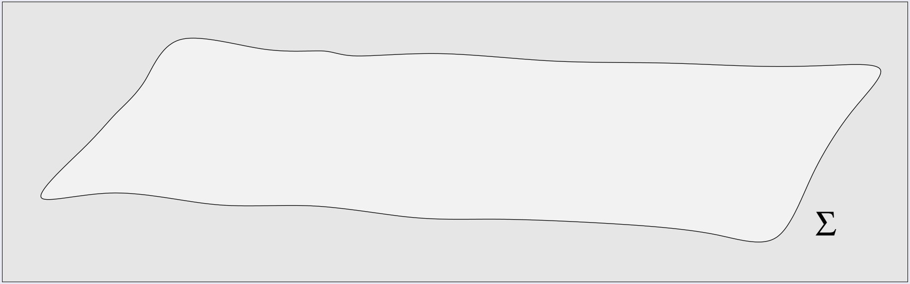

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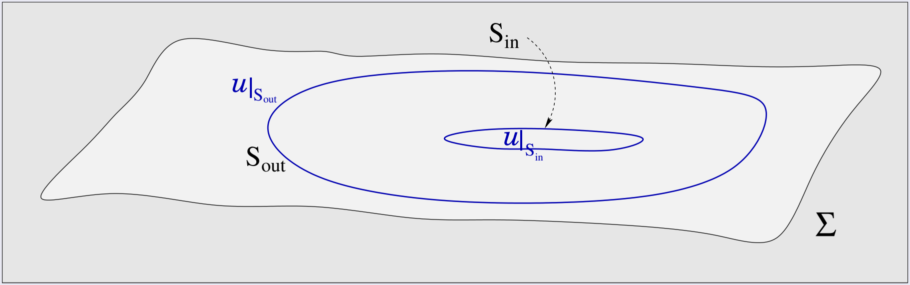

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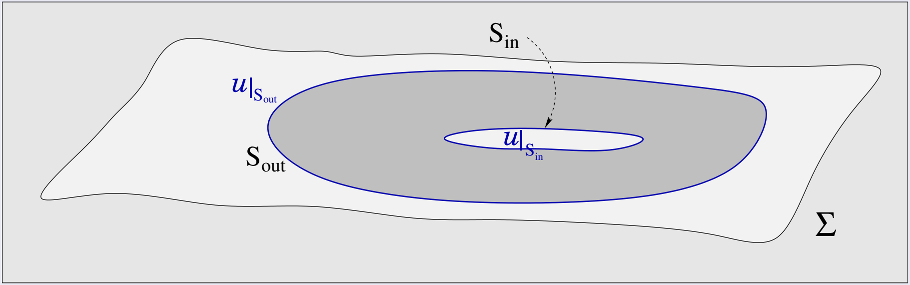

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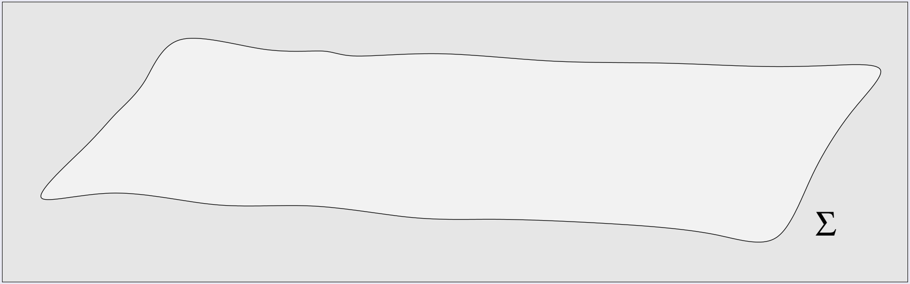

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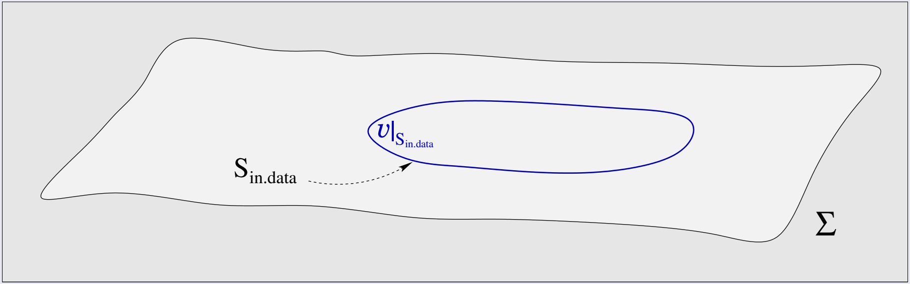

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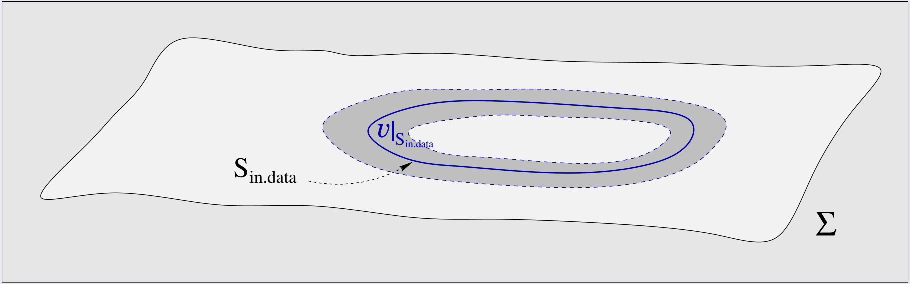

