On the use of evolutionary methods in spaces of Euclidean signature

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The main message:

some of the arguments and techniques developed originally and applied so far exclusively only in the Lorentzian case do also apply to Riemannian spaces

Based on some recent works:

- I. Rácz: *Is the Bianchi identity always hyperbolic?*, CQG **31** 155004 (2014)
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All the involved results are valid for arbitrary dimension: i.e. for $dim(M) = n\ (\geq 4)$. Nevertheless, for the sake of simplicity attention will be restricted to the case of $n = 4$.

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Outline:

- **Einsteinian spaces:** (M, g_{ab})
  - First part
  - Second part

  - in both cases metrics of Euclidean signature will be involved
  - no gauge condition

    ... arbitrary choice of foliations & "evolutionary" vector field

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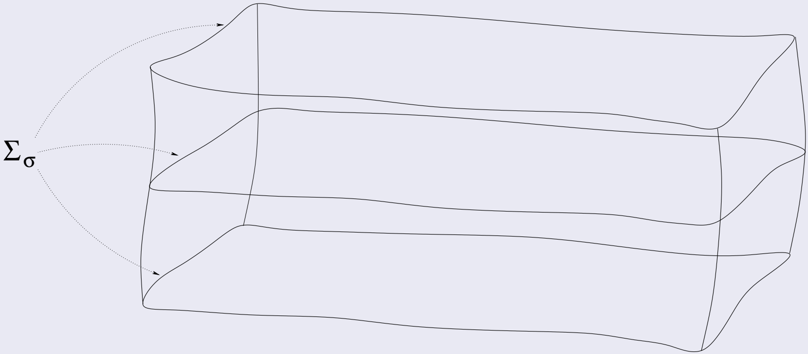

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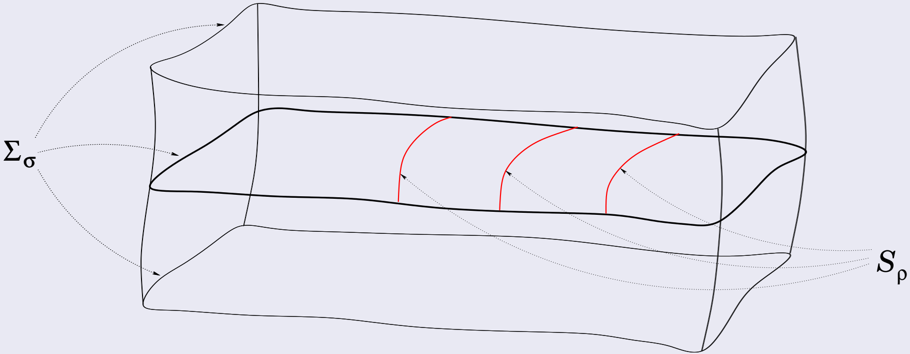

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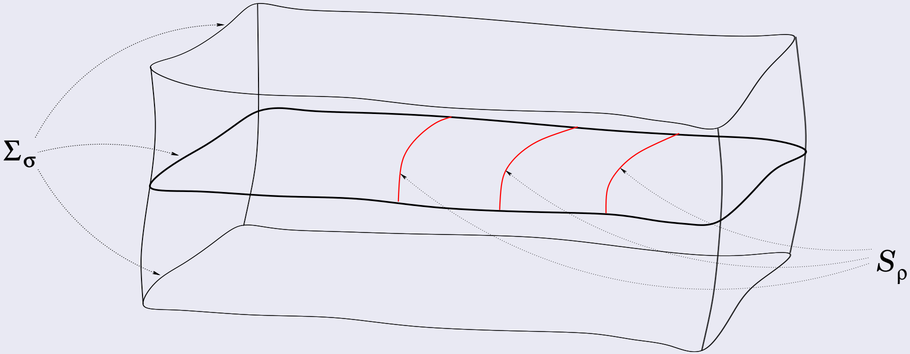

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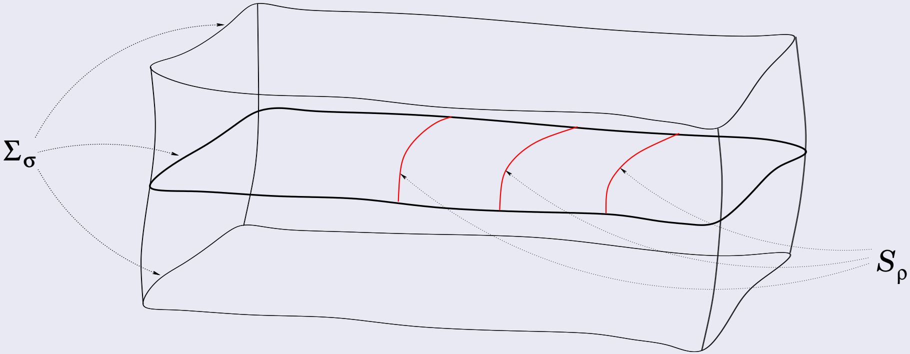

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- **Einsteinian spaces:** (M, g_{ab})
  - M : 4-dimensional, smooth, paracompact, connected, orientable manifold
  - g_{ab}: smooth Lorentzian$_{(-,+,+,+)}$ or Riemannian$_{(+,+,+,+)}$ metric

- **Einstein's equations:**

$$\boxed{G_{ab} - \mathscr{G}_{ab} = 0} \quad \text{with source term:} \quad \boxed{\nabla^a \mathscr{G}_{ab} = 0}$$

  - $\boxed{\nabla_a}$ denotes the covariant derivative operator associated with g_{ab}.
  - in a more familiar setup: **Einstein's equations** with cosmological constant Λ

$$\boxed{[R_{ab} - \tfrac{1}{2}\, g_{ab}\, R] + \Lambda\, g_{ab} = 8\pi\, T_{ab}}$$

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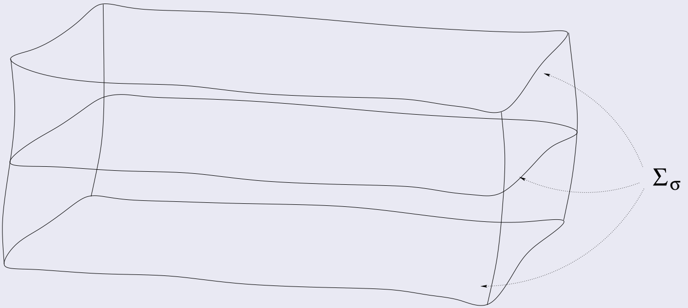
The primary splitting

- **Assume:** M is smoothly foliated by a one-parameter family of homologous hypersurfaces, i.e. $M \simeq \mathbb{R} \times \Sigma$, for some three-dimensional manifold Σ.
  - known to **hold for globally hyperbolic spacetimes** (Lorentzian case)
  - **equivalent to** the existence of a smooth function $\sigma : M \to \mathbb{R}$ with non-vanishing gradient $\partial_a \sigma$ such that the $\sigma = const$ level surfaces $\Sigma_\sigma = \{\sigma\} \times \Sigma$ comprise the one-parameter foliation of M.
  - $$n_a \sim \partial_a \sigma \quad \ldots \; \& \ldots \; g^{ab} \quad \longrightarrow \quad n^a = g^{ab} n_b$$