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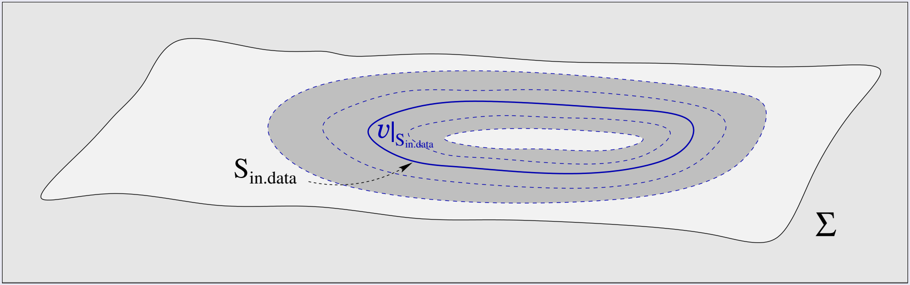

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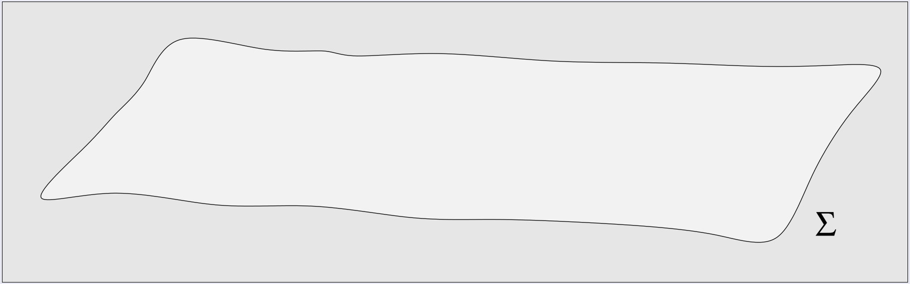

A simple example:

It is a parabolic equation for w on $\Sigma \approx \mathbb{R}^2$:

$$(\partial_\chi^2 + \partial_\xi^2)\boldsymbol{u} + (\partial_\chi - \partial_\xi)\boldsymbol{v} + (a\,\partial_\chi - \partial_\xi^2)w + \boldsymbol{z} = 0$$

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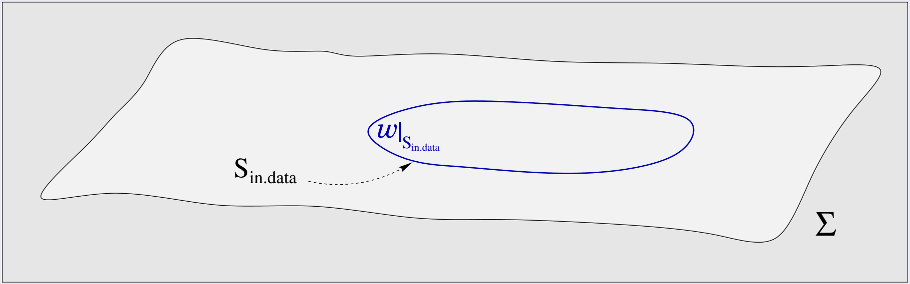

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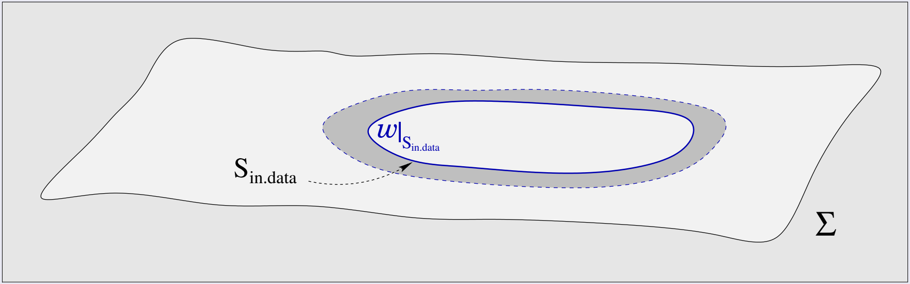

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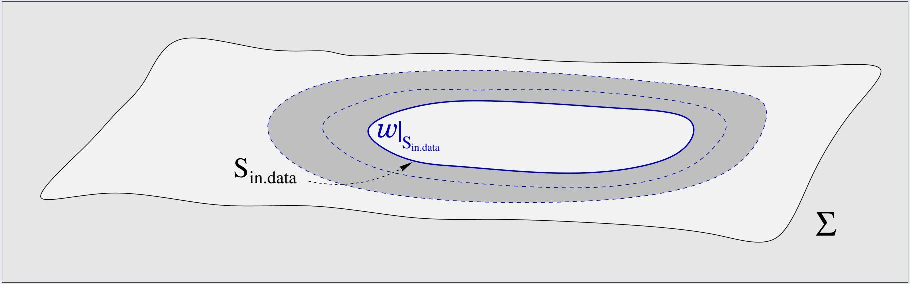

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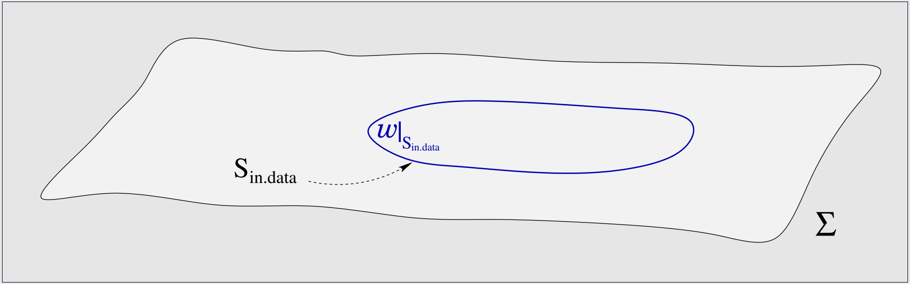