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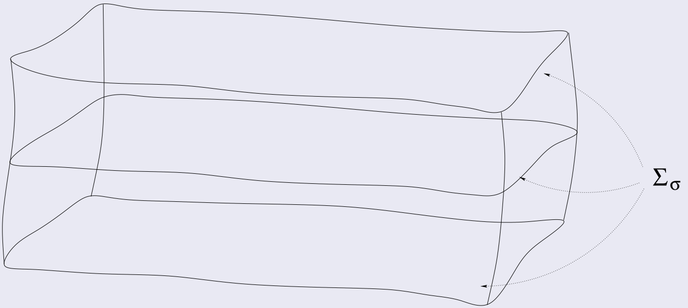

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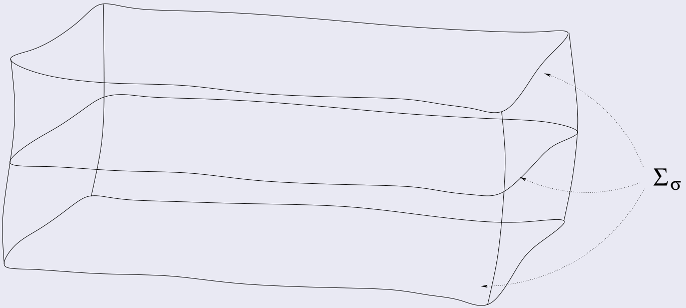

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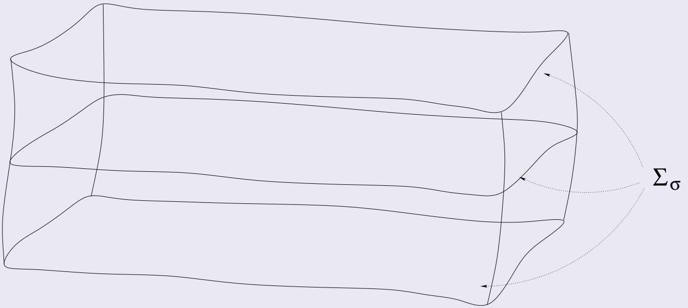

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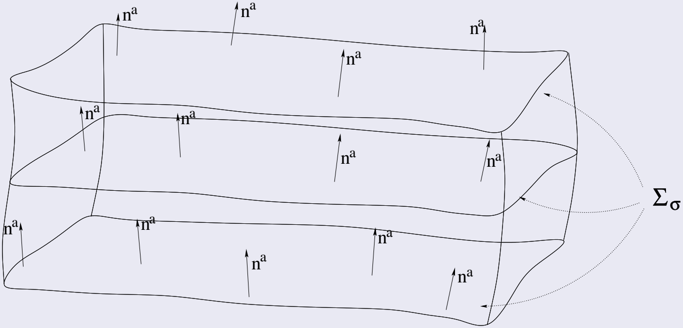

Projections:

The projection operator:

- n^a the 'unit norm' vector field that is normal to the Σ_σ level surfaces

$$n^a n_a = \epsilon$$

  - the sign is not fixed: ϵ takes the value -1 or $+1$ for Lorentzian or Riemannian metric g_{ab}, respectively
- **the projection operator**

$$h^a{}_b = \delta^a{}_b - \epsilon\, n^a n_b$$

to the level surfaces of $\sigma : M \to \mathbb{R}$.

- **the induced metric** on the $\sigma = const$ level surfaces

$$h_{ab} = h^e{}_a h^f{}_b \, g_{ef}$$

  - D_a denotes the covariant derivative operator associated with h_{ab}

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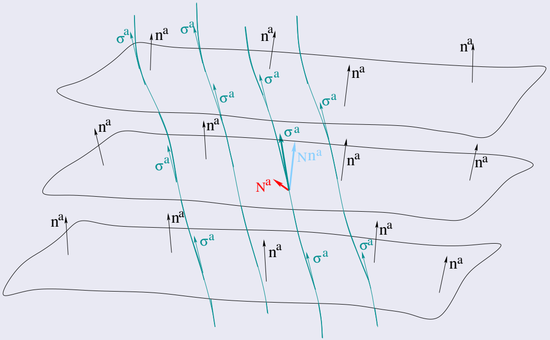
σ^a is "time evolution vector field" **if**:

- the integral curves of σ^a meet the $\sigma = const$ level surfaces precisely once
- $\boxed{\sigma^e \nabla_e \sigma = 1}$



- where N and N^a denotes the **lapse** and **shift** of σ^a:

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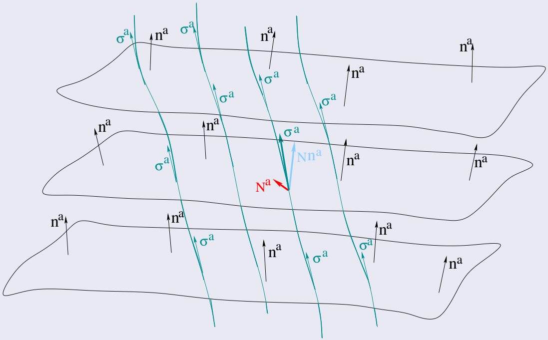

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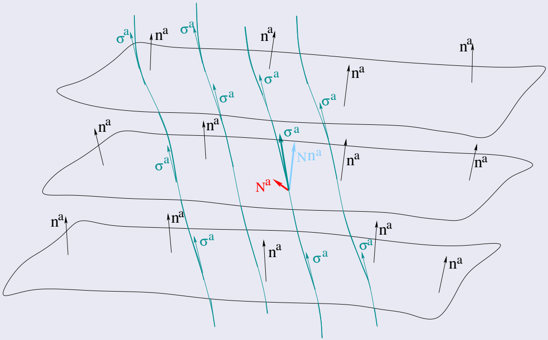

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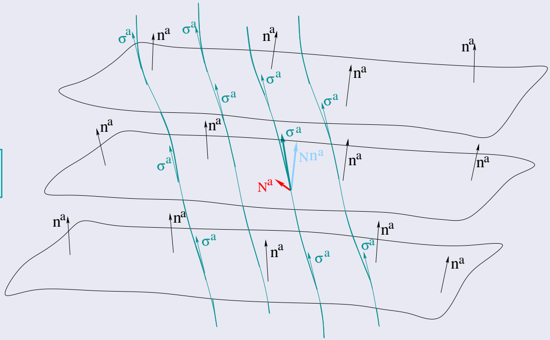

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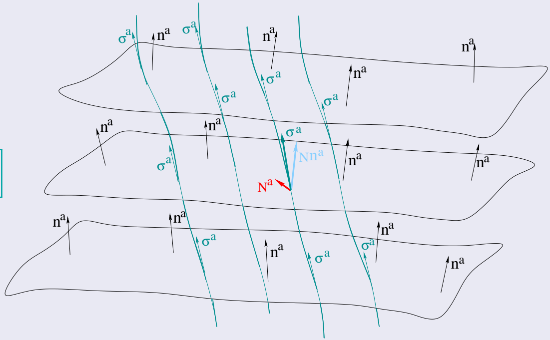

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Decompositions of various fields:

Any symmetric tensor field P_{ab} can be decomposed

in terms of n^a and fields intrinsic to the individual $\sigma = const$ level surfaces as

$$P_{ab} = \boldsymbol{\pi}\, n_a n_b + [n_a\, \mathbf{p}_b + n_b\, \mathbf{p}_a] + \mathbf{P}_{ab}$$

It is also rewarding to inspect the decomposition of the cov. divergence $\nabla^a P_{ab}$:

$$\epsilon\, (\nabla^a P_{ae})\, n^e = \mathscr{L}_n \boldsymbol{\pi} + D^e \mathbf{p}_e + [\boldsymbol{\pi}\,(K^e{}_e) - \epsilon\, \mathbf{P}_{ef} K^{ef} - 2\,\epsilon\, \dot{n}^e \mathbf{p}_e]$$
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where $$\boldsymbol{\pi} = n^e n^f\, P_{ef}\,, \quad \mathbf{p}_a = \epsilon\, h^e{}_a n^f\, P_{ef}\,, \quad \mathbf{P}_{ab} = h^e{}_a h^f{}_b\, P_{ef}$$

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It is also rewarding to inspect the decomposition of the cov. divergence $\nabla^a P_{ab}$:

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back

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back

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Examples:

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$$g_{ab} = \epsilon\, n_a n_b + h_{ab}$$

- the "source term"

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The main result of the first part:

Theorem

Let (M, g_{ab}) *be an Einsteinian space as specified and assume that the metric* h_{ab} *induced on the* $\sigma = const$ *level surfaces is Riemannian. Then,* **regardless whether** g_{ab} **is of Lorentzian or Euclidean signature**, *any solution to the reduced equations* $E^{(\mathcal{EVOL})}_{ab} = 0$ *is also a solution to the full set of field equations* $G_{ab} - \mathscr{G}_{ab} = 0$ *provided that the constraint expressions* $E^{(\mathcal{H})}$ *and* $E^{(\mathcal{M})}_a$ *vanish on one of the* $\sigma = const$ *level surfaces.*

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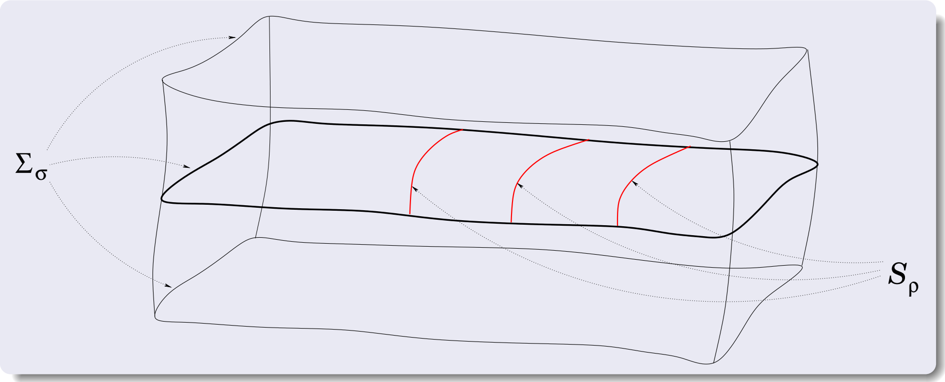
PART II:



The explicit form of the constraints:

The constraint expressions are projections of $E_{ab} = G_{ab} - \mathscr{G}_{ab}$:

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Consider the underdetermined equation on $\Sigma \approx \mathbb{R}^2$ with some coordinates (χ, ξ)

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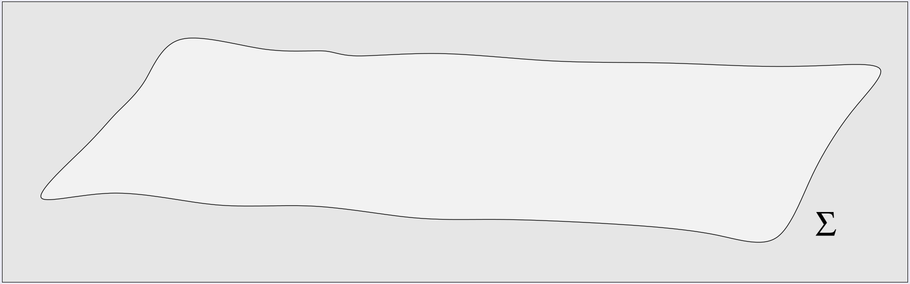

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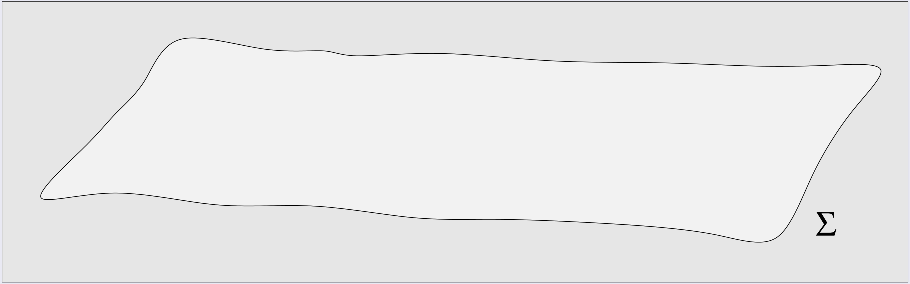

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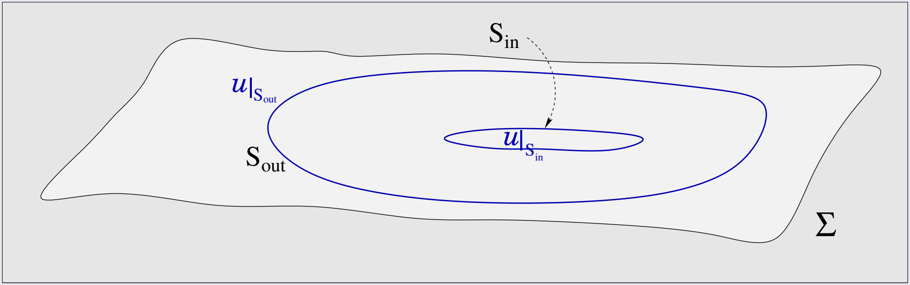