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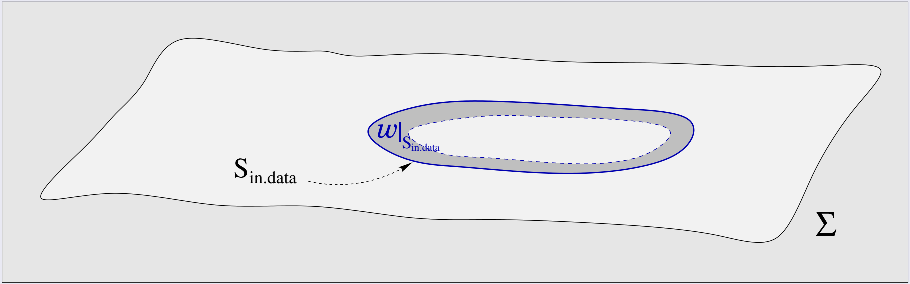

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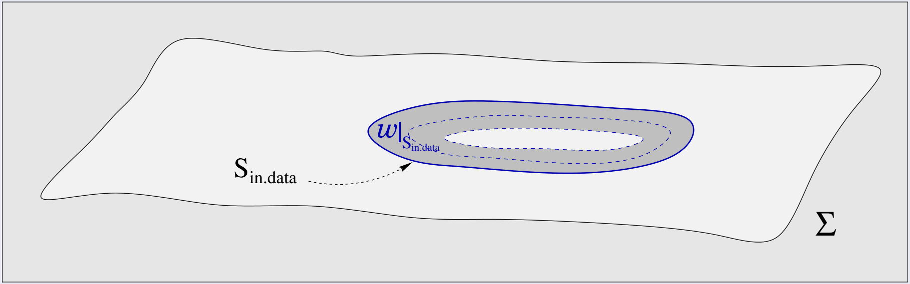

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It is an algebraic equation for z :

$$(\partial_\chi^2 + \partial_\xi^2)u + (\partial_\chi^2 - \partial_\xi^2)v + (a\,\partial_\chi - \partial_\xi^2)w + z = 0$$

- once the variables u, v, w are specified on $\mathbb{R}^2$ the solution is determined as

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It is an algebraic equation for $\boldsymbol{z}$:

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$$\boxed{\rho^i = \widehat{N}\,\widehat{n}^i + \widehat{N}^i\,, \quad \text{where} \quad \widehat{N} = \rho^j\widehat{n}_j \quad \text{and} \quad \widehat{N}^i = \widehat{\gamma}^i{}_j\,\rho^j}$$

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Splitting of the symmetric tensor field K_{ij}:

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- $$(h_{ij}, K_{ij}) \iff (\widehat{N}, \widehat{N}^i, \widehat{\gamma}_{ij}; \boldsymbol{\kappa}, \mathbf{k}_i, \mathbf{K}^l{}_l, \mathring{\mathbf{K}}_{ij})$$

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The momentum constraint:

$$D_e K^e{}_a - D_a K^e{}_e - \epsilon\, \mathfrak{p}_a = 0$$

First order symmetric hyperbolic system:

- contract (1) with $2\,\widehat{N}\,\widehat{\gamma}^{ij}$ and mult. (2) by $\widehat{N}$, when writing them out in coordinates (ρ, x^2, x^3), adopted to the foliation $\mathscr{S}_\rho$ and the vector field ρ^i,
- a first order **symmetric hyperbolic** system for the vector valued variable

$$(\mathbf{k}_B, \mathbf{K}^E{}_E)^T$$

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$$\mathscr{L}_{\hat n}\mathbf{k}_i - \tfrac{1}{2}\,\widehat{D}_i(\mathbf{K}^l{}_l) - \widehat{D}_i\boldsymbol{\kappa} + \widehat{D}^l\overset{\circ}{\mathbf{K}}_{li} + (\widehat{K}^l{}_l)\,\mathbf{k}_i + \boldsymbol{\kappa}\,\dot{\hat n}_i - \dot{\hat n}^l\,\mathbf{K}_{li} - \epsilon\,\mathfrak{p}_l\,\hat\gamma^l{}_i = 0$$

back: str.hyp.sys.

$$\mathscr{L}_{\hat n}(\mathbf{K}^l{}_l) - \widehat{D}^l\mathbf{k}_l - \boldsymbol{\kappa}\,(\widehat{K}^l{}_l) + \mathbf{K}_{kl}\widehat{K}^{kl} + 2\,\dot{\hat n}^l\,\mathbf{k}_l + \epsilon\,\mathfrak{p}_l\,\hat n^l = 0$$

First order symmetric hyperbolic system:

- contract (1) with $2\,\widehat{N}\,\hat\gamma^{ij}$ and mult. (2) by $\widehat{N}$, when writing them out in coordinates (ρ, x^2, x^3), adopted to the foliation $\mathscr{S}_\rho$ and the vector field ρ^i,
- a first order **symmetric hyperbolic** system for the vector valued variable

$$(\mathbf{k}_B, \mathbf{K}^E{}_E)^T$$

The momentum constraint:

$\dot{\hat{n}}_i := \hat{n}^l D_l \hat{n}_i = -\widehat{D}_i \ln \widehat{N}$

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$$\mathscr{L}_{\hat{n}}(\mathbf{K}^l{}_l) - \widehat{D}^l \mathbf{k}_l - \boldsymbol{\kappa}\, (\widehat{K}^l{}_l) + \mathbf{K}_{kl} \widehat{K}^{kl} + 2\, \dot{\hat{n}}^l\, \mathbf{k}_l + \epsilon\, \mathfrak{p}_l\, \hat{n}^l = 0$$

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$$\left\{\begin{pmatrix} 2\,\hat\gamma^{AB} & 0 \\ 0 & 1\end{pmatrix}\partial_\rho + \begin{pmatrix} -2\,\widehat{N}^K\,\hat\gamma^{AB} & -\widehat{N}\,\hat\gamma^{AK} \\ -\widehat{N}\,\hat\gamma^{BK} & -\widehat{N}^K\end{pmatrix}\partial_K\right\}\begin{pmatrix}\mathbf{k}_B \\ \mathbf{K}^E{}_E\end{pmatrix} + \begin{pmatrix}\mathscr{B}^A_{(\mathbf{k})} \\ \mathscr{B}_{(\mathbf{K})}\end{pmatrix} = 0$$

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- a first order **symmetric hyperbolic** system for the vector valued variable

$$(\mathbf{k}_B, \mathbf{K}^E{}_E)^T$$

!!! ρ **plays the role of 'time'**

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$$(\mathbf{k}_B, \mathbf{K}^E{}_E)^T$$

!!! ρ **plays the role of 'time'** regardless of the value of $\epsilon = \pm 1$

The Hamiltonian constraint:

The Hamiltonian constraint in terms of the new variables:

- $$E^{(\mathcal{H})} = n^e n^f E_{ef} = \tfrac{1}{2}\{-\epsilon\,{}^{(3)}\!R + (K^e{}_e)^2 - K_{ef}K^{ef} - 2\,\mathfrak{e}\} = 0$$

- using $${}^{(3)}\!R = \widehat{R} - \left\{2\,\mathscr{L}_{\widehat{n}}(\widehat{K}^l{}_l) + (\widehat{K}^l{}_l)^2 + \widehat{K}_{kl}\widehat{K}^{kl} + 2\,\widehat{N}^{-1}\widehat{D}^l\widehat{D}_l\widehat{N}\right\}$$

  $\widehat{R}$ and $\widehat{K}_{kl}$ denote the scalar and extrinsic curvature of $\widehat{\gamma}_{kl}$, respectively

- $$-\epsilon\,\widehat{R} + \epsilon\left\{2\,\mathscr{L}_{\widehat{n}}(\widehat{K}^l{}_l) + (\widehat{K}^l{}_l)^2 + \widehat{K}_{kl}\,\widehat{K}^{kl} + 2\,\widehat{N}^{-1}\widehat{D}^l\widehat{D}_l\widehat{N}\right\} + 2\,\boldsymbol{\kappa}\,\mathbf{K}^l{}_l + \tfrac{1}{2}\,(\mathbf{K}^l{}_l)^2 - 2\,\mathbf{k}^l\mathbf{k}_l - \mathring{\mathbf{K}}_{kl}\,\mathring{\mathbf{K}}^{kl} - 2\,\mathfrak{e} = 0$$

Alternative choices yielding evolutionary systems:

- it is a **parabolic equation** for $\widehat{N}$ (the sign of $\widehat{K}^l{}_l$ plays a role)
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The parabolic-hyperbolic system:

The Hamiltonian constraint as a parabolic equation for $\widehat{N}$:

$$-\epsilon\,\widehat{R} + \epsilon\left\{2\,\boxed{\mathscr{L}_{\widehat{n}}(\widehat{K}^l{}_l)} + (\widehat{K}^l{}_l)^2 + \widehat{K}_{kl}\,\widehat{K}^{kl} + 2\,\boxed{\widehat{N}^{-1}\,\widehat{D}^l\widehat{D}_l\widehat{N}}\right\} + 2\,\boldsymbol{\kappa}\,\mathbf{K}^l{}_l + \tfrac{1}{2}\,(\mathbf{K}^l{}_l)^2 - 2\,\mathbf{k}^l\mathbf{k}_l - \mathring{\mathbf{K}}_{kl}\,\mathring{\mathbf{K}}^{kl} - 2\,\mathfrak{e} = 0$$