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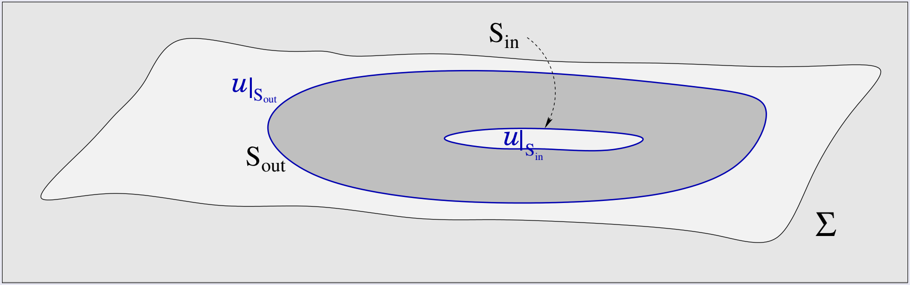
- in solving this equation the variables $\boldsymbol{v}, \boldsymbol{w}$ and $\boldsymbol{z}$ have to be specified on $\mathbb{R}^2$
- the variable $\boldsymbol{u}$ has also to be fixed at the boundaries $\mathrm{S}_{\mathrm{out}}$ and S_{in}



A simple example:

It is a hyperbolic equation for v on $\Sigma \approx \mathbb{R}^2$:

$$(\partial_\chi^2 + \partial_\xi^2)\boldsymbol{u} + (\partial_\chi - \partial_\xi)\boldsymbol{v} + (a\,\partial_\chi - \partial_\xi^2)\boldsymbol{w} + \boldsymbol{z} = 0$$

- in solving this equation the variables u, w and z have to be specified on $\mathbb{R}^2$
- the variable v has also to be fixed at the initial data surface $\mathrm{S}_{\mathrm{in.data}}$

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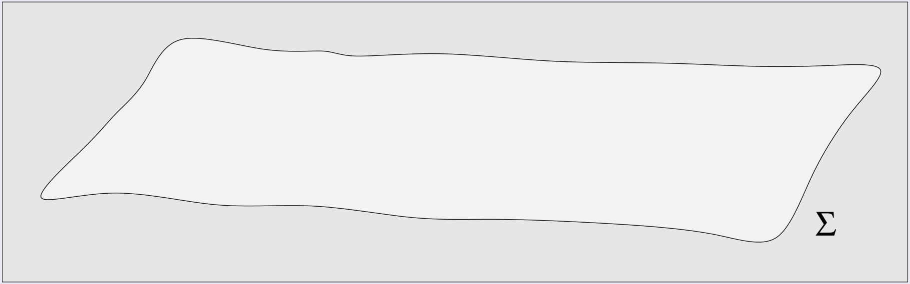

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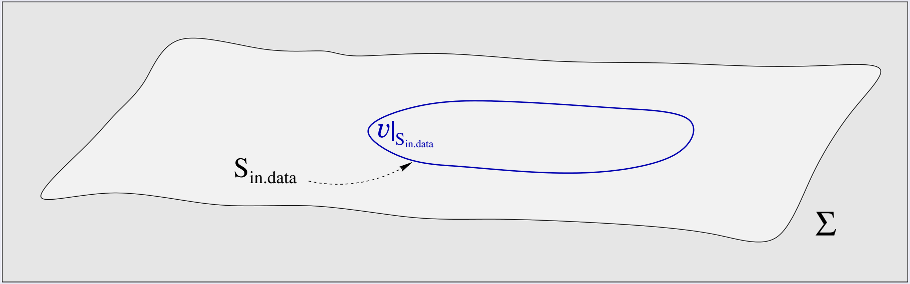

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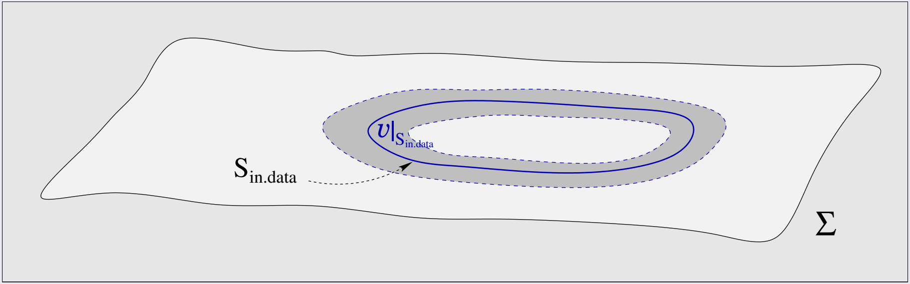

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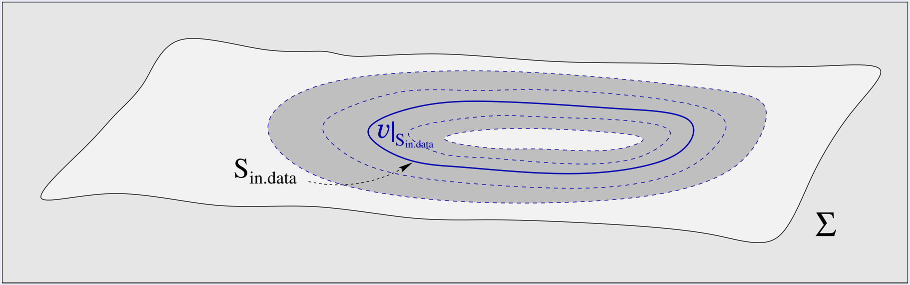

A simple example:

It is a parabolic equation for w on $\Sigma \approx \mathbb{R}^2$:

$$(\partial_\chi^2 + \partial_\xi^2)\boldsymbol{u} + (\partial_\chi - \partial_\xi)\boldsymbol{v} + (a\,\partial_\chi - \partial_\xi^2)w + \boldsymbol{z} = 0$$

- in solving this equation the variables u, v and z have to be fixed on $\mathbb{R}^2$: $a > 0$
- the variable w has also to be fixed at the initial data surface $\mathrm{S}_{\mathrm{in.data}}$

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It is a parabolic equation for w on $\Sigma \approx \mathbb{R}^2$:

$$(\partial_\chi^2 + \partial_\xi^2)\boldsymbol{u} + (\partial_\chi - \partial_\xi)\boldsymbol{v} + (a\,\partial_\chi - \partial_\xi^2)w + \boldsymbol{z} = 0$$

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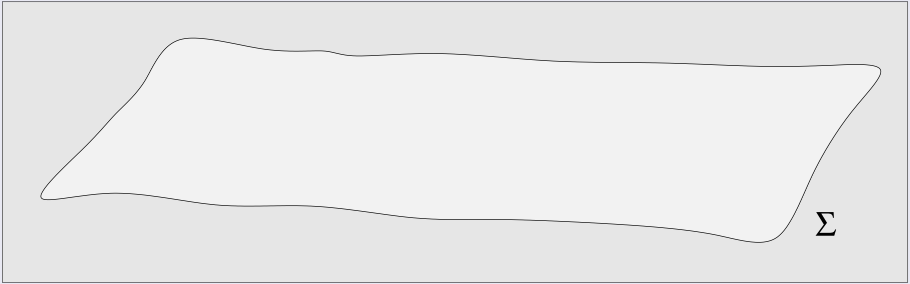

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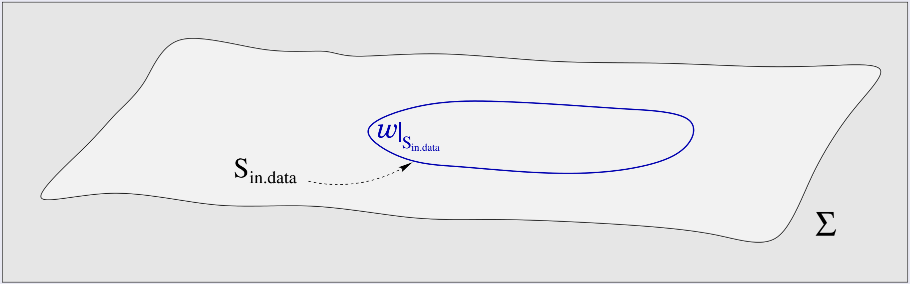

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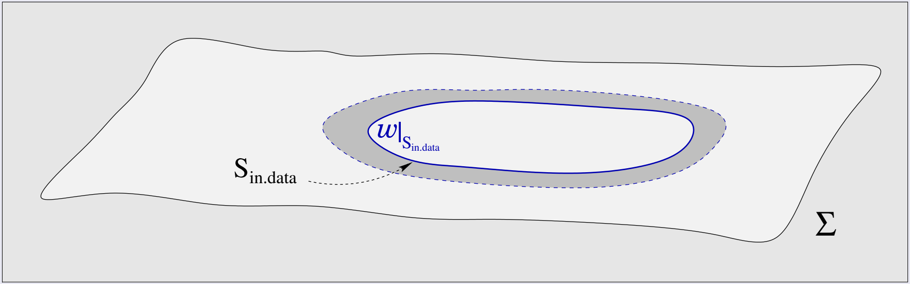

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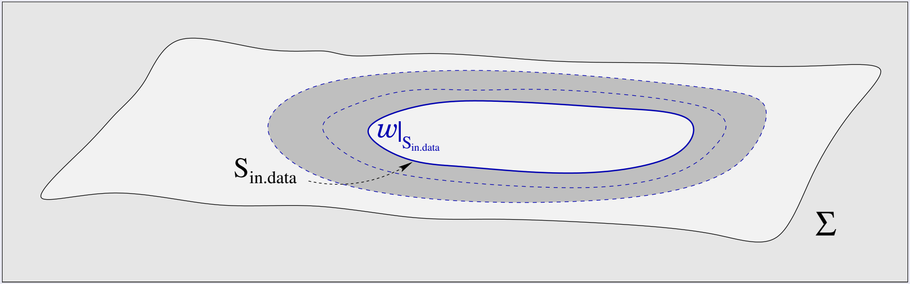

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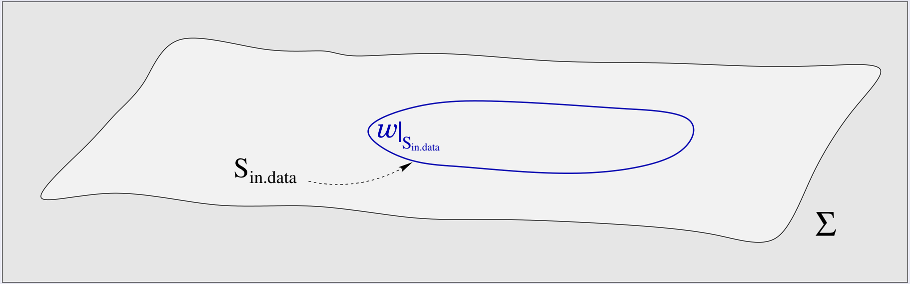

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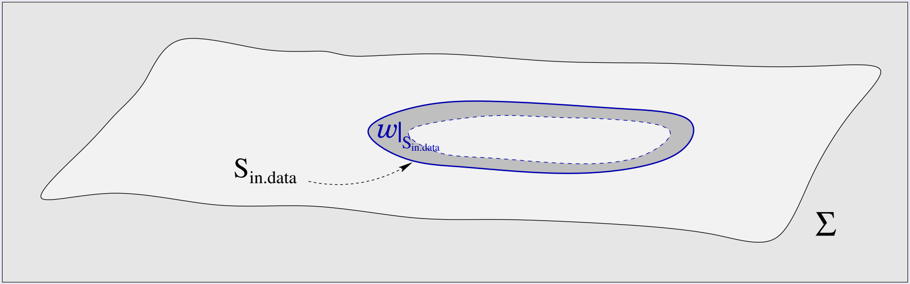

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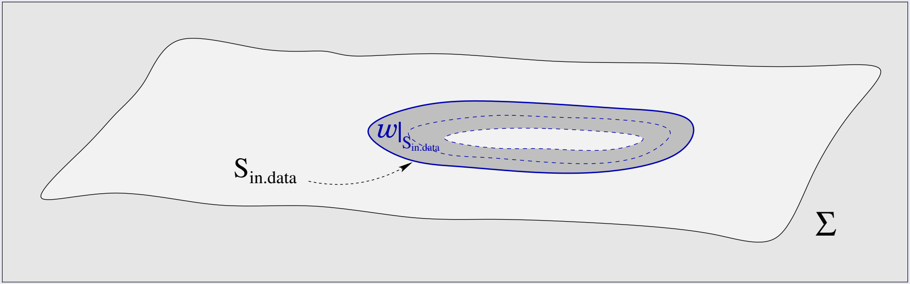

A simple example:

It is an algebraic equation for z :

$$(\partial_\chi^2 + \partial_\xi^2)u + (\partial_\chi^2 - \partial_\xi^2)v + (a\,\partial_\chi - \partial_\xi^2)w + z = 0$$

- once the variables u, v, w are specified on $\mathbb{R}^2$ the solution is determined as

$$z = -\left[(\partial_\chi^2 + \partial_\xi^2)u + (\partial_\chi^2 - \partial_\xi^2)v + (a\,\partial_\chi - \partial_\xi^2)w\right]$$

A simple example:

It is an algebraic equation for $\boldsymbol{z}$:

$$(\partial_\chi^2 + \partial_\xi^2)\boldsymbol{u} + (\partial_\chi^2 - \partial_\xi^2)\boldsymbol{v} + (a\,\partial_\chi - \partial_\xi^2)w + \boldsymbol{z} = 0$$

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New variables by applying $2+1$ decompositions:

Splitting of the metric h_{ij}:

- choose ρ^i to be a flow field on Σ: the integral curves... & $\rho^i\partial_i\rho = 1$
- 'lapse' and 'shift' of ρ^i

$$\rho^i = \widehat{N}\,\widehat{n}^i + \widehat{N}^i\,, \quad \text{where} \quad \widehat{N} = \rho^j\widehat{n}_j \quad \text{and} \quad \widehat{N}^i = \widehat{\gamma}^i{}_j\,\rho^j$$