- $\widehat{K}^l{}_l = \widehat{\gamma}^{ij}\,\widehat{K}_{ij} = \widehat{N}^{-1}[\,\frac{1}{2}\,\widehat{\gamma}^{ij}\,\mathscr{L}_\rho\widehat{\gamma}_{ij} - \widehat{D}_j\widehat{N}^j\,] = \widehat{N}^{-1}\overset{\star}{K}$ as $\widehat{n}^i = \widehat{N}^{-1}[\,\rho^i - \widehat{N}^i\,]$
- $\mathscr{L}_{\widehat{n}}(\widehat{K}^l{}_l) = -\widehat{N}^{-3}\overset{\star}{K}\,[\,(\partial_\rho\widehat{N}) - (\widehat{N}^l\widehat{D}_l\widehat{N})\,] + \widehat{N}^{-2}[\,(\partial_\rho\overset{\star}{K}) - (\widehat{N}^l\widehat{D}_l\overset{\star}{K})\,]$
- using
$$\mathcal{A} = 2\,[\,(\partial_\rho\overset{\star}{K}) - \widehat{N}^l(\widehat{D}_l\overset{\star}{K})\,] + \overset{\star}{K}{}^2 + \overset{\star}{K}_{kl}\overset{\star}{K}{}^{kl}$$
$$\mathcal{B} = -\widehat{R} + \epsilon\,[\,2\,\boldsymbol{\kappa}\,(\mathbf{K}^l{}_l) + \tfrac{1}{2}\,(\mathbf{K}^l{}_l)^2 - 2\,\mathbf{k}^l\mathbf{k}_l - \mathring{\mathbf{K}}_{kl}\,\mathring{\mathbf{K}}^{kl} - 2\,\mathfrak{e}\,]$$
- it gets to be a **Bernoulli-type parabolic partial differential equation** provided that $\overset{\star}{K}$ ...
- $$2\,\overset{\star}{K}\,[\,(\partial_\rho\widehat{N}) - \widehat{N}^l(\widehat{D}_l\widehat{N})\,] = 2\,\widehat{N}^2(\widehat{D}^l\widehat{D}_l\widehat{N}) + \mathcal{A}\,\widehat{N} + \mathcal{B}\,\widehat{N}^3$$ & momentum constr.
- in highly specialized cases of "quasi-spherical" foliations with $\widehat{\gamma}_{ij} = r^2\,\mathring{\gamma}_{ij}$ and with time symmetric initial data $K_{ij} \equiv 0$ R. Bartnik (1993), G. Weinstein & B. Smith (2004)

The parabolic-hyperbolic system:

The Hamiltonian constraint as a parabolic equation for $\widehat{N}$:

$$-\epsilon\,\widehat{R} + \epsilon\left\{2\,\boxed{\mathscr{L}_{\widehat{n}}(\widehat{K}^l{}_l)} + (\widehat{K}^l{}_l)^2 + \widehat{K}_{kl}\,\widehat{K}^{kl} + 2\,\boxed{\widehat{N}^{-1}\,\widehat{D}^l\widehat{D}_l\widehat{N}}\right\} + 2\,\boldsymbol{\kappa}\,\mathbf{K}^l{}_l + \tfrac{1}{2}\,(\mathbf{K}^l{}_l)^2 - 2\,\mathbf{k}^l\mathbf{k}_l - \mathring{\mathbf{K}}_{kl}\,\mathring{\mathbf{K}}^{kl} - 2\,\mathfrak{e} = 0$$

- $\widehat{K}^l{}_l = \widehat{\gamma}^{ij}\,\widehat{K}_{ij} = \widehat{N}^{-1}[\,\tfrac{1}{2}\,\widehat{\gamma}^{ij}\mathscr{L}_\rho\widehat{\gamma}_{ij} - \widehat{D}_j\widehat{N}^j\,] = \widehat{N}^{-1}\overset{\star}{K}$ as $\widehat{n}^i = \widehat{N}^{-1}[\,\rho^i - \widehat{N}^i\,]$
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$$\mathcal{A} = 2\,[\,(\partial_\rho\overset{\star}{K}) - \widehat{N}^l(\widehat{D}_l\overset{\star}{K})\,] + \overset{\star}{K}{}^2 + \overset{\star}{K}_{kl}\overset{\star}{K}{}^{kl}$$
$$\mathcal{B} = -\widehat{R} + \epsilon\,[\,2\,\boldsymbol{\kappa}\,(\mathbf{K}^l{}_l) + \tfrac{1}{2}\,(\mathbf{K}^l{}_l)^2 - 2\,\mathbf{k}^l\mathbf{k}_l - \mathring{\mathbf{K}}_{kl}\,\mathring{\mathbf{K}}^{kl} - 2\,\mathfrak{e}\,]$$
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The parabolic-hyperbolic system:

The Hamiltonian constraint as a parabolic equation for $\widehat N$:

$$-\epsilon\,\widehat R+\epsilon\left\{2\,\boxed{\mathscr{L}_{\hat n}(\widehat K^l{}_l)}+(\widehat K^l{}_l)^2+\widehat K_{kl}\,\widehat K^{kl}+2\,\boxed{\widehat N^{-1}\,\widehat D^l\widehat D_l\widehat N}\right\}$$
$$+\,2\,\boldsymbol{\kappa}\,\mathbf{K}^l{}_l+\tfrac12\,(\mathbf{K}^l{}_l)^2-2\,\mathbf{k}^l\mathbf{k}_l-\mathring{\mathbf{K}}_{kl}\,\mathring{\mathbf{K}}^{kl}-2\,\mathfrak{e}=0$$

- $\widehat K^l{}_l=\widehat\gamma^{ij}\,\widehat K_{ij}=\widehat N^{-1}[\,\tfrac12\,\widehat\gamma^{ij}\mathscr{L}_\rho\widehat\gamma_{ij}-\widehat D_j\widehat N^j\,]=\widehat N^{-1}\overset{\star}{K}$ as $\hat n^i=\widehat N^{-1}[\,\rho^i-\widehat N^i\,]$
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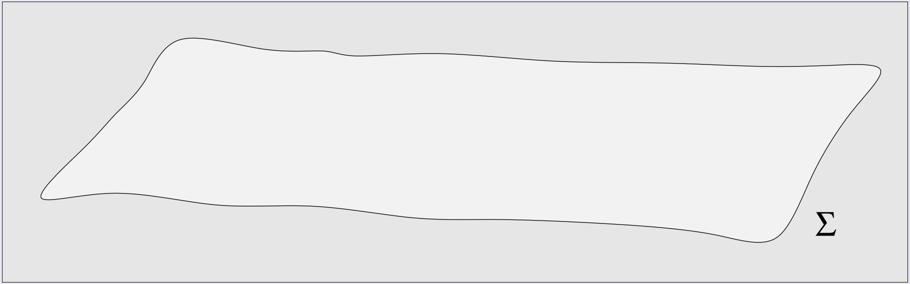