- induced metric, extrinsic curvature and acceleration of the $\mathscr{S}_\rho$ level surfaces:

$$\widehat{\gamma}_{ij} = \widehat{\gamma}^k{}_i\,\widehat{\gamma}^l{}_j\,h_{kl} \qquad \widehat{K}_{ij} = \tfrac{1}{2}\,\mathscr{L}_{\widehat{n}}\widehat{\gamma}_{ij} \qquad \dot{\widehat{n}}_i := \widehat{n}^l D_l\widehat{n}_i = -\widehat{D}_i \ln \widehat{N}$$

- the metric h_{ij} can then be given as

$$h_{ij} = \widehat{\gamma}_{ij} + \widehat{n}_i\widehat{n}_j \qquad \Longleftrightarrow \qquad \{\widehat{N}, \widehat{N}^i, \widehat{\gamma}_{ij}\}$$

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$2+1$ decompositions:

Splitting of the symmetric tensor field K_{ij}:

- $$K_{ij} = \boldsymbol{\kappa}\,\widehat{n}_i\widehat{n}_j + [\widehat{n}_i\,\mathbf{k}_j + \widehat{n}_j\,\mathbf{k}_i] + \mathbf{K}_{ij}$$

  where

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- the **trace** and **trace free** parts of $\mathbf{K}_{ij}$

  $$\mathbf{K}^l{}_l = \widehat{\gamma}^{kl}\,\mathbf{K}_{kl} \quad \text{and} \quad \mathring{\mathbf{K}}_{ij} = \mathbf{K}_{ij} - \tfrac{1}{2}\,\widehat{\gamma}_{ij}\mathbf{K}^l{}_l$$

The new variables:

- $$(h_{ij}, K_{ij}) \quad \Longleftrightarrow \quad (\widehat{N}, \widehat{N}^i, \widehat{\gamma}_{ij}; \boldsymbol{\kappa}, \mathbf{k}_i, \mathbf{K}^l{}_l, \mathring{\mathbf{K}}_{ij})$$

- these variables retain the physically distinguished nature of h_{ij} and K_{ij}

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- $$(h_{ij}, K_{ij}) \quad \Longleftrightarrow \quad (\widehat{N}, \widehat{N}^i, \widehat{\gamma}_{ij}; \boldsymbol{\kappa}, \mathbf{k}_i, \mathbf{K}^l{}_l, \mathring{\mathbf{K}}_{ij})$$

- these variables retain the physically distinguished nature of h_{ij} and K_{ij}

$2+1$ decompositions:

Splitting of the symmetric tensor field K_{ij}:

- $$K_{ij} = \boldsymbol{\kappa}\, \widehat{n}_i \widehat{n}_j + [\widehat{n}_i\, \mathbf{k}_j + \widehat{n}_j\, \mathbf{k}_i] + \mathbf{K}_{ij}$$

  where

  $$\boldsymbol{\kappa} = \widehat{n}^k \widehat{n}^l\, K_{kl}\,, \quad \mathbf{k}_i = \widehat{\gamma}^k{}_i \widehat{n}^l\, K_{kl} \quad \text{and} \quad \mathbf{K}_{ij} = \widehat{\gamma}^k{}_i \widehat{\gamma}^l{}_j\, K_{kl}$$

- the **trace** and **trace free** parts of $\mathbf{K}_{ij}$

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The momentum constraint:

$$D_e K^e{}_a - D_a K^e{}_e - \epsilon\, \mathfrak{p}_a = 0$$

First order symmetric hyperbolic system:

- contract "(1)" with $2\,\widehat{N}\,\widehat{\gamma}^{ij}$ and mult. "(2)" by $\widehat{N}$, when writing them out in coordinates (ρ, x^2, x^3), adopted to the foliation $\mathscr{S}_\rho$ and the vector field ρ^i,

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!!! ρ **plays the role of 'time'**

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!!! ρ **plays the role of 'time'** regardless of the value of $\epsilon = \pm 1$

The Hamiltonian constraint:

The Hamiltonian constraint in terms of the new variables:

- $$E^{(\mathcal{H})} = n^e n^f E_{ef} = \tfrac{1}{2}\,\{-\epsilon\,{}^{(3)}\!R + (K^e{}_e)^2 - K_{ef}K^{ef} - 2\,\mathfrak{e}\} = 0$$

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Alternative choices yielding evolutionary systems:

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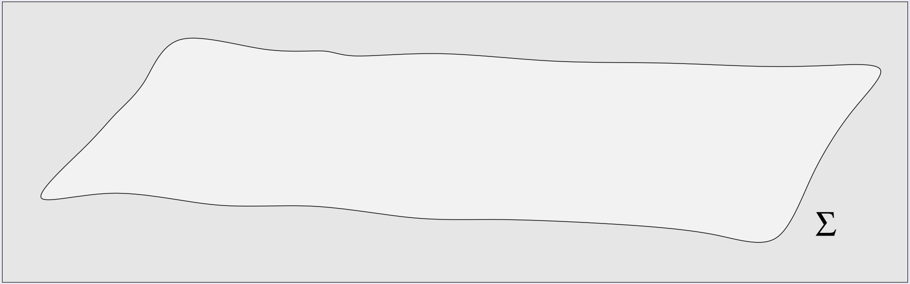

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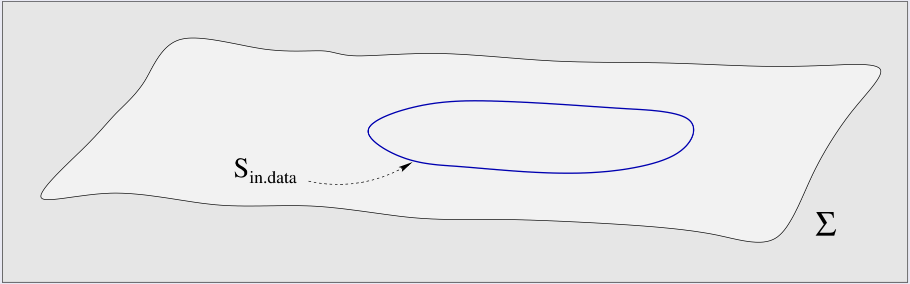

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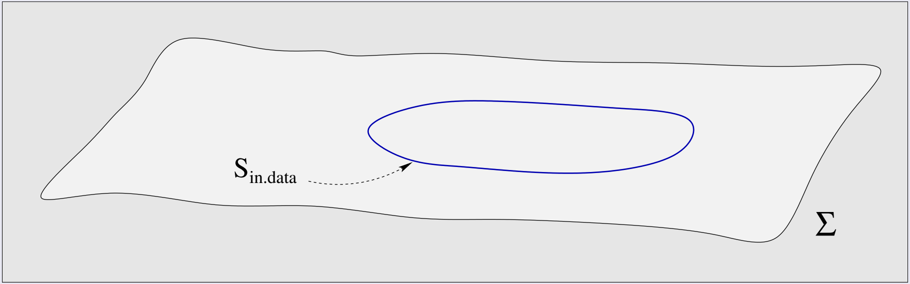

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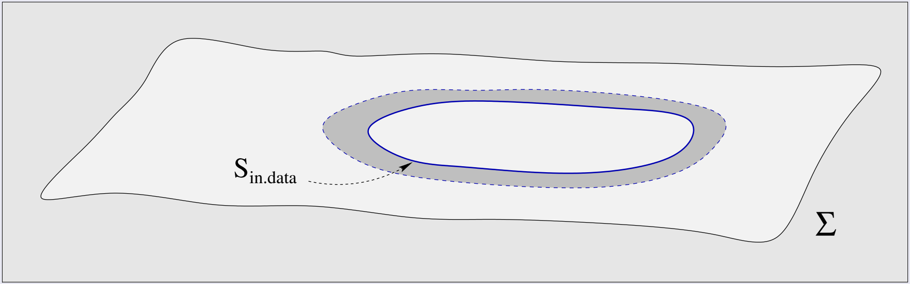

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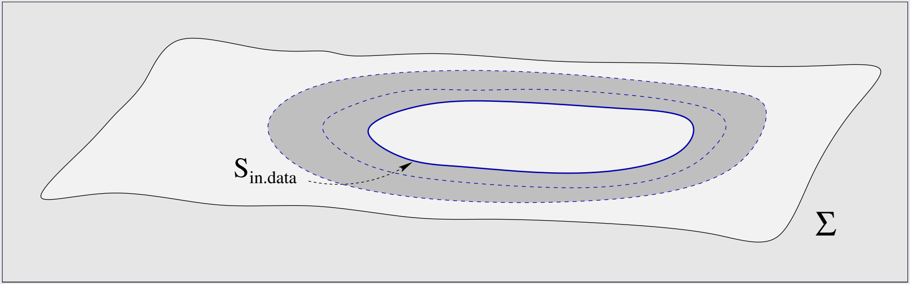

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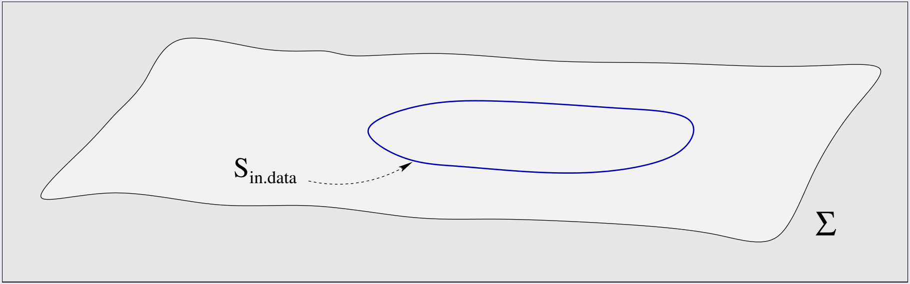

Constraints as evolutionary systems I.

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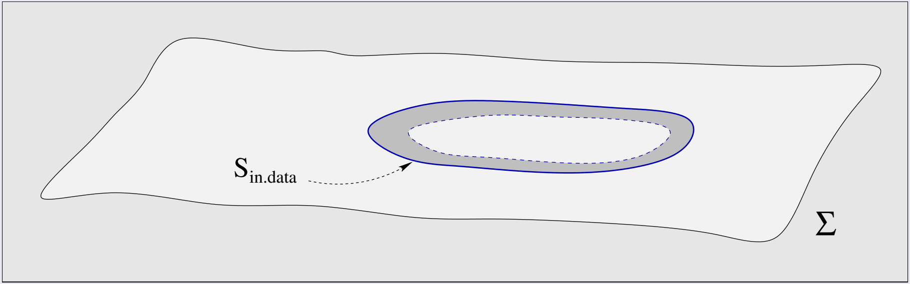

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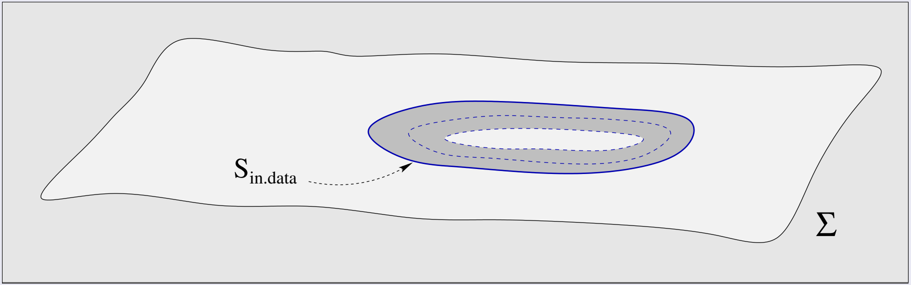

The strongly hyperbolic system:

The Hamiltonian constraint as an algebraic equation for $\boldsymbol\kappa$:

$$-\epsilon\,\widehat{R} + \epsilon\left\{2\,\mathscr{L}_{\widehat{n}}(\widehat{K}^l{}_l)\;+(\widehat{K}^l{}_l)^2 + \widehat{K}_{kl}\,\widehat{K}^{kl} + 2\,\widehat{N}^{-1}\,\widehat{D}^l\widehat{D}_l\widehat{N}\right\} + 2\,\boxed{\boldsymbol\kappa}\,\mathbf{K}^l{}_l + \tfrac{1}{2}\,(\mathbf{K}^l{}_l)^2 - 2\,\mathbf{k}^l\mathbf{k}_l - \mathring{\mathbf{K}}_{kl}\,\mathring{\mathbf{K}}^{kl} - 2\,\mathfrak{e} = 0$$

whence $\boldsymbol\kappa = (2\,\mathbf{K}^l{}_l)^{-1}[\,2\,\mathbf{k}^l\mathbf{k}_l - \tfrac{1}{2}\,(\mathbf{K}^l{}_l)^2 - \boldsymbol\kappa_0\,]\,,\quad \boldsymbol\kappa_0 = -\epsilon\,{}^{(3)}\!R - \mathring{\mathbf{K}}_{kl}\,\mathring{\mathbf{K}}^{kl} - 2\,\mathfrak{e}$

- by eliminating $\widehat{D}_i\boldsymbol\kappa$ from the momentum constraint mom. constr. one gets

$$\mathscr{L}_{\widehat{n}}\mathbf{k}_i + (\mathbf{K}^l{}_l)^{-1}[\,\boldsymbol\kappa\,\widehat{D}_i(\mathbf{K}^l{}_l) - 2\,\mathbf{k}^l\,\widehat{D}_i\mathbf{k}_l\,] + (2\,\mathbf{K}^l{}_l)^{-1}\,\widehat{D}_i\boldsymbol\kappa_0 + (\widehat{K}^l{}_l)\,\mathbf{k}_i + [\,\boldsymbol\kappa - \tfrac{1}{2}\,(\mathbf{K}^l{}_l)\,]\,\dot{\widehat{n}}_i - \dot{\widehat{n}}^l\,\mathring{\mathbf{K}}_{li} + \widehat{D}^l\mathring{\mathbf{K}}_{li} - \epsilon\,\mathfrak{p}_l\,\widehat{\gamma}^l{}_i = 0\,,$$
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Constraints as evolutionary systems II:

The strongly hyperbolic system:

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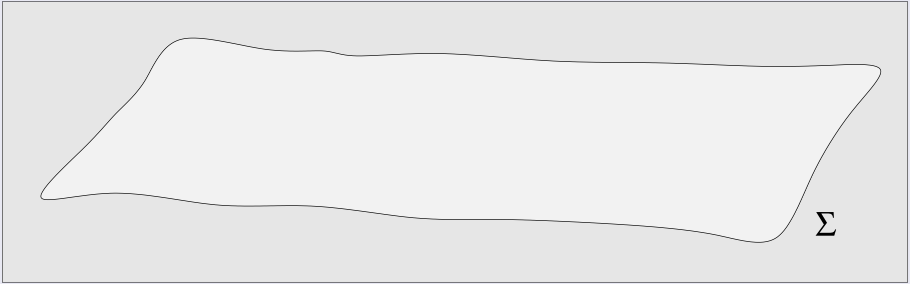

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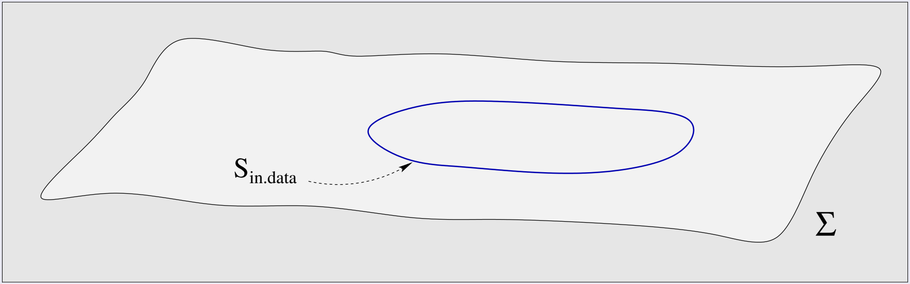

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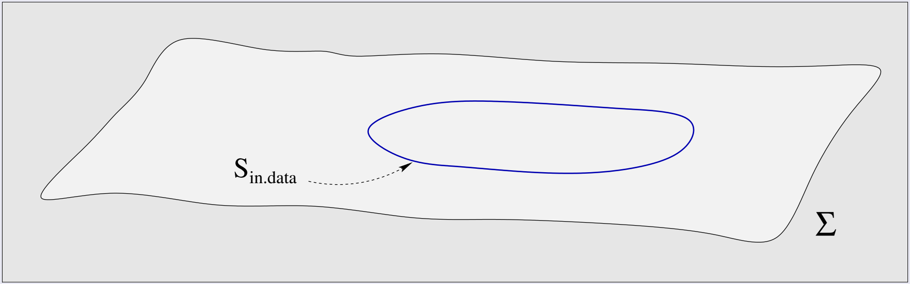

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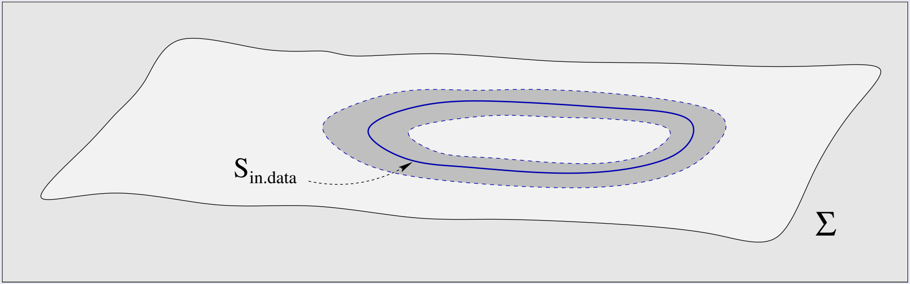

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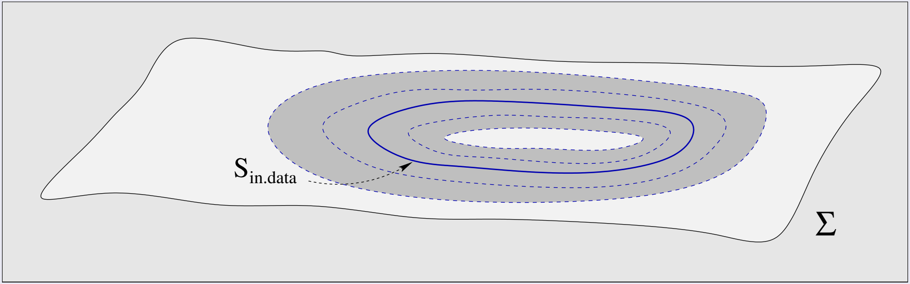

Summary:

4-dimensional **Riemannian and Lorentzian spaces** satisfying Einstein's equations, and some mild topological assumptions, were considered. **!!!** $[n(\geq 4)]$

1. it was shown that **the constraint expressions** satisfy a **FOSH system** that is linear and homogeneous $\Longrightarrow$ (the constraints propagate)
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On contrary to the folklore, in the considered two explicit examples, **evolutionary methods can be applied in spaces with metric of Euclidean signature where, in principle, there is no room for 'time'**

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4-dimensional **Riemannian and Lorentzian spaces** satisfying Einstein's equations, and some mild topological assumptions, were considered. **!!!** $[n(\geq 4)]$

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The first order symmetric hyperbolic system for $(E^{(\mathcal{H})}, E_i^{(\mathcal{M})})^T$

- Characteristic directions associated with the FOSH system governing the evolution of the constraint expressions are determined as

$$[\,h^{ij} - n^i n^j\,]\,\xi_i\xi_j = [\,g^{ij} - (1+\epsilon)\,n^i n^j\,]\,\xi_i\xi_j = 0$$

The momentum constraint: first order symmetric hyperbolic system

- with characteristic cone given as

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- ... given a Riemannian metric $\mathfrak{g}_{ij}$, a unit form field $\mathfrak{n}_i$ and a positive real function α

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- ... given a Riemannian metric $\mathfrak{g}_{ij}$, a unit form field $\mathfrak{n}_i$ and a positive real function α $\Longrightarrow$ a metric of Lorentzian signature can be defined as

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The conformal (elliptic) method:

Lichnerowicz A (1944) and York J W (1972):

- replace

$$h_{ij} = \phi^4\,\widetilde{h}_{ij} \quad \text{and} \quad K_{ij} - \tfrac{1}{3}\,h_{ij}\,K^l{}_l = \phi^{-2}\,\widetilde{K}_{ij}$$

- using these variables the constraints are put into a **semilinear elliptic system**

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