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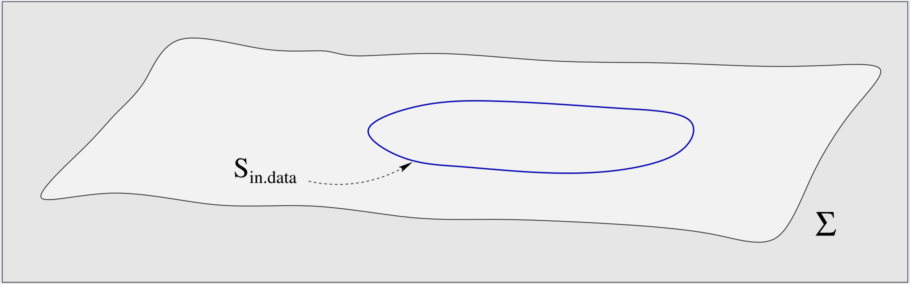

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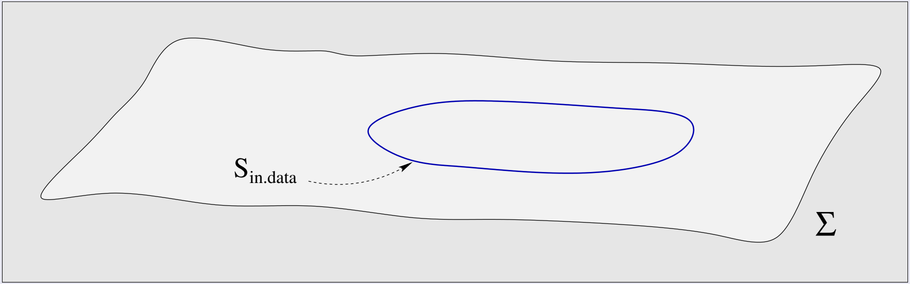

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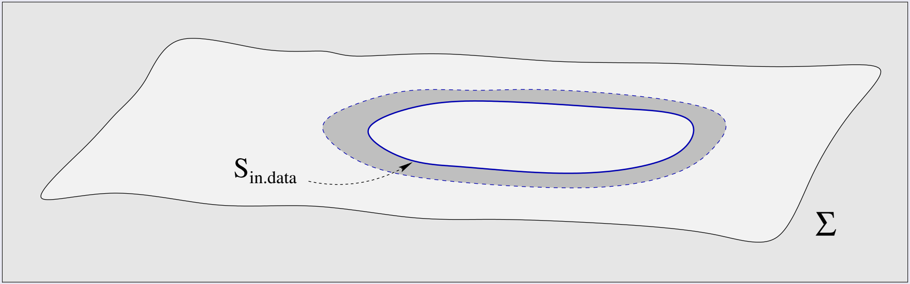

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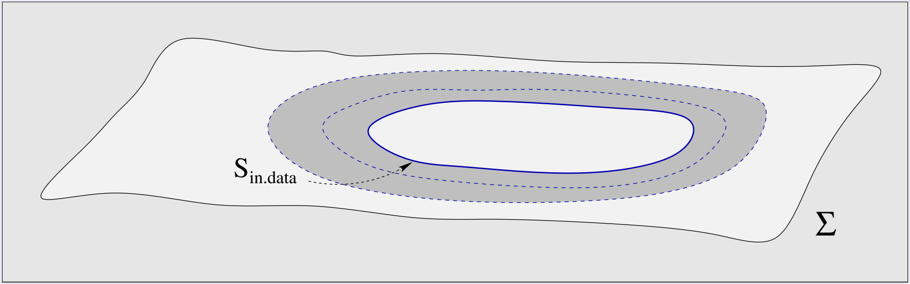

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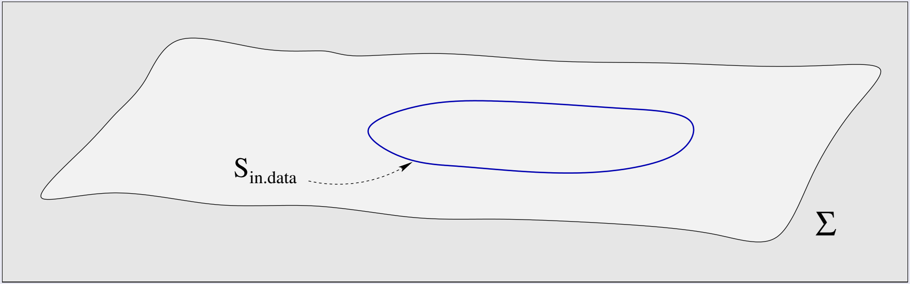

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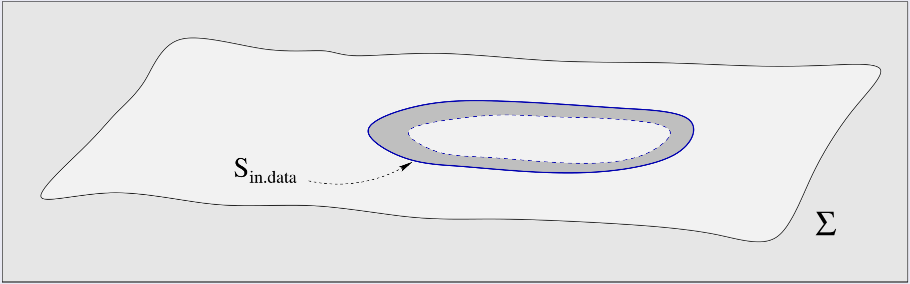

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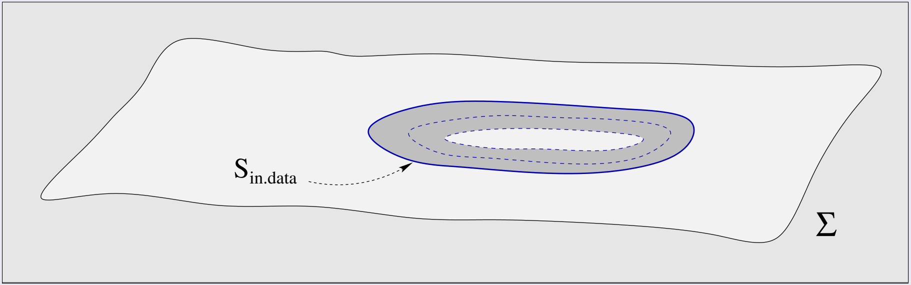

The strongly hyperbolic system:

The Hamiltonian constraint as an algebraic equation for $\boldsymbol{\kappa}$:

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whence $\boldsymbol{\kappa} = (2\,\mathbf{K}^l{}_l)^{-1}[\,2\,\mathbf{k}^l\mathbf{k}_l - \tfrac{1}{2}\,(\mathbf{K}^l{}_l)^2 - \boldsymbol{\kappa}_0\,]\,,\quad \boldsymbol{\kappa}_0 = -\epsilon\,{}^{(3)}\!R - \mathring{\mathbf{K}}_{kl}\,\mathring{\mathbf{K}}^{kl} - 2\,\mathfrak{e}$

- by eliminating $\widehat{D}_i\boldsymbol{\kappa}$ from the momentum constraint mom. constr. one gets

$$\mathscr{L}_{\widehat{n}}\mathbf{k}_i + (\mathbf{K}^l{}_l)^{-1}[\,\boldsymbol{\kappa}\,\widehat{D}_i(\mathbf{K}^l{}_l) - 2\,\mathbf{k}^l\,\widehat{D}_i\mathbf{k}_l\,] + (2\,\mathbf{K}^l{}_l)^{-1}\,\widehat{D}_i\boldsymbol{\kappa}_0 + (\widehat{K}^l{}_l)\,\mathbf{k}_i + [\,\boldsymbol{\kappa} - \tfrac{1}{2}\,(\mathbf{K}^l{}_l)\,]\,\dot{\widehat{n}}_i - \dot{\widehat{n}}^l\,\mathring{\mathbf{K}}_{li} + \widehat{D}^l\mathring{\mathbf{K}}_{li} - \epsilon\,\mathfrak{p}_l\,\widehat{\gamma}^l{}_i = 0\,,$$

$$\mathscr{L}_{\widehat{n}}(\mathbf{K}^l{}_l) - \widehat{D}^l\mathbf{k}_l - \boldsymbol{\kappa}\,(\widehat{K}^l{}_l) + \mathbf{K}_{kl}\widehat{K}^{kl} + 2\,\dot{\widehat{n}}^l\,\mathbf{k}_l + \epsilon\,\mathfrak{p}_l\,\widehat{n}^l = 0$$

- the above system is a **strongly hyperbolic** one for $(\mathbf{k}_i, \mathbf{K}^l{}_l)^T$ provided that $\boldsymbol{\kappa}\cdot\mathbf{K}^l{}_l < 0$
- $\boldsymbol{\kappa}$ is determined algebraically once $\mathbf{k}_i$ and $\mathbf{K}^l{}_l$ are known !!!
- the entire three-metric $h_{ij} = \widehat{\gamma}_{ij} + \widehat{n}_i\widehat{n}_j$ is freely specifiable. !!!

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- by eliminating $\widehat{D}_i\boldsymbol{\kappa}$ from the momentum constraint ◂ mom. constr. one gets

$$\begin{aligned}
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- (h_{ij}, K_{ij}) represented by the variables $(\widehat{N}, \widehat{N}^i, \widehat{\gamma}_{ij}; \boldsymbol{\kappa}, \mathbf{k}_i, \mathbf{K}^l{}_l, \mathring{\mathbf{K}}_{ij})$
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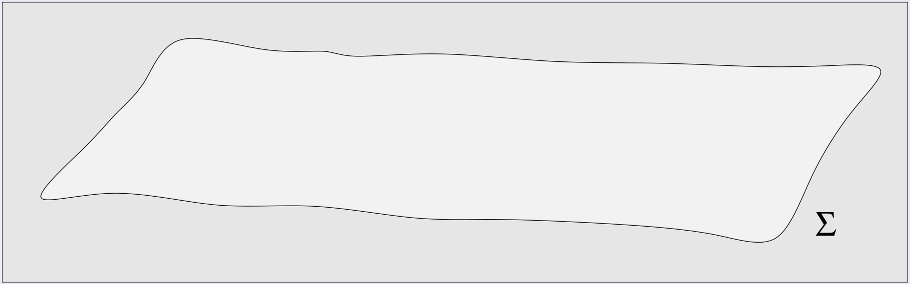

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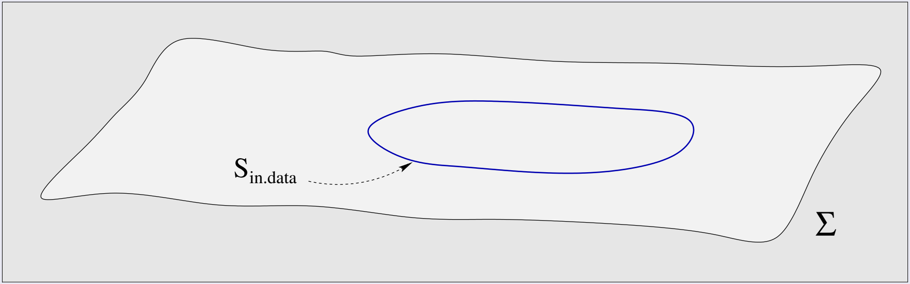

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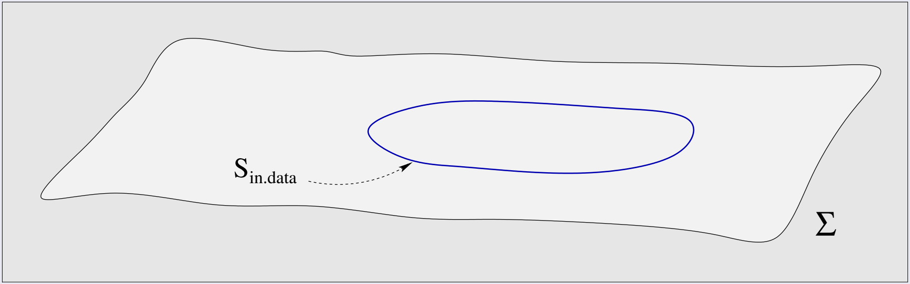

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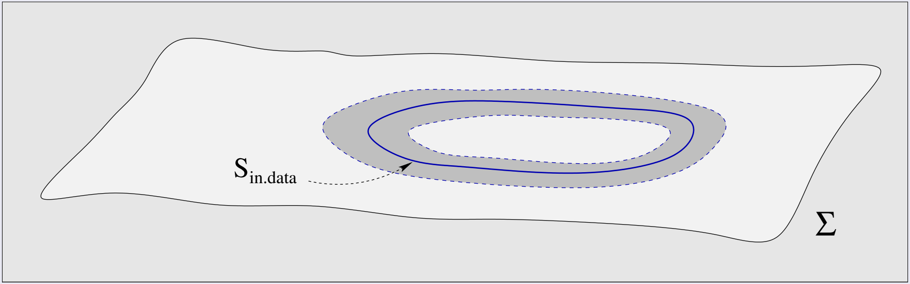

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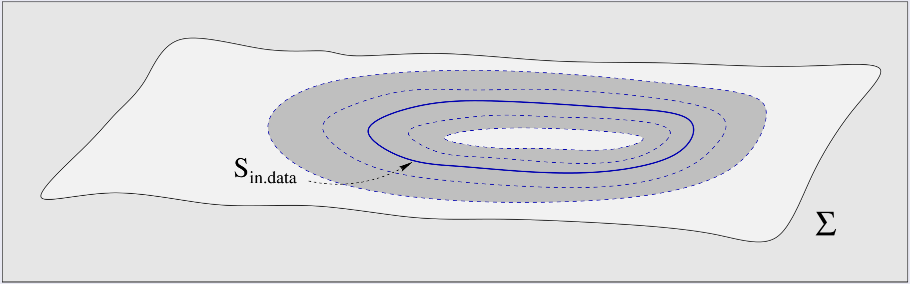

Summary:

4-dimensional **Riemannian and Lorentzian spaces** satisfying Einstein's equations, and some mild topological assumptions, were considered. **!!!** $[n(\geq 4)]$

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The roots of the evolutionary aspects

The first order symmetric hyperbolic system for $(E^{(\mathcal{H})}, E_i^{(\mathcal{M})})^T$

- Characteristic directions associated with the FOSH system governing the evolution of the constraint expressions are determined as

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- with characteristic cone given as

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- ... given a Riemannian metric $\mathfrak{g}_{ij}$, a unit form field $\mathfrak{n}_i$ and a positive real function α

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Deriving a Lorentzian metric from a Riemannian one

- ... given a Riemannian metric $\mathfrak{g}_{ij}$, a unit form field $\mathfrak{n}_i$ and a positive real function α $\Longrightarrow$ a metric of Lorentzian signature can be defined as

$$\breve{\mathfrak{g}}_{ij} = \mathfrak{g}_{ij} - (1+\alpha)\,\mathfrak{n}_i\mathfrak{n}_j$$

The conformal (elliptic) method:

Lichnerowicz A (1944) and York J W (1972):

- replace

$$h_{ij} = \phi^4\, \widetilde{h}_{ij} \quad \text{and} \quad K_{ij} - \tfrac{1}{3}\, h_{ij}\, K^l{}_l = \phi^{-2}\, \widetilde{K}_{ij}$$

- using these variables the constraints are put into a **semilinear elliptic system**

$$\widetilde{D}^l \widetilde{D}_l \phi + \epsilon\, \tfrac{1}{8}\, \widetilde{R}\, \phi + \tfrac{1}{8}\, \widetilde{K}_{ij} \widetilde{K}^{ij}\, \phi^{-7} - \left[\tfrac{1}{12}\, (K^l{}_l)^2 - \tfrac{1}{4}\, \mathfrak{e}\right] \phi^5 = 0$$

where $\widetilde{D}_l$, $\widetilde{R}$, ... $\widetilde{h}_{ij}$

